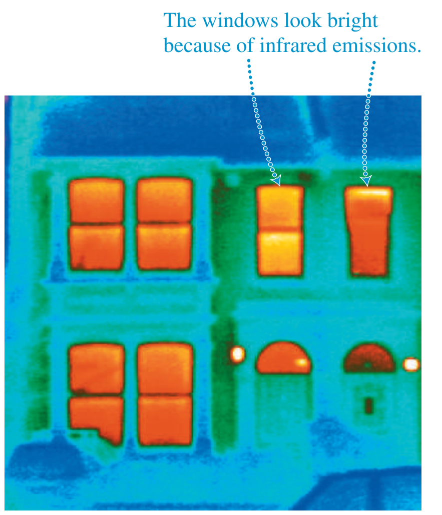
27.1 Black body radiation

In the last quarter of the 19th century, physicists became interested in how hot objects such as stars and the coals of a fire emit thermal radiation (what we now call **infrared radiation**). Scientists studied the absorption and emission of infrared radiation and visible light from a hot object by modeling that object as a so-called **black body**. A black body absorbs all incident (incoming) light and converts the light's energy into thermal energy (no light is reflected). The black body then radiates electromagnetic (EM) waves based solely on its temperature.

What characterizes a black body?

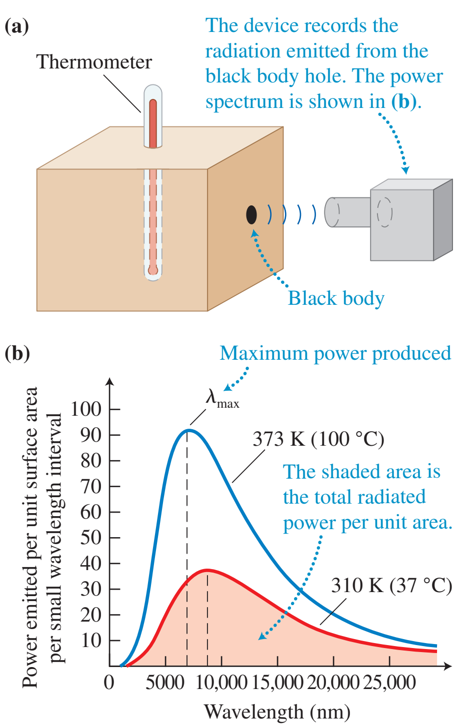
FIGURE 27.1 An infrared photo taken at night with the house lights off.



To understand how the black body model works, imagine a small window in a house whose lights are off, or a small opening in a box, or the pupil of the human eye. Small openings like these look black to an outside observer because light entering the openings is not reflected—it is trapped inside. A small window, for example, admits sunlight. That light is absorbed inside the room, making the room warmer. As a result, more and more infrared radiation also leaves the room through the window. Eventually, the rate at which sunlight energy enters the room through the window equals the rate at which infrared radiation energy leaves the room. At this point the window becomes "a black body." In **Figure 27.1** an infrared camera has detected the infrared radiation being emitted through the windows of an unlit house.

We will model a black body as the surface of a small hole in one side of a closed container (**Figure 27.2a**). Imagine that this container also has a thermometer inside. The hole looks black, and the box is at temperature $T_1 = 310$ K. If you measure the power output per unit radiating surface area of the EM radiation coming from the hole at different wavelengths, you find that the hole emits a continuous EM spectrum (the lower curve in Figure 27.2b).

FIGURE 27.2 A black body spectrum.



The graph in Figure 27.2b is called a **spectral curve**. The quantity on the vertical axis is the power output per unit radiating surface area of the black body per small wavelength interval in units of $W/m^2/m$. On the horizontal axis is the radiation's wavelength. The *total* power output per unit radiating surface area is the area under the black body spectral curve; this quantity is also known as the **intensity** (I) of the EM radiation. (Notice that at a particular wavelength we see the greatest intensity of EM radiation coming from the hole.)

