Calculus p. 170 Derivatives of Inverse Trig. Functions

1) $\frac{-1}{\sqrt{1-\left(x^2\right)^2}}\left(2x\right)=\frac{-2x}{\sqrt{1-x^4}}$

2) $\frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(\frac{-1}{x^2}\right)=\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}=\frac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}}=\frac{1}{x^2\frac{\sqrt{x^2-1}}{|x|}}=\frac{1}{|x|\sqrt{x^2-1}}$

3) $\frac{1}{\sqrt{1-\left(\sqrt{2}t\right)^2}}\left(\sqrt{2}\right)=\frac{\sqrt{2}}{\sqrt{1-2t^2}}$

4) $\frac{1}{\sqrt{1-\left(1-t\right)^2}}\left(-1\right)=\frac{-1}{\sqrt{1-\left(1-2t+t^2\right)}}=\frac{-1}{\sqrt{2t-t^2}}$

5) $\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\left(\frac{-6}{t^3}\right)=\frac{-6}{t^3\sqrt{1-\frac{9}{t^4}}}=\frac{-6}{t^3\sqrt{\frac{t^4-9}{t^4}}}=\frac{-6}{t^3\frac{\sqrt{t^4-9}}{t^2}}=\frac{-6}{t\sqrt{t^4-9}}$

6) $s\left(\frac{1}{2\sqrt{1-s^2}}\right)\left(-2s\right)+(1)\sqrt{1-s^2}+\frac{-1}{\sqrt{1-s^2}}\left(1\right)=\frac{-s^2}{\sqrt{1-s^2}}+\frac{1-s^2}{\sqrt{1-s^2}}+\frac{-1}{\sqrt{1-s^2}}=\frac{-2s^2}{\sqrt{1-s^2}}$

7) $\left(1\right)\sin^{-1}x+x\left(\frac{1}{\sqrt{1-x^2}}\right)=\frac{1}{2\sqrt{1-x^2}}\left(-2x\right)=\sin^{-1}x+\frac{x-x}{\sqrt{1-x^2}}=\sin^{-1}x$

8) $y=\left(\sin^{-1}(2x)\right)^{-1}$

$y'=-1\left(\sin^{-1}(2x)\right)^{-2}\left(\frac{1}{\sqrt{1-(2x)^2}}\left(2\right)\right)=\frac{-2}{\left(\sin^{-1}(2x)\right)^2\sqrt{1-4x^2}}$

9) $x'(t)=v(t)=\frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}}\left(\frac{1}{4}\right)=\frac{1}{4\sqrt{1-\frac{t^2}{16}}}$

$v\left(3\right)=\frac{1}{4\sqrt{1-\frac{9}{16}}}=\frac{1}{4\sqrt{\frac{7}{16}}}=\frac{1}{4\frac{\sqrt{7}}{4}}=\frac{1}{\sqrt{7}}=\frac{\sqrt{7}}{7}$

10) $x(t) = \sin^{-1}\left(\frac{\sqrt{t}}{4}\right), \quad t = 4$

$$x'(t) = v(t) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{t}}{4}\right)^2}}\left(\frac{1}{8\sqrt{t}}\right)$$

$$v(4) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{4}}{4}\right)^2}}\left(\frac{1}{8\sqrt{4}}\right) = \frac{1}{\sqrt{1-\frac{1}{4}}}\left(\frac{1}{16}\right) = \frac{1}{\frac{\sqrt{3}}{2}}\left(\frac{1}{16}\right) = \frac{1}{8\sqrt{3}} = \frac{\sqrt{3}}{24}$$

11) $x(t) = \tan^{-1} t, \quad t = 2$

$$x'(t) = v(t) = \frac{1}{1+t^2}$$

$$v(2) = \frac{1}{1+2^2} = \frac{1}{5}$$

12) $x(t) = \tan^{-1}\left(t^2\right), \quad t = 1$

$$x'(t) = v(t) = \frac{1}{1+t^2}$$

$$v(2) = \frac{1}{1+2^2} = \frac{1}{5}$$

13) $y = \sec^{-1}\left(2s+1\right)$

$$\frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\left(2\right) = \frac{2}{|2s+1|\sqrt{4s^2+4s}} = \frac{2}{|2s+1|\sqrt{4\left(s^2+s\right)}} = \frac{2}{|2s+1|\left(2\right)\sqrt{s^2+s}} = \frac{1}{|2s+1|\sqrt{s^2+s}}$$

14) $y = \sec^{-1} 5s$

$$\frac{dy}{ds} = \frac{1}{|5s|\sqrt{(5s)^2-1}}\left(5\right) = \frac{5}{5|s|\sqrt{25s^2-1}} = \frac{1}{|s|\sqrt{25s^2-1}}$$

15) $y = \csc^{-1}\left(x^2+1\right), x > 0$

$$\frac{dy}{dx} = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\left(2x\right) = \frac{-2x}{|x^2+1|\sqrt{x^4+2x^2}} = \frac{-2x}{|x^2+1|\sqrt{x^2\left(x^2+2\right)}} = \frac{-2x}{|x^2+1|x\sqrt{x^2+2}} = \frac{-2}{|x^2+1|\sqrt{x^2+2}}$$

