Introduction

At A Glance

Students explore different ways to slice a three-dimensional figure and examine the two-dimensional shapes resulting from the slice. They are introduced to the term **cross-section.** Students explore the different shapes created by slices that are parallel, perpendicular, or at an angle to the base of a rectangular pyramid.

Step By Step

- Work through **Think It Through** as a class.
- Introduce the Question at the top of the page.

▶ English Language Learners

- Read the description of a cross-section. You may want to use a manipulative such as a building block to demonstrate (using a slicing motion with your hand) the possible slices—direction and angle—you could make through a rectangular prism.
- Explain that a three-dimensional figure can have many different cross-sections depending on the direction or angle.
- Read the **Think** statement as a class. Remind students what it means for a plane to be perpendicular or parallel to another plane.
- Have students circle the words or phrases that describe the types of slices you can make.

▶ Mathematical Discourse 1 and 2

PS TIP Use Tools
Students use appropriate tools strategically when they use mathematical models and vary assumptions such as how the slice of a three-dimensional model determines the shape of the cross-section. Using a variety of concrete representations to represent abstract concepts such as cross-sections builds a deeper understanding of the concepts. *(PS 5)*

Lesson 25 Introduction

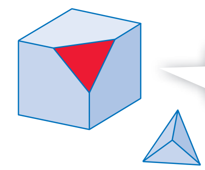
Previews: 8.GM.1

Understand Plane Sections of Prisms and Pyramids

Think It Through

What is a cross-section of a three-dimensional figure?

You can think about a cross-section as a slice through a three-dimensional figure. Picture slicing a cube made up of clay with a string. The two-dimensional shape that you get is called a **cross-section**. When you slice through a cube, one of the shapes that you could get is a triangle.

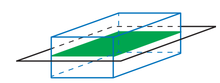


A three-dimensional figure can have many different cross-sections. It all depends on the direction or angle of the slice.

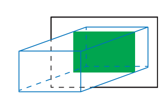
Think Different slices through a rectangular prism create different shapes

One way to slice a rectangular prism is with a plane parallel to a base of the prism.

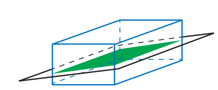
Circle the words or phrases that describe the types of slices you can make.



Another way to slice a rectangular prism is with a plane perpendicular to a base of the prism.



The slice could also look like this. It doesn't have to be parallel or perpendicular to a face of the prism.



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▶ Mathematical Discourse

1 *How are cross-sections of three-dimensional figures different from faces of three-dimensional figures?*

Faces are the surfaces of the three-dimensional figure. There are a defined number of faces. Cross-sections are 'inside' the figure, and there are an infinite number.

2 *How are cross-sections of three-dimensional figures similar to faces of three-dimensional figures?*

Both faces and cross-sections are two-dimensional shapes such as rectangles, squares, triangles, pentagons, etc. Faces and cross-sections are parts of the three-dimensional figure.

▶ English Language Learners

Tell students you are going to define and illustrate the definitions of three similar word concepts for this lesson.

Draw a cube on the board with a shaded triangle cross section. Sketch and label a **plane** above the cube. Write the word **slice** between the plane and the cube with an arrow showing the plane slicing the cube. Write the word **cross-section** on the shaded part of the cube. Follow the diagram from the plane to the shaded region while explaining to students that a **plane slices** the figure to make a **cross-section.**

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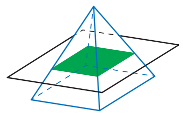
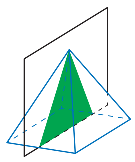
Think **Different slices through a rectangular pyramid create different shapes.**

One way to slice a rectangular pyramid is with a plane parallel to the base of the pyramid.

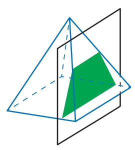


The slice can be parallel to the base, perpendicular to the base, or at a completely different angle.

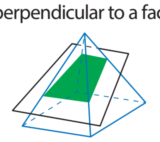
Another way to slice a rectangular pyramid is with a plane perpendicular to the base. If the plane goes through the top vertex, the slice could look like this.



If the plane is perpendicular to the base but doesn't go through the top vertex, the slice could look like this.



The slice doesn't have to be parallel or perpendicular to a face of the pyramid. It could look like this.