Now imagine that you place the box on a hot stove—the box's temperature rises and the thermometer in the box indicates a higher temperature $T_2 = 373$ K. If we again measure the spectrum of the EM radiation being emitted from the hole, we find that

- the total power output (intensity multiplied by the area of the hole) from the hole is now greater,
- the spectral curve rises (the upper curve in Figure 27.2b) at all wavelengths, and
- the peak of the spectral curve shifts to a shorter wavelength.

As the temperature of the box increases further, these patterns persist. The higher the temperature, the shorter the wavelength at which the maximum of the black body spectral curve occurs, and the greater the power radiated at all wavelengths.

A black body is not a real object; it is a model. It turns out, however, that many real objects emit EM radiation similar to that of a black body. For example, the metal filament of an incandescent lightbulb gets very warm when electric current is passed through it. The bulb radiates some visible light, a lot of invisible infrared radiation, and a little bit of UV. The spectral curve of a lightbulb is very close to the spectral curve of a black body of the same temperature. Thus, studying black body radiation might allow us to understand how objects emit and absorb light and how the radiation they emit might relate to the objects' temperatures.

Studies of black body radiation

Several physicists in the late 1800s used the black body model to determine characteristics of EM radiation. Among their findings were the total power output of a black body and the wavelength at which the peak of the emitted radiation power occurs.

Total power output In 1879 a Slovenian physicist, Joseph Stefan (1835–1893), using his own experiments and data collected by John Tyndall, investigated the power of light emission from a black body—the total power output per unit of surface area—which is equal to the area under the black body spectral curve (see **Figure 27.3**). As the graph lines for different temperatures show, the total radiation output (power per unit area of the emitting object) increases dramatically as the temperature of the black body increases. Stefan found that the total power output per unit of surface area in Figure 27.3 is proportional to the fourth power of the temperature. For example, if you double the temperature of the object (in kelvins), the total radiation power increases by 16 times. The area under the 6000-K curve in Figure 27.3 is 16 times the area under the 3000-K curve.

The total energy emitted by a 1-m^2 black body every second can be written as $I \propto T^4$ or $I = \sigma T^4$, where σ (Greek letter sigma) is the coefficient of proportionality. To find the power P radiated by the whole object, we multiply the intensity of its emitted radiation at its surface by the radiating surface area A of the object. In 1884 Ludwig Boltzmann (1844–1906) derived theoretically the empirical relationship found by Stefan by modeling a black body as a thermodynamic engine with light instead of gas.

The work of Stefan and Boltzmann resulted in the following relationship between the total emitted power P in watts (W), the temperature of the black body T in kelvins, and its surface area A in square meters (**Stefan-Boltzmann's law**):

$$P = IA = \sigma T^4 A \qquad (27.1)$$

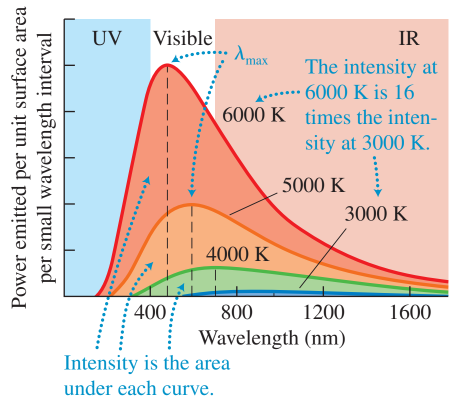
where sigma (σ) stands for the coefficient of proportionality (Stefan-Boltzmann's constant): $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}$.

Wavelength at which maximum intensity occurs In 1893, the German physicist Wilhelm Wien (1864–1928) quantified a second aspect of black body radiation using data similar to those in the graphs in Figures 27.2 and 27.3. The wavelength λ_{max} at which a black body emits radiation of maximum power per wavelength depends on the temperature of the black body:

$$\lambda_{max} = \frac{2.90 \times 10^{-3}\,\mathrm{m \cdot K}}{T} \qquad (27.2)$$

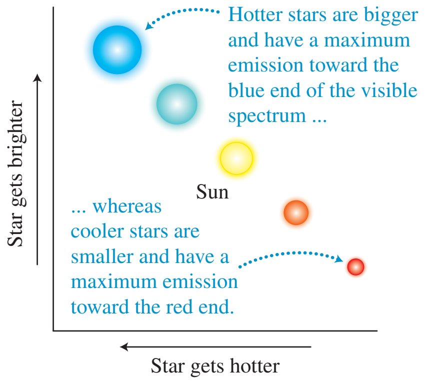
This became known as **Wien's law.** Note that the wavelength λ_{max} at which the black body emits maximum power per wavelength interval becomes shorter as the black body temperature T becomes greater.

FIGURE 27.3 Black body radiation at different temperatures. The total power is proportional to the fourth power of the temperature in kelvins.



TIP A black body (the system) emits electromagnetic waves with power given by Stefan-Boltzmann's law even if it is at the same temperature as the environment—but it also absorbs energy at the same rate. If the body is hotter than the environment, it radiates more energy than it absorbs. If it is cooler, it absorbs more than it radiates. The black body continues to emit and radiate different amounts of energy until it reaches thermal equilibrium with the environment. After that, emission and absorption continue, but their rates become the same.

FIGURE 27.4 The color of stars. The color of a star is an indication of how hot it is. Hotter stars are bigger and appear bluer than cooler stars.



As we saw in the "Try it yourself" question of Quantitative Exercise 27.1, the Earth radiates EM waves primarily in the infrared part of the electromagnetic spectrum, invisible to our eyes. Thus, when we see Earth and most objects on its surface, we are seeing sunlight reflected from them. The same mechanism (reflection of sunlight) makes the Moon visible to us. The Sun and other stars, however, do not need an external source of EM waves to illuminate them.

A star that has a lower surface temperature than the Sun's radiates with a maximum intensity wavelength that is longer than 510 nm, and the star looks red instead of yellow. A star that is hotter than the Sun radiates with maximum intensity at shorter wavelengths, a combination of blue and other visible wavelengths that cause it to look whiter. Very hot stars (around 10,000 K) look blue. Stars even hotter than these are generally invisible to our eyes, but they can be detected with instruments sensitive to ultraviolet radiation.

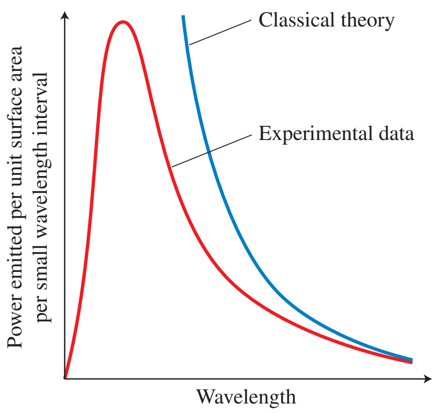
Figure 27.4 shows a pattern characteristic of the majority of the stars. You can see that hotter stars are bigger and cooler stars are smaller. This relationship is true for stars of ages similar to that of the Sun. Older stars can have lower temperatures and large sizes (red giants) or very high temperatures and smaller sizes (white dwarfs). The fascinating topic of stellar evolution is unfortunately beyond the scope of this book.

The ultraviolet catastrophe

By the end of the 19th century, physicists had devised a mechanism to explain how objects emitted electromagnetic radiation, namely, that the atoms and molecules inside objects are made of charged particles, and when these charged particles vibrate, they radiate electromagnetic waves. The higher the vibration frequency, the higher the frequency of the electromagnetic waves that are emitted. The atoms and molecules in a black body at temperature T vibrate at many different frequencies centered on the frequency of the peak radiation on the black body spectrum curve.

Maxwell's electromagnetic wave theory (Chapter 25), together with some assumptions about the structure of matter, predicts that the intensity of the radiation emitted by a black body should increase with the frequency of the radiation—greater intensity for ultraviolet light than for visible light; greater intensity for X-rays than for ultraviolet light. However, actual spectral curves show that at high frequencies (short wavelengths) the emitted intensity actually drops off (**Figure 27.5**). No models built on the physics of the late 19th century made predictions that were consistent with this drop. This problem became known as the **ultraviolet catastrophe.**

FIGURE 27.5 Classical black body radiation theory failed to match the experimental data at short wavelengths.



Planck's hypothesis

After working on the problem for several years, Max Planck (1858–1947) devised an ingenious model that correctly predicted the spectral curve for a black body. In Planck's model, the charged particles inside radiating objects were still responsible for producing electromagnetic radiation. However, they could radiate energy only in discrete portions called **quanta** (one portion is one quantum). Each quantum of emitted energy was equal to a multiple of some fundamental portion of energy: E_0, $2E_0$, $3E_0$, $4E_0$, etc. This idea is known as **Planck's hypothesis**.

Although the idea of this so-called **energy quantization** was completely revolutionary, the idea of quantization itself was not new to physics. Think of mechanical waves. When waves travel back and forth on a string of finite length with fixed ends, the string can vibrate only at discrete standing wave frequencies:

$$f_n = n\frac{v}{2L}$$

where $n = 1, 2, 3, \ldots$. The frequencies depend on the speed v of the waves on the string and the string length L. The standing wave frequencies are therefore said to be **quantized**. The energy of these standing waves can take on any positive value, however, because the energy of the wave depends on the *amplitude* of the wave and not on its frequency.

Planck's hypothesis is different. He suggested that it was *the energy of the oscillator* that was quantized, rather than its frequency.

Using his energy quantization model, Planck was able to derive a mathematical function describing the black body spectral curve. The derivation is complicated, so we won't show it. When he matched the calculated values from the function to experimental results, he was able to determine the constant of proportionality between the oscillating frequency of a charged particle f and its smallest fundamental energy E_0. The relationship is

$$E_0 = hf \tag{27.3}$$

The proportionality constant h became known as **Planck's constant**. The units of h in the SI system are $\text{J/Hz} = \text{J}/(1/\text{s}) = \text{J} \cdot \text{s}$. Its value was determined experimentally to be $h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$.

Planck viewed his energy quantization model as a mathematical trick of some sort rather than an explanation of actual physical phenomena. How could something oscillate smoothly at a particular frequency yet emit waves of that frequency only in discrete energy portions, as though it were emitting some sort of energy "particle"? He was almost embarrassed to have suggested such a model, and for much of his life he harbored doubts that real molecules or atoms radiated energy in this way. However, other scientists, including Albert Einstein, successfully applied this idea to explain other phenomena.

Planck's quantum idea added a new twist to the study of light, and this divergence made him uncomfortable even though he knew the mathematics was correct. Table 27.1 presents a summary of that history. Note that as of the early 20th century, there was no explanation for the absorption of light.

TABLE 27.1 Evolution of ideas concerning the nature of light

Approximate time period	Model	Experimental evidence that the model explains
1600s	*Particle (bullet) model* Light can be modeled as a stream of bullet-like particles.	Reflection, shadows, and possibly refraction
Early 1800s	*Wave model* Particle model insufficient. Light modeled as wave. Wave mechanism unknown.	Double-slit interference, diffraction, light colors, reflection, and refraction
Middle 1800s	*Electromagnetic wave (ether model)* Light modeled as transverse vibrations of ether medium.	Polarization and electromagnetic wave transmission and all of the above evidence
Late 1800s	*Electromagnetic wave (no-ether model)* Light modeled as transverse vibrations of electric and magnetic fields. No medium required.	Michelson-Morley experiment
Early 1900s	*Quantum model* Electromagnetic wave model insufficient. Light modeled as stream of discrete energy quanta.	Emission of light by black bodies

Today, Planck's quantum model is understood as the birth of quantum physics. To understand how this transformation from classical to quantum physics occurred and what model of light is currently accepted, we now turn the discussion to a new phenomenon.