16) $y = \csc^{-1}\left(x/2\right)$

$$\frac{dy}{dx} = \frac{-1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}}\left(\frac{1}{2}\right) = \frac{-1}{\frac{1}{2}|x|\sqrt{\frac{x^2}{4}-1}}\left(\frac{1}{2}\right) = \frac{-1}{|x|\sqrt{\frac{x^2-4}{4}}} = \frac{-1}{|x|\frac{\sqrt{x^2-4}}{\sqrt{4}}} = \frac{-2}{|x|\sqrt{x^2-4}}$$

17) $y=\sec^{-1}\left(\frac{1}{t}\right),\ 0<t<1$

$$\frac{dy}{dt}=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\left(-\frac{1}{t^2}\right)=\frac{-1}{t\sqrt{\frac{1}{t^2}-1}}=\frac{-1}{t\sqrt{\frac{1-t^2}{t^2}}}=\frac{-1}{t\frac{\sqrt{1-t^2}}{\sqrt{t^2}}}=\frac{-1}{\sqrt{1-t^2}}$$

18) $y=\cot^{-1}\sqrt{t}$

$$\frac{dy}{dt}=\frac{-1}{1+\left(\sqrt{t}\right)^2}\left(\frac{1}{2\sqrt{t}}\right)=\frac{-1}{(1+t)\left(2\sqrt{t}\right)}$$

19) $y=\cot^{-1}\sqrt{t-1}$

$$\frac{dy}{dt}=\frac{-1}{1+\left(\sqrt{t-1}\right)^2}\left(\frac{1}{2\sqrt{t-1}}\right)=\frac{-1}{(t)\left(2\sqrt{t-1}\right)}=\frac{-1}{2t\sqrt{t-1}}$$

20) $y=\sqrt{s^2-1}-\sec^{-1}s$

$$\frac{dy}{ds}=\frac{1}{2\sqrt{s^2-1}}(2s)-\frac{1}{|s|\sqrt{s^2-1}}=\frac{s}{\sqrt{s^2-1}}-\frac{1}{|s|\sqrt{s^2-1}}=\frac{s|s|-1}{|s|\sqrt{s^2-1}}$$

21) $y=\tan^{-1}\sqrt{x^2-1}+\csc^{-1}x,\ x>1$

$$\frac{dy}{dx}=\frac{1}{1+\left(\sqrt{x^2-1}\right)^2}\left(\frac{1}{2\sqrt{x^2-1}}\right)(2x)+\frac{-1}{|x|\sqrt{x^2-1}}=\frac{1}{x\sqrt{x^2-1}}+\frac{-1}{|x|\sqrt{x^2-1}}=0$$

22) $y=\cot^{-1}\left(\frac{1}{x}\right)-\tan^{-1}x$

$$\frac{dy}{dx}=\frac{-1}{1+\left(\frac{1}{x}\right)^2}\left(\frac{-1}{x^2}\right)-\frac{1}{1+x^2}=\frac{1}{x^2\left(1+\frac{1}{x^2}\right)}-\frac{1}{1+x^2}=\frac{1}{x^2+1}-\frac{1}{1+x^2}=0$$

23) $y=\sec^{-1}x,\ x=2$

$$\frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}}\qquad @x=2,\ \frac{dy}{dx}=\frac{1}{2\sqrt{4-1}}=\frac{1}{2\sqrt{3}}$$

Equation of tangent: $y-\sec^{-1}(2)=\frac{1}{2\sqrt{3}}(x-2),\ or\ y=0.289x+0.470$

24) $y=\tan^{-1}x,\ x=2$

$$\frac{dy}{dx}=\frac{1}{1+x^2}\qquad @x=2,\ \frac{dy}{dx}=\frac{1}{1+4}=\frac{1}{5}$$

Equation of tangent: $y-\tan^{-1}(2)=\frac{1}{5}(x-2),\ or\ y=0.2x+0.707$

25) $y = \sin^{-1}\left(\frac{x}{4}\right),\ x = 3$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{x^2}{16}}}\left(\frac{1}{4}\right) \quad @x = 3,\ \frac{dy}{dx} = \frac{1}{4\sqrt{1-\frac{9}{16}}} = \frac{1}{4\sqrt{\frac{7}{16}}} = \frac{1}{4\left(\frac{\sqrt{7}}{4}\right)} = \frac{\sqrt{7}}{7}$$

Equation of tangent: $y - \sin^{-1}\left(\frac{3}{4}\right) = \frac{\sqrt{7}}{7}(x-3),\ or\ y = 0.378x - 0.286$

26) $y = \tan^{-1}\left(x^2\right),\ x = 1$

$$\frac{dy}{dx} = \frac{1}{1+x^4}(2x) \quad @x = 1,\ \frac{dy}{dx} = \frac{1}{1+1}(2(1)) = 1$$

Equation of tangent: $y - \tan^{-1}(1) = 1(x-1),\ or\ y = x - 0.215$