Reflect

1 How can you use the formula for the volume of a cylinder to remember the formulas for the volume of a cone and the volume of a sphere?

Possible answer: You can get triangles, squares, trapezoids, parallelograms, and rectangles.

Step By Step

- Read the **Think** statement as a class.
- Explain to students that they will examine the various shapes created when slices of different angles are made with the base of a rectangular pyramid.
- Ask: *Do you recall how we define a rectangular pyramid?* [It is a three-dimensional shape with four triangular faces and a square or rectangular base.]
- Direct students to compare the cross-sections that result from slices that are parallel, perpendicular, or at an angle to the base.
- Point out that a perpendicular slice can go through the vertex or any other part of the top of the pyramid.

▶ Mathematical Discourse 3–5

- Have students read and reply to the **Reflect** directive.

▶ Hands-On Activity

Ready® **Mathematics** PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 305–306** after students have completed this section.

▶ Hands-On Activity

Create 2-D and 3-D models.

Materials: drinking straws, scissors, cardboard

- Have students cut a rectangle and a triangle out of the cardboard. The cutouts should be 2 to 4 inches at the longest dimension.
- Next, have students cut a one-inch slit in the end of the drinking straw.
- Insert the cardboard cutout shapes one at a time into the straw. Have students spin the straw between their fingers and observe the three-dimensional solid of revolution generated by the two-dimensional cutout.
- Have students explain how the cutout represents a cross-section while the rotating cutout represents a three-dimensional figure.

▶ Mathematical Discourse

3 *What are some real-world examples of rectangular prisms and rectangular pyramids?*

Encourage students to think about architecture or containers. Examples of rectangular prisms are buildings, packing boxes, and blocks. Show students The Pyramid Arena in Memphis for a real-world pyramid.

4 *Can you think of an example of a real-world cross-section of a real-world prism or pyramid?*

Floors and ceilings could be thought of as cross-sections in buildings.

5 *What kind of slice would give you the real-world cross-section of the real-world prism or pyramid?*

Most answers will be that the slice is parallel to the base.

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Guided Instruction

At A Glance

Students visualize the two-dimensional cross-sections that are made by slices that are parallel, perpendicular, or at an angle to the base of a three-dimensional figure. Students predict the shape of the cross-section based on whether the slice is parallel, perpendicular, or at an angle to the base.

Step By Step

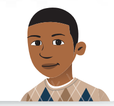
Let's Explore the Idea

- Tell students that they will have time to work individually on the problems on this page and then share their responses in groups. You may choose to work through the first problem together as a class.
- Ask: *Show me with your hand what a parallel or horizontal slice will look like. Show me with your hand what a perpendicular or vertical slice will look like.* (You may need to pay special attention to ELL students to make sure they understand horizontal versus vertical.)
- Ask a student volunteer to describe how to cut the prism to get a triangle for the cross-section.
- Check students' answers to problem 4. If students need more support, continue to stress that visualizing the slice through the prism can help describe the cross-sections.
- Have students do the last two problems. Take note of students who are still having difficulty and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

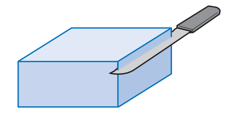
▶ **Mathematical Discourse 1 and 2**

Think About Plane Sections of Three-Dimensional Figures

Let's Explore the Idea Visualizing the cut can help you describe the cross-sections of three-dimensional figures.

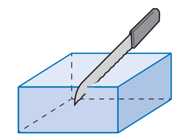


The figures shown are right rectangular prisms.



2 Think about slicing a cake or a block of cheese. If your slice is horizontal, or parallel to the base of the prism, what shape do you think the cross-section will be?

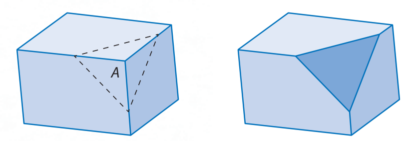
I think it will be a rectangle.



3 If your slice is vertical, or perpendicular to the base of the prism, what shape do you think the cross-section will be?

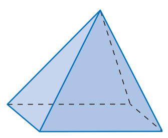
I think it will be a rectangle.

4 Look at a cross-section of a rectangular prism shown below. Describe how you could cut the prism to get a triangle for the cross-section.



You could cut off the corner that's labeled with an A.

