Discovering the Binomial Theorem **Name:**

When you multiply a binomial times itself multiple times, you are performing what we call a <u>*binomial expansion*</u>*. To do this, we usually do TONS of distributing. But today we're going to learn a cool shortcut. Look at the binomial expansions below. Find some patterns to help you predict what the next expansions would be. Then answer the questions below.*

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$(x+y)^5 =$

$(x+y)^6 =$

Describe any pattern(s) that you see in the <u>exponents</u> of the x's and y's.

Describe any pattern(s) that you see in the <u>coefficients</u> of each term.

Hey check this out! This is called Pascal's Triangle. Do you see anything familiar?

1 — row 0
1 1 — row 1
1 2 1 — row 2
1 3 3 1 — row 3
1 4 6 4 1 — row 4
1 5 10 10 5 1 — row 5
1 6 15 20 15 6 1 — row 6
1 7 21 35 35 21 7 1 — row 7

Expanding Binomials

When we expand a binomial, we're really just writing out a series! Look at this:

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

This series is sometimes called the Binomial Theorem. Here it is expanded:

$$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \ldots + {}_nC_n a^0 b^n$$

Let's try some!

Expand each expression below. Use any method you prefer; you may use the Binomial Theorem or use Pascal's Triangle and follow the patterns we discovered. After you expand the expression, simplify each term. Look at the example below.

ex: $(3x+2)^5 = (3x)^5 + 5(3x)^4 \cdot 2 + 10(3x)^3 \cdot 2^2 + 10(3x)^2 \cdot 2^3 + 5(3x) \cdot 2^4 + 2^5$

$= 243x^5 + 810x^4 + 1080x^3 + 720x^2 + 240x + 32$

1) Expand $(a+b)^7$

2) Expand $(x-3)^4$

3) Expand $(x^2-5)^3$

4) Expand $(3x+2y^3)^5$

5) What would the 4th term of $(3x-2)^8$ be?