

Part I: Functions: Is the Function Continuous or Discrete?

	Continuous Function	Discrete Function
Step 1	Domain → use an inequality to represent $\frac{\text{Low}}{\text{Low}} < x < \frac{\text{High}}{\text{High}}$	Domain → list all of your x-values
Step 2	Range → use an inequality to represent $\frac{\text{Low}}{\text{Low}} < y < \frac{\text{High}}{\text{High}}$	Range → List all of your y-values
Step 3	Remember to adjust the inequality signs depending on open or closed circles Open Circles: $< >$ Closed Circles: $\leq \geq$	Is it a function? Does every x go to one y? (Your x value can not repeat or go to 2 different y's)

Use these steps from the table above to complete the following problems.

1. What is the domain of the relation? What is the range? $\{(-7, 6), (1, -3), (4, 3), (5, 9)\}$

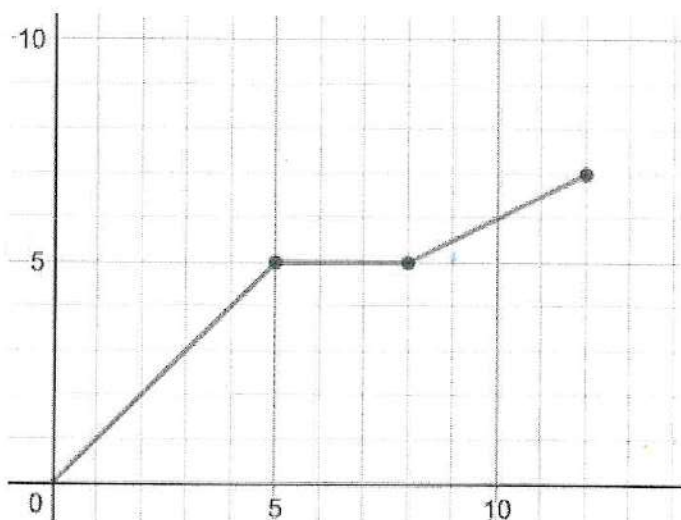
D: -7, 1, 4, 5

R: 6, -3, 3, 9

2. State the Domain and Range of the function represented by the graph.

a. The domain is: $0 \leq x \leq 12$

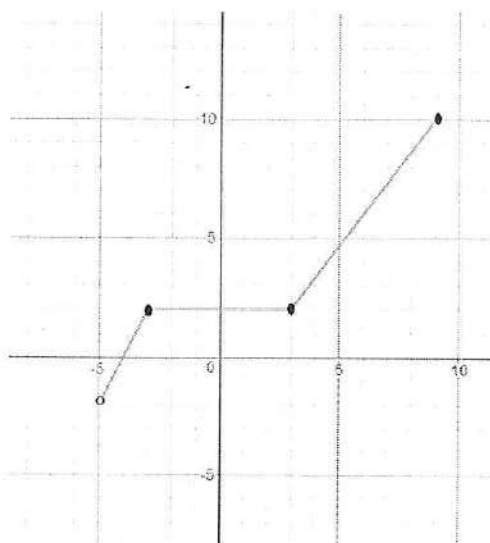
b. The range is: $0 \leq y \leq 7$



3. State the Domain and Range of the function represented by the graph.

a. The domain is: $-5 < x \leq 9$

b. The range is: $-2 < y \leq 10$



Part II: Evaluating Functions and Expressions:

Step 1: Replace the variable with the assigned number.

Step 2: Perform the operations in the expression using the correct order of operations. (PEMDAS)

Step 3: On a graph, if given an x find the y value. If given a y , find the x value.

Use these steps to complete the following problems.

5. Evaluate $f(x) = 2x + 3$ when $x = 4$.

$$\begin{aligned} f(4) &= 2(4) + 3 \\ f(4) &= 8 + 3 \end{aligned}$$

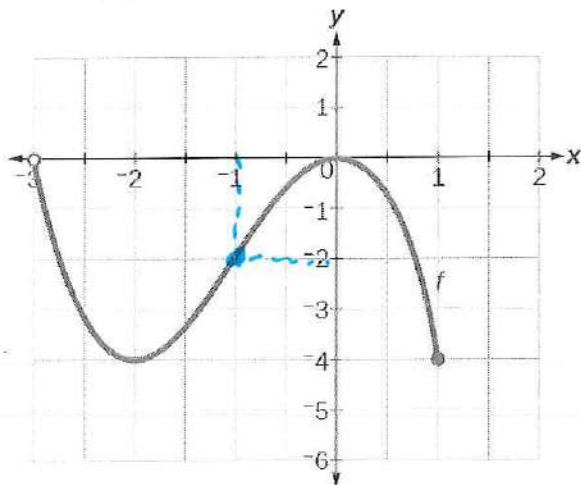
$$f(4) = 11$$

6. Evaluate $f(x) = 4x - 6$ when $x = 9$.

$$\begin{aligned} f(9) &= 4(9) - 6 \\ f(9) &= 36 - 6 \end{aligned}$$

$$f(9) = 30$$

7. Find $f(-1)$ on the graph



$$\begin{aligned} f(-1) \quad & -1 \text{ is} \\ \text{replacing } x, \text{ so } & x = -1 \\ \text{find } y & \\ f(-1) &= -2 \end{aligned}$$

Linear or Nonlinear Functions:

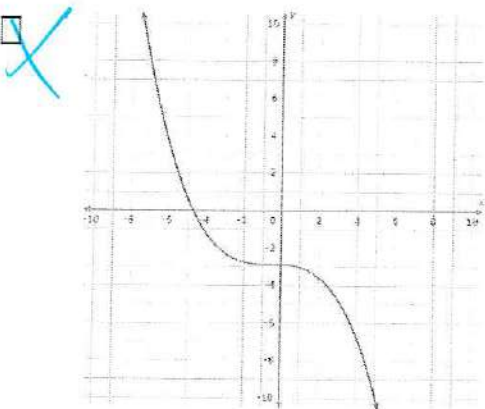
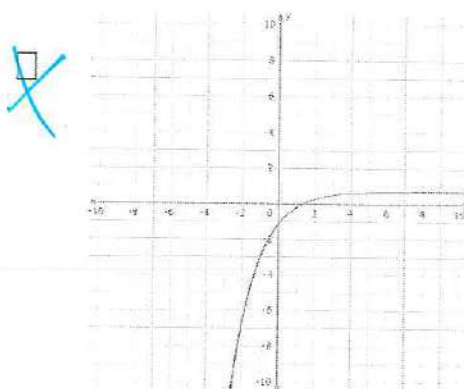
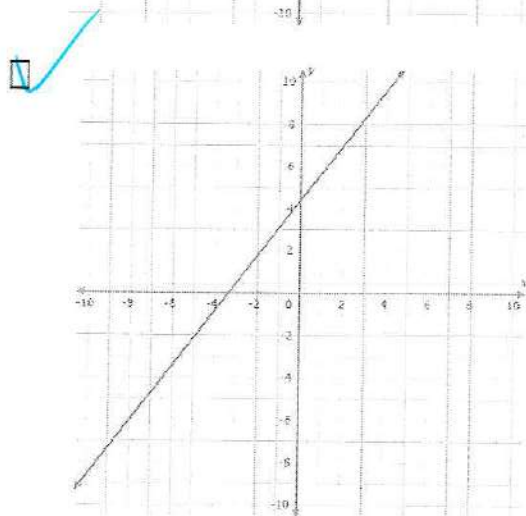
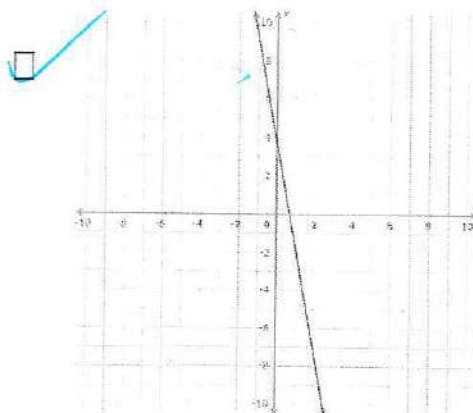
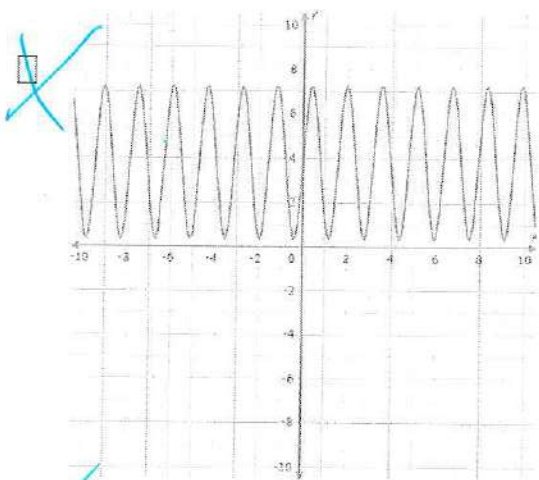
A function is linear if the following is true:

Step 1: The equation can be rewritten in Slope-Intercept Form, $y = mx + b$

Step 2: The graph is a line i.e. vertical, horizontal, or diagonal

Step 3: There is a constant rate

9. Which of the following tables, graphs, and equations are linear functions? Select all that apply.



☒

x	y
0	20
5	16
10	12
15	8

+5, +5, +5, -4, -4, -4

☒

x	0	2	4	6
y	0	2	8	18

☒

x	3	6	9	12
y	12	10	8	6

+3, -2, -2, -2

☒ $y = 9$

☒ $y = x + 4$

☒ $y = \frac{6}{x} + 5$

☒ $y = \frac{1}{2}x$

☒ $y = x^2 + 8$

Part IV: Solving Equations:

Step 1: See if you can distribute anything.

Step 2: What is the order of operations being performed on the variable you're solving for? What is the result of these operations?

Step 3: How would you undo each operation?

Step 4: Work backwards undoing each operation to solve for your variable.

Use these steps to complete the following problems.

10. Find a if: $x + 2a = 4z$

$$\begin{array}{r} x + 2a = 4z \\ -x \quad -x \\ \hline 2a = 4z - x \\ \frac{2a}{2} = \frac{4z - x}{2} \end{array}$$

$$a = 2z - \frac{x}{2}$$

11. Solve for y: $7x + 3y = 8$

$$\begin{array}{r} -7x \quad -7x \\ 3y = -7x + 8 \\ \frac{3y}{3} = \frac{-7x + 8}{3} \end{array} \quad y = \frac{-7}{3}x + \frac{8}{3}$$

12. Devon is solving the equation $8x - 1 = 23 - 4x$. Here are the first steps of her solution?

$$8x - 1 = 23 - 4x$$

$$12x - 1 = 23$$

$$12x = 24$$

$$x = 2$$

What property did Devon do to get $x = 2$?

- a. Addition Property of Equality
- b. Subtraction Property of Equality
- c. Multiplication Property of Equality
- d. Division Property of Equality

Part V: Writing Linear Equations and Inequalities:

Use the following steps to help you complete the following problems.

Linear Equations:

Step 1: Identify Slope or calculate slope using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

A. Parallel Lines: have the same slope

B. Perpendicular Lines have negative reciprocal slopes

Step 2: Substitute a point and the slope into Point-Slope Formula: $y - y_1 = m(x - x_1)$

Step 3: Distribute

Step 4: Solve for y

Step 5: Write an equation in slope-intercept form: $y = mx + b$

Use point slope form

Slope or Intercept

13. Write an equations of the line that passes through: (2,-4), parallel to the line $y=3x+2$.

parallel lines have the same slope $m=3$

$$y - -4 = 3(x - 2)$$

$$y + 4 = 3x - 6$$

$$y = 3x - 10$$

or

$$y = mx + b$$

$$y = 3x + b$$

$$-4 = 3(2) + b$$

$$-4 = 6 + b$$

$$-6 - 6$$

$$-10 = b$$

$$y = 3x - 10$$

14. Write an equations of the line that passes through: (1,-5), perpendicular to the line $y = \frac{1}{8}x + 2$.

perpendicular has opposite reciprocal slopes

$m = \frac{1}{8}$ new slope is $m = -\frac{8}{1}$ or -8

$$y = mx + b$$

$$-5 = -8(1) + b$$

$$+8 \quad +8$$

$$3 = b$$

$$y = -8x + 3$$

15. Write a linear equation through the points (0, 3) and (-4, -1).

need to use point slope form $y - y_1 = m(x - x_1)$

Find slope 1st

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - -1}{0 - -4} = \frac{3 + 1}{0 + 4} = \frac{4}{4} = 1$$

$$y - 3 = 1(x - 0)$$

$$y - 3 = 1x$$

$$+3 \quad +3$$

$$y = 1x + 3$$

16. Howard decided to start jogging every day at the track. He completes the following chart for the first month (4 weeks) that he jogs every day.

Time (weeks)	0	1	2	3
Laps	4	5	6	7

Linear

Determine linear, quadratic or Exponential

Use the table to write an equation to represent the number of laps l Howard runs if t is the time in weeks since he began jogging. Interpret the slope and the y-intercept.

Equation:

$$y = mx + b$$

$$y = 1x + 4$$

Slope means:

the slope is 1, it means each week Howard increases 1 lap around the track

y-intercept means:

y-intercept is 4 it means he starts off jogging 4 laps at the beginning of the month

17. The function below shows the cost of a hamburger with different numbers of toppings (t).

$$f(t) = 1.90 + 1.40t$$

a. What is the y-intercept, and what does it mean?

y-int = 1.90 means that the base price of the hamburger is \$1.90

b. What is the slope, and what does it mean?

slope is 1.40 means that for each topping there is an upcharge of \$1.40

c. If Jodi paid \$3.30 for a hamburger, how many toppings were on Jodi's hamburger?

$$\begin{array}{r} 3.30 = 1.90 + 1.40t \\ -1.90 \quad -1.90 \end{array}$$

$$\begin{array}{r} 1.4 = 1.40t \\ \underline{1.4} \quad \underline{1.40} \\ 1 = t \end{array} \quad \begin{array}{l} \text{he got} \\ 1 \\ \text{topping.} \end{array}$$

Linear Inequalities:

- At least – means greater than or equal to
- No more than – means less than or equal to
- More than – means greater than
- Less than – means less than

Step 1: Read through the entire problem.

Step 2: Highlight the important information and keywords that you need to write the inequality.

Step 3: Identify your variables.

Step 4: Write the equation or inequality.

18. Keith has \$500 in a savings account at the beginning of the summer. He wants to have at least \$200 in the account by the end of summer. He withdraws (takes out) \$25 a week for his cell phone bill.

Write an inequality that represents Keith's situation.

$$-25x + 500 \geq 200 \quad \text{or} \quad 500 - 25x \geq 200$$

19. The high school student council is selling candy as a fundraiser. You have at most \$10 to spend. Hershey bars cost \$1.50 each and each Skittles bag costs \$1.75. Write an inequality for the number of Hershey bars and SKittles bags you can purchase.

$$1.50h + 1.75s \leq 10$$

20. Suppose that you are running a concession stand when a person gives you \$18 and asks for six soft drinks and as many hot dogs as the remaining money will buy. If soft drinks are \$1.00 and hot dogs are \$1.75, write an inequality for the maximum number of hot dogs the person can buy?

$$1.00s + 1.75h \leq 18$$

$$\begin{array}{l} \text{or } 6.00 + 1.75h \leq 18 \\ \hline \text{or } 1.75h \leq 12 \end{array}$$

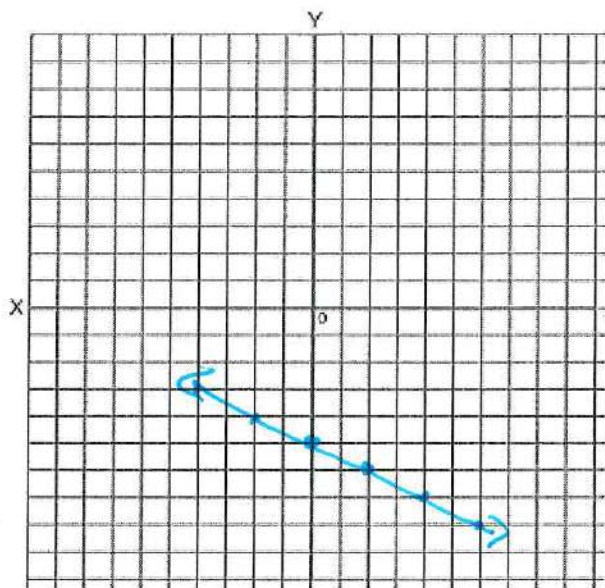
Part VI: Graphing Linear Equations:

Step 1: Plot the y-intercept

Step 2: use the slope (rise/run) to plot additional points

Step 3: connect points with a line

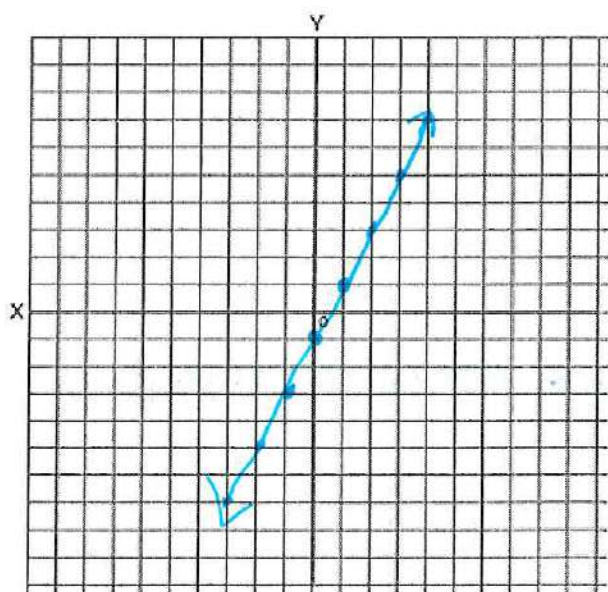
16. Graph the function $y = -\frac{1}{2}x - 5$



$$y = -\frac{1}{2}x - 5$$

↑ slope ↑ y-int

17. Graph the function $y = 2x - 1$



$$y = 2x - 1$$

↑ slope ↑ y-int

$$\frac{2}{1} = \frac{\text{rise}}{\text{run}}$$

Part VII: Recursive Sequence:

Use the following steps to help you complete the following problems.

Arithmetic:

Step 1: What is your starting point? ($a_0 = y$ intercept)

Step 2: How do you one term to get to the next term? ($d =$ constant rate)

Geometric:

Step 1: What is your first term? ($a_1 =$ first term)

Step 2: How do you one term to get to the next term? $a_n = r * a_{n-1}$

($r =$ common ratio and how you get from one term to the next)

18. Consider the sequence 20, 17, 14, 11, ...

$a_n = dn + a_0$

$a_n = \underline{-3}n + \underline{23}$

a. Find the next 3 terms in the sequence.

8, 5, 2

19. Write the next three terms of the arithmetic sequence.

-7, -3, 1, 5, ...

$+4, +4, +4$

9, 13, 17

20. Write the first six terms of the sequence.

$$a_n = 1.5 \cdot a_{n-1}$$

$$a_1 = 6$$

← Common ratio means multiply

$$6 \cdot 1.5 =$$

$$a_1 = 6$$

$$a_2 = 9$$

$$a_3 = 13.5$$

$$a_4 = 20.25$$

$$a_5 = 30.375$$

$$a_6 = 45.5625$$

Part VIII: Solving and Graphing Inequalities:

Step 1: Use the same steps for solving an equation

Step a: See if you can distribute anything.

Step b: What is the order of operations being performed on the variable you're solving for? What is the result of these operations?

Step c: How would you undo each operation?

Step d: Work backwards undoing each operation to solve for your variable.

Step 2: Remember to flip your inequality sign if you multiply or divide by a negative number.

Use these steps to complete the following problems.

Graphing Inequalities:

Step 1: Graph the inequality like an equation.

- Plot the y-intercept and use slope for additional points.
-

Solid Line	Dashed Line	Shade Above	Shade Below
\geq \leq	$>$ $<$	\geq $>$	\leq $<$

Writing Inequalities from a graph:

Step 1: Write an equation of the line in slope intercept form $y=mx+b$

Step 2: Identify slope and y-intercept

Step 3: Change the equal sign into the correct inequality sign using the table above

20. Solve : $-7x + 7 \leq -56$

$$\begin{array}{r} -7 \quad -7 \\ -7x \leq -63 \\ -7 \quad -7 \end{array}$$

$$x \geq 9$$

21. Solve $-6x - 4 < -2x + 8$

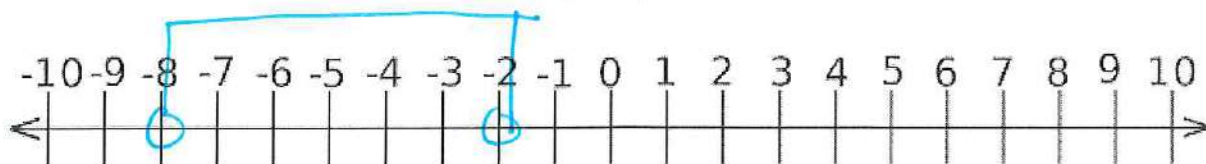
$$\begin{array}{r} +2x \quad +2x \\ -4x - 4 < 8 \\ +4 \quad +4 \\ \hline -4x < 12 \end{array}$$

$$\begin{array}{r} -4x < 12 \\ -4 \quad -4 \\ \hline x > -3 \end{array}$$

22. Find and graph the solution: $-50 < 7k + 6 < -8$

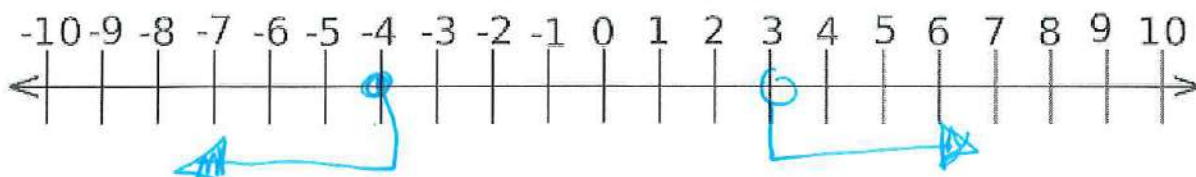
$$\begin{array}{r} -6 \quad -6 \quad -6 \\ -50 < 7k < -14 \\ \hline \frac{-50}{7} < \frac{7k}{7} < \frac{-14}{7} \end{array}$$

$$-8 < k < -2$$



23. Find and graph the solution: $2y + 7 > 13$ or $-3y - 2 \geq 10$

$$\begin{array}{l|l} -7 & -7 \\ \hline 2y > \frac{6}{2} \\ y > 3 \end{array} \quad \begin{array}{l|l} +2 & +2 \\ \hline -3y \geq \frac{12}{-3} \\ y \leq -4 \end{array}$$



25. Which of the following is a solution to the inequality $-2x+3y>6$? Select all that apply.

- a. (4,4) ☒
- b. (0,2) ☒
- c. (-3,2) ☒
- d. (3,5) ☒
- e. (-6,-2) ☒

plug in and see if there is a true statement.

$$\begin{aligned} -2(-6) + 3(-2) &> 6 \\ 6 &> 6 \end{aligned}$$

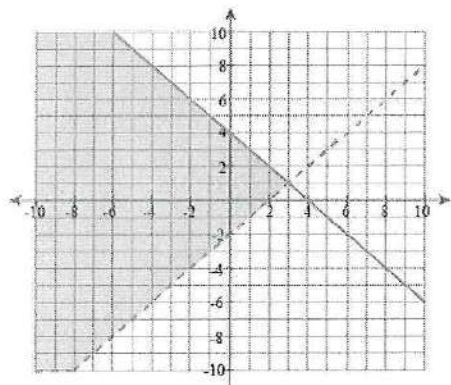
$$\begin{aligned} -2(4) + 3(4) &> 6 \\ 4 &> 6 \\ -2(0) + 3(2) &> 6 \\ 6 &> 6 \end{aligned}$$

$$\begin{aligned} -2(-3) + 3(2) &> 6 \\ 6 + 6 &> 6 \\ -2(3) + 3(5) &> 6 \end{aligned}$$

27. Complete the system of inequalities based on the graph below

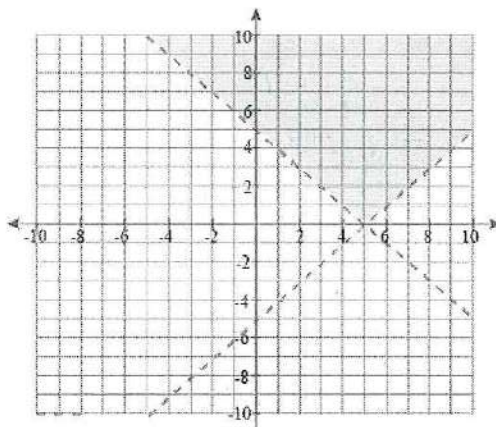
a.

b.



$$y \leq -1x + 4$$

$$y \boxed{>} 1x - 2$$



$$y > -1x + 5$$

$$y \boxed{>} 1x - 5$$

Part VIII: Creating Systems of Equations and Inequalities:

Step 1: Define your variables.

Step 2: Write an equation for the quantities given in the problem.

Step 3: Write a second equation for the quantities given in the problem.

Step 4: Solve the system by either graphing, elimination, or substitution. (Graphing may be easiest). Make sure you find the solution for **both** your variables.

Use these steps to complete the following problems.

27. A roofing contractor buys 30 bundles of shingles and 4 rolls of roofing paper for \$1040. In a second purchase (at the same prices), the contractor buys 8 bundles of shingles for \$256. Find the price per bundle of shingles and the price per roll of roofing paper.

$x = \text{shingles}$
 $y = \text{paper}$

it cost \$20
for roofing paper

$$\begin{aligned} 30x + 4y &= 1040 \\ \frac{8x}{8} &= \frac{256}{8} \end{aligned}$$

$$\begin{aligned} x &= 32 \\ 30(32) + 4y &= 1040 \end{aligned}$$

$$960 + 4y = 1040$$

$$\begin{aligned} 4y &= \frac{80}{4} \\ y &= 20 \end{aligned}$$

28. A business with two locations buys seven large delivery vans and five small delivery vans. Location A receives five large vans and two small vans for a total cost of \$235,000. Location B receives two large vans and three small vans for a total cost of \$160,000. What is the cost of each type of van?

Large vans = x
Small vans = y

$$\begin{aligned} A: 5x + 2y &= 235,000 \\ B: 2x + 3y &= 160,000 \end{aligned}$$

$$\begin{aligned} 15x + 6y &= 705,000 \\ -4x - 6y &= -320,000 \\ \hline 11x &= 385,000 \end{aligned}$$

(use elimination method)

$$x = 35,000$$

$$\begin{aligned} 5(35,000) + 2y &= 235,000 \\ 175,000 + 2y &= 235,000 \\ -175,000 & \quad -175,000 \end{aligned}$$

$$\begin{aligned} 2y &= 60,000 \\ \frac{2y}{2} &= \frac{60,000}{2} \\ y &= 30,000 \end{aligned}$$

29. Solve the system of linear equations:

$$\begin{aligned} -10x + 3y &= 1 \\ -5x - 6y &= 23 \end{aligned}$$

$$\begin{aligned} -5x - 6y &= 23 \\ -10x + 6y &= 2 \end{aligned}$$

$$\begin{aligned} -25x &= 25 \\ -25 & \quad -25 \end{aligned}$$

$$x = -1$$

plug in to find y

$$\begin{aligned} -5(-1) - 6y &= 23 \\ 5 - 6y &= 23 \end{aligned}$$

$$(-1, -3)$$

$$\begin{aligned} -6y &= 18 \\ -6 & \quad -6 \\ y &= -3 \end{aligned}$$

30.

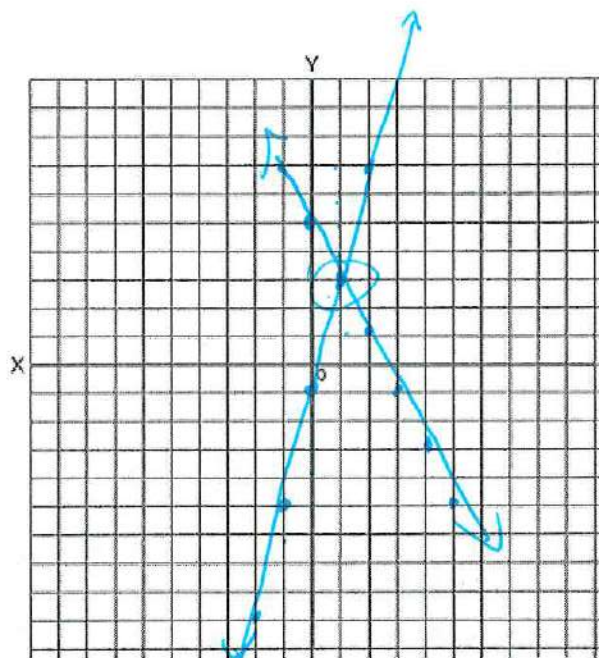
Solve the system of linear equations by graphing.

$y = -2x + 5$

Equation 1

$y = 4x - 1$

Equation 2

 $(1, 3)$ 

31. Identify how many solutions each system of equations has: 1 solution, no solution, infinitely many

a.
$$\begin{array}{l} 5(3x - 2y = -5) \\ 2(4x + 5y = 47) \end{array}$$

$$\begin{array}{r} 15x - 10y = -25 \\ + 8x + 10y = 94 \\ \hline 23x = 69 \\ \frac{23x}{23} = \frac{69}{23} \end{array}$$

 1 solution

$x = 3$

b.
$$\begin{array}{l} -2(x - y = 0) \\ 5x - 2y = 6 \end{array}$$

$$\begin{array}{r} -2x + 2y = 0 \\ + 5x - 2y = 6 \\ \hline 3x = 6 \\ \frac{3x}{3} = \frac{6}{3} \end{array}$$

 1 solution

$x = 2$

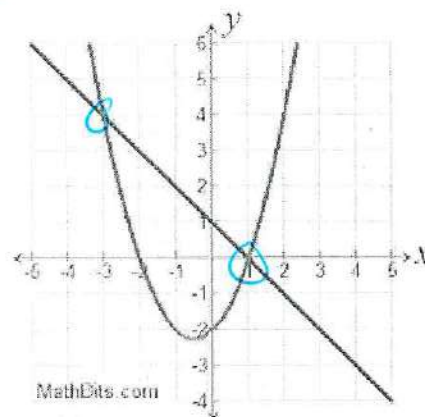
c.
$$\begin{array}{l} 3(-2x + 4y = 1) \\ 2(3x - 6y = 9) \end{array}$$

$$\begin{array}{r} -6x + 12y = 3 \\ 6x - 12y = 18 \\ \hline 0 = 21 \end{array}$$

 $0 = 21 \text{ No solution}$

31. Identify the solutions of the following system of equations given the graph

$(-3, 4)$
and
 $(1, 0)$



32. Given the situation below which would be the correct system of inequalities?

You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time can you spend at each location?

$$x+y \leq 8$$

$$x \geq 2$$

$$y > 4$$

$$x+y \geq 8$$

$$x \leq 2$$

$$y > 4$$

Part X: Absolute Value Transformations:

$$y = a|x - h| + k$$

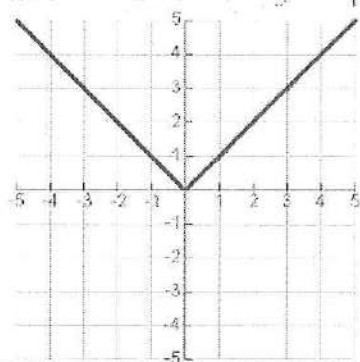
#1: vertical stretch ($|a| > 1$) or shrink ($0 < |a| < 1$)

*negative: vertical reflection

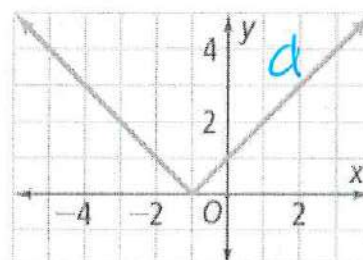
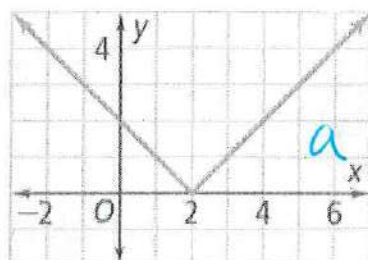
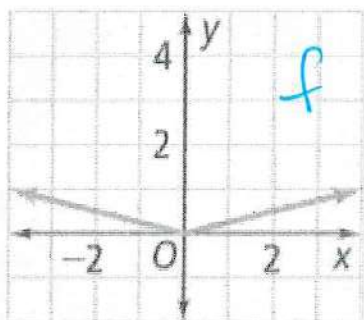
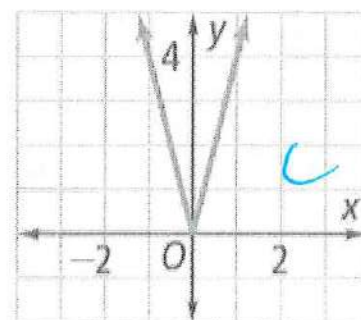
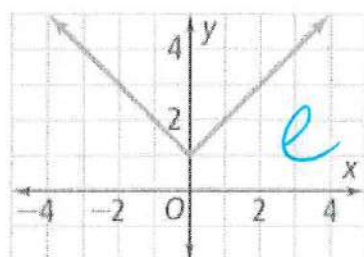
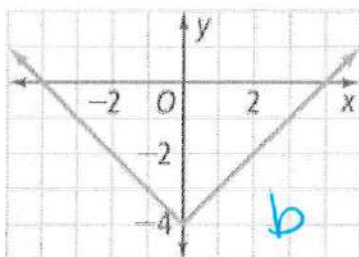
#2: horizontal translation (opposite)

#3: vertical translation

Parent function $y = |x|$



33. Match the following absolute value functions with its graph



a. $f(x) = |x - 2|$ ✓

b. $f(x) = |x| - 4$ ✓

c. $f(x) = 4|x|$ ✓

d. $f(x) = |x + 1|$ ✓

e. $f(x) = |x| + 1$ ✓

f. $f(x) = \frac{1}{4}|x|$ ✓

Part XI: Operations with Polynomials

Adding and Subtracting Polynomials Section

Step 1: Line up like terms on top of each other

***Note:** if the sign is subtraction do Keep Change Flip first

Step 2: add like terms together

***Note:** only add coefficients the exponent and variable stays the same

Find the Sum or Difference of each polynomials and write your answer in Standard Form.

***Hint:** identify like terms and use the operation between the 2 polynomials to combine them

34. $(5x^3 - 7x^2 - 8) - (4x^2 + 5x - 6)$

KFC

$$\begin{array}{r} 5x^3 - 7x^2 - 8 \\ + \quad \downarrow \quad -4x^2 + 6 - 5x \\ \hline 5x^3 - 11x^2 - 2 - 5x \end{array}$$

36. $(9x^5 - 6x^3 + 7x^2) - (7x^3 - 6x^5 + 2x^2)$

$$5x^3 - 11x^2 - 5x - 2$$

35. $(4x^3 - 9x + 3) + (5x^2 - 4x + 7)$

$$\begin{array}{r} 4x^3 - 9x + 3 \\ + \quad \downarrow \quad -4x + 7 + 5x^2 \\ \hline 4x^3 - 13x + 10 + 5x^2 \\ 4x^3 + 5x^2 - 13x + 10 \end{array}$$

Multiplying Polynomials Section (7.2-7.3)

Step 1: Use any of the following methods to multiply

a. Use the Box Method

b. Distributive Property to find the product.

Step 2: Combine any like terms

Step 3: Write your answer in Standard Form.

37. Find $(3x+2)(x-1)$

$$\begin{array}{r} 3x + 2 \\ \times \quad x - 1 \\ \hline 3x^2 + 2x \\ -3x - 2 \\ \hline 3x^2 - x - 2 \end{array}$$

$$3x^2 - x - 2$$

38.

$(2x-5)(4x^2-7x+3)$

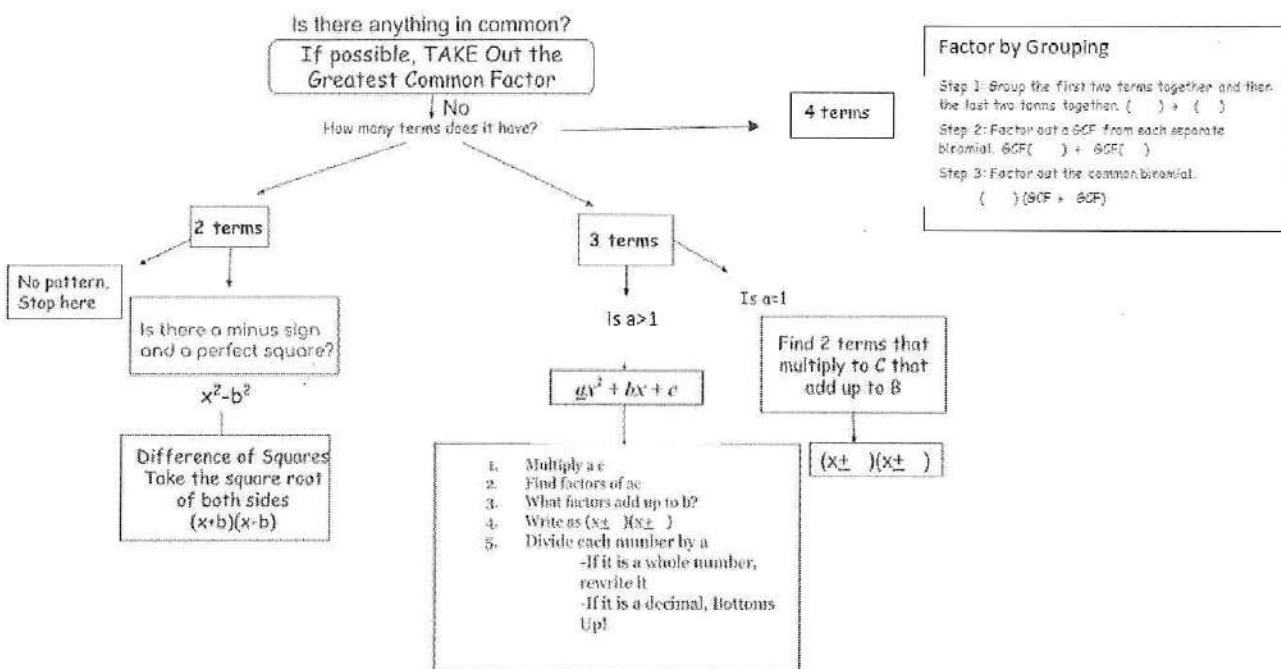
$$\begin{array}{r} 4x^2 - 7x + 3 \\ 2x \quad 8x^3 - 14x^2 + 6x \\ -5 \quad -20x^2 + 35x - 15 \\ \hline 8x^3 - 34x^2 + 41x - 15 \end{array}$$

39. $(5-2x)(5+2x)$

$$\begin{array}{r} 25 + 10x - 10x - 4x^2 \\ \hline 25 - 4x^2 \end{array}$$

Part XII: Factoring and Solving by Factoring:

Factoring Flow Chart



40. Factor the Following:

a.13. $2x^2 + 7x + 3$

$a \cdot c = 2 \cdot 3 = \frac{6}{1 \cdot 6}$ Add to 7
 $\frac{6}{1 \cdot 6}$

$(x+1)(x+3)$
 $(x+1)(x+3)$

d. $2x^2 + 8x + 3x + 12$

$(2x^2 + 8x) + (3x + 12)$
 $2x(x+4) + 3(x+4)$
 $(2x+3)(x+4)$

e. $9x^2 - 1$

$(3x-1)(3x+1)$

b. $x^2 + 5x - 6$

$\frac{-6}{2 \cdot 3}$ Add to 5
 $\frac{-6}{-1 \cdot 6}$
 $(x-1)(x+6)$

d. $(24x^3 - 6x^2) + (8x - 2)$

$6x^2(4x-1) + 2(4x-1)$
 $(4x-1)(6x^2+2)$

f. $n^4 - 49$

$(n-7)(n+7)$

g.

$$x^2 - 24x + 144$$

$$(x-12)(x-12)$$

$$\begin{array}{r} 144 \\ -12 \quad -12 \\ \hline \end{array} \quad \text{Add to 24}$$

h.

$$49n^2 - 56n + 16$$

$$(7n-4)(7n-4) \quad \text{or} \quad (7n-4)^2$$

Factoring using GCF and Zero Product Property

Follow the Steps: Step 1: Make sure equation is set equal to zero

$$5x^2 = 25x$$

$$\underline{-25x \quad -25x}$$

$$5x^2 - 25x = 0$$

Step 2: Find the GCF and factor it out

GCF is 5x (Both numbers have 5 as the greatest factor and you can take an x out of each term)

$$5x^2 - 25x = 0 \quad \text{----- GCF: } 5x$$

Factor out GCF

$$5x(x - 5) = 0$$

Step 3: Use the Zero Product Property to set each factor equal to zero and solve for x.

$$5x(x - 5) = 0$$

$$\underline{5x=0}$$

$$5 \quad 5$$

$$x = 0$$

$$x - 5 = 0$$

$$\underline{+5 \quad +5}$$

$$x = 5 \quad \text{the roots are 0 and 5}$$

$$41. \quad 6x^2 = -3x$$

$$\begin{array}{r} +3x \\ \hline 6x^2 + 3x = 0 \end{array}$$

$$3x(2x+1)=0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 3x=0 \quad 2x+1=0 \\ \underline{\quad \quad} \quad \underline{-1 \quad -1} \\ \frac{3}{3} \quad \frac{2x}{2} = \frac{-1}{2} \\ x=0 \quad x = -\frac{1}{2} \end{array}$$

$$42. \quad 4y^2 - 20y = 0$$

$$4y(y-5)=0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 4y=0 \quad y-5=0 \\ \underline{\quad \quad} \quad \underline{+5 \quad +5} \\ y=0 \quad y=5 \end{array}$$

Part XII: Converting from Vertex to Standard form of Quadratic Functions:

Step 1: Expand the parenthesis

Step 2: Multiply the two binomials () ()

Step 3: Combine Like terms

Step 4: Make sure your answer is in standard form

43. Rewrite the following equations in standard form:

a. $f(x) = (x+2)^2 - 1$

$$f(x) = (x+2)(x+2) - 1$$

$$f(x) = x^2 + 4x + 4 - 1$$

$$f(x) = x^2 + 4x + 3$$

b. $f(x) = (x-5)^2 + 3$

$$f(x) = (x-5)(x-5) + 3$$

$$f(x) = x^2 - 10x + 25 + 3$$

$$f(x) = x^2 - 10x + 28$$

Part XIII: Solving Quadratic Equations by Graphing, Square Roots, and Quadratic Formula

Solving Quadratic Equations by Graphing

Step 1: Write the equation in standard form

(terms must all be on the same side of the equal sign and in order from greatest to least)

Step 2: Graph the equation in Standard form from Step 1

Step 3: Find the x-intercepts

Solving by Square Roots

Step 1: Isolate the perfect square. Either

Step 2: Take the square root of each side. Don't forget the \pm .

Step 3: Simplify the radical.

Step 4: Get x by itself.

Solving by Quadratic Formula

Step 1: Identify a, b, and c and plug them into the quadratic formula. In this case $a = 1$, $b = -8$, and $c = 14$.

Step 2: Use the order of operations to simplify the quadratic formula.

Step 3: Simplify the radical, if you can.

Step 4: Reduce the problem, if you can

44. Solving Following using Square roots:

a.

$$9n^2 + 10 = 91$$

$$\begin{array}{r} -10 \quad -10 \\ \hline 9n^2 = 81 \end{array}$$

$$\frac{9n^2}{9} = \frac{81}{9}$$

$$\sqrt{n^2} = \sqrt{9}$$

$$n = \pm 3$$

c.

$$13 - 8n^2 = -1139$$

$$\begin{array}{r} -13 \quad -13 \\ \hline -8n^2 = -1152 \end{array}$$

$$\frac{-8n^2}{-8} = \frac{-1152}{-8}$$

$$\frac{-8n^2}{-8} = \frac{-1152}{-8}$$

$$\sqrt{n^2} = \sqrt{144}$$

$$n = \pm 12$$

b.

$$7v^2 + 1 = 29$$

$$\begin{array}{r} -1 \quad -1 \\ \hline 7v^2 = 28 \end{array}$$

$$\frac{7v^2}{7} = \frac{28}{7}$$

$$\sqrt{v^2} = \sqrt{4}$$

$$v = \pm 2$$

43. Solve the following problems using the Quadratic Formula

a.

b.

$$2m^2 + 2m - 12 = 0$$

$$2x^2 - 3x - 5 = 0$$

$$\begin{aligned} a &= 2 \\ b &= 2 \\ c &= -12 \\ X &= \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-12)}}{2(2)} \\ X &= 2 \text{ or } X = -3 \end{aligned}$$

$$\begin{aligned} a &= 2 \\ b &= -3 \\ c &= -5 \\ X &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} \\ X &= \frac{3}{2} \text{ or } X = -1 \end{aligned}$$

Part XIII: Simplifying Radicals

Step 1: Break up the number inside the radical into its prime factors. Start by dividing the number by the first prime number 2 and continue dividing by 2. Then divide by 3, 5, 7, etc. until the only numbers left are prime numbers. Also factor any variables inside the radical by writing them out example: .

Step 2: Determine the index or root of the radical. The index tells you how many of a kind you need in each group to move outside the radical.

Step 3: Remember for each group only one number or variable comes outside. Whatever is not in a group stays inside the radical.

Step 4: Simplify the expressions both inside and outside the radical by multiplying. Multiply all numbers and variables inside the radical together. Multiply all numbers and variables outside the radical together.

44.

$$\begin{aligned} \sqrt{108} &= \sqrt{2 \cdot 54} \\ &= \sqrt{2 \cdot 2 \cdot 27} \\ &= \sqrt{2 \cdot 2 \cdot 3 \cdot 9} \\ &= \sqrt{(2 \cdot 2) \cdot (3 \cdot 3) \cdot 3} \\ &= 6\sqrt{3} \end{aligned}$$

46.

$$\begin{aligned} \sqrt{80p^3} &= \sqrt{4 \cdot 20 p p p} \\ &= \sqrt{2 \cdot 2 \cdot 2 \cdot 10 p p p} \\ &= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2 \cdot 5) \cdot (p \cdot p \cdot p)} \\ &= 4p\sqrt{5p} \end{aligned}$$

45.

$$\begin{aligned} \sqrt{150} &= \sqrt{25 \cdot 6} \\ &= \sqrt{(5 \cdot 5) \cdot 6} \\ &= 5\sqrt{6} \end{aligned}$$

47.

$$\begin{aligned} \sqrt{200m^4n} &= \sqrt{2 \cdot 100 m m m m n} \\ &= \sqrt{2 \cdot (10 \cdot 10) m m m m n} \\ &= 10m^2\sqrt{2n} \end{aligned}$$

Key.

Part XXI: Exponential Functions

Exponential functions are in the form $y = ab^x$

a = the initial amount or constant multiplier b = common ratio

There are two types of Exponential functions:

1. Growth when $b > 1$ use the formula $b = 1 + r$ (where r is the rate in decimal form)