Now try these problems.



5 Suppose you cut this rectangular pyramid by slicing it parallel to its base. What shape do you think the cross-section will be?

I think it will be a rectangle.

6 How could you slice the pyramid and get a triangle for the cross-section?

You could cut off any of the bottom corners.

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▶ Mathematical Discourse

1 *Is it easier to visualize certain cross-sections over others? Which ones?*

Most students will find it easiest to visualize the square and rectangle cross-sections that are made by making a slice parallel to the base. Students may say that slices that are perpendicular produce cross-sections that are easy to visualize. Slices at an angle to the base can be harder to visualize.

2 *What do you think it is about these cross-sections that make them easier to visualize?*

When a slice is parallel to the base, it makes the same shape as the base. Once a slice is angled within the three-dimensional figure, it gets challenging to visualize the shape of the resulting cross-section. Encourage students to describe visualizing angled slices.

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Let's Talk About It
Solve the problems below as a group.

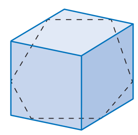
7 Look at problem 2. When a slice is made parallel to the base of a rectangular prism, do you think you will always get a rectangle? If your answer is yes, describe the rectangle. If your answer is no, describe the other types of polygons you can get.

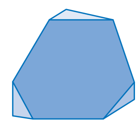
Possible answer: Yes; The rectangle will be the same size and shape as the base.

8 Look at problem 4. Do you think you can get different types and sizes of triangles when you slice a rectangular prism? Explain your thinking.

Possible answer: Yes; the type and size of triangle depends on the location and angle of the slice.

9 Look at this cross section of a cube. Describe how you can slice a cube and get a hexagon for a cross-section.



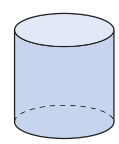


Slice so that the cut goes through each face of the cube.

Try It Another Way
Work with your group to explore cross-sections of cylinders and cones.

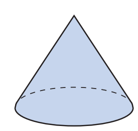
10 Describe the shapes you would get if you slice a cylinder with a plane parallel to the base or with a plane perpendicular to the base.

When you slice a cylinder parallel to the base, you get a circle, the same size and shape as the base. When you slice a cylinder perpendicular to the base, you get a rectangle.



11 What can you say about the shapes you could get if you slice a cone with a plane parallel to the base?

You get circles of different sizes, depending where you slice.



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▶ Mathematical Discourse

3 *Can you think of a job where you might slice into a three-dimensional figure to get a cross-section?*

Bakers cut into cakes. Builders cut into solid figures. Construction workers and engineers visualize the cross-sections they will create when cutting into pipes. Contemporary architects design buildings with tops that appear to be cross-sections.

4 *What is a benefit—mathematical or real-world—of being able to visualize the cross-section before you make the slice?*

Answers will vary. Try to get students to discuss the benefits of spatial reasoning demonstrated by visualizing. Visualizing helps you plan, predict, and confirm.

Step By Step

Let's Talk About It

- Organize students into pairs or groups. Work through the first **Let's Talk About It** problem together as a class. Remind students the question refers to the questions from the previous page.
- Walk around to each group, listen to, and join in on discussions at different points. Use the Mathematical Discourse to support or extend students' thinking.

▶ Mathematical Discourse 3 and 4

- Students may find problem 7 much easier than problem 8. Encourage students to compare the difficulty of visualizing cross-sections that result from parallel and perpendicular slices to cross-sections that result from angled slices.
- You may want to do problem 9 as a class. Ask: *Would you have guessed that a cross-section can have more sides than any of the faces of a figure?* Explain the connection between the number of faces you slice and the number of sides of the cross-section polygon. [They are the same.]

Try It Another Way

- Have groups work through **Try It Another Way.**
- Ask: *What is the difference in the cross-sections from a parallel slice in the figure in problem 10 compared to those in the figure in problem 11?* [In problem 10, all of the cross-sections made by a slice will be the same-sized circle whereas in problem 11, the cross-sections will be circles of different sizes.]

PS TIP Look for Structure
Students are looking closely to discern a pattern given the three-dimensional figure, its base, and the slice. They recognize the significance of the angle and direction of a slice for solving problems. *(PS 7)*

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 307–308** after students have completed this section.

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Guided Practice

At A Glance

Students demonstrate their understanding of how the shape of a cross-section of a three-dimensional figure is determined by the angle of the slice to the base. Students identify some possible cross-section shapes that can be made by slicing through a cube.

Step By Step

- Discuss each **Connect** problem as a class using the discussion points outlined below.

Compare

- You may want to ask: *No matter where you make a slice parallel to the base on the rectangular pyramid, what is the shape of the cross-section?* [Rectangle.] *Is it the same size no matter where you slice?* [No, it gets smaller as you move away from the base.]
- You may want to ask a similar series of questions for the cone.
- Ask students to discuss with a partner the similarities and differences between cross-sections of the pyramid and the cone.

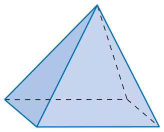
Analyze

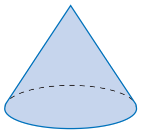
- The second problem focuses on how the size and shape of a cross-section of a pyramid is determined by the angle and location of the slice.
- Read the problem together as a class. Ask students to continue to work in pairs to discuss and write their responses to the question.
- For the second part of problem 13, you may want to sketch a trapezoid on the board and define it. (A trapezoid is a quadrilateral with at least one pair of parallel sides.)

Connect Plane Sections of Prisms and Pyramids

Talk through these problems as a class, then write your answers below.

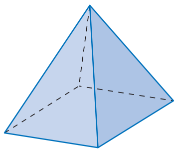
12 Compare Picture slicing a rectangular pyramid and a cone with a plane parallel to the base. How are the cross-sections the same? How are they different?





Possible answer: For both the pyramid and the cone, the cross-sections have the same shape as the base, just different sizes. The cross-section for the pyramid will be a rectangle and the cross-section for the cone will be a circle.

13 Analyze The base of the pyramid shown is a square with an area of 64 square inches.



a. Suppose that the pyramid is sliced so that the cross-section is a square with an area of 49 square inches. What can you say about how to slice the pyramid to get that square?

Possible answer: To get a square, the slice through the pyramid has to be parallel to the base. Since the areas are not very far apart, the slice would have to be closer to the base of the pyramid than to the top vertex.

b. Suppose the pyramid is sliced so that the cross-section is a trapezoid. What can you say about how to slice the pyramid to get a trapezoid?

Possible answer: The slice through the pyramid could be perpendicular to the base, but not through the vertex of the pyramid.

Teacher Notes

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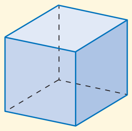
Apply Plane Sections of Prisms and Pyramids

14 Put It Together Use what you have learned to complete this task.

The figure shown below is a cube.



Part A Which of the following cross-sections can you get when you slice a cube? Write *possible* or *not possible* next to each figure named.

i square **possible**

ii circle **not possible**

iii rectangle that is not a square **possible**

iv pentagon **possible**

Part B Choose two of the shapes that you decided are possible to get. Circle your choices. Then describe how you would slice the cube to get that cross-section. You can use a drawing to help explain, if needed.

Answers will vary. Possible answers:

I could slice the cube with a slice parallel to the base to get a square.

I could slice at an angle from the top so that I cut off two corners to get a rectangle.

I could slice down through two adjacent edges from the top at an angle such that I slice through five sides to get a pentagon.

Step By Step

Put It Together

- Direct students to complete the **Put It Together** task on their own.
- For Part A, encourage students to visualize the cross-sections made by slices that are parallel, perpendicular, or at an angle to the base.
- As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.
- If time permits, have students **Pair/Share** their answers to Part B.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 309–310** after students have completed this section.

Scoring Rubrics

See student facsimile page for possible student answers.

Part A

Points	Expectations
2	The student selected a square and a rectangle.
1	The student selected a square and one other cross-section.
0	The student selected only a square.

Part B

Points	Expectations
2	The student's response demonstrates a complete understanding of how the angle of the slice determines the shape of the cross-section. The student uses appropriate math terms such as parallel, perpendicular, and base.
1	The student's response demonstrates an understanding of the concept of how the angle of the slice determines the shape of the cross-section but lacks complete accuracy. The student uses limited math terms such as parallel, perpendicular, and base.
0	The student's response demonstrates a complete lack of understanding of how the angle of the slice determines the shape of the cross-section. The student makes no attempt to use the appropriate math terms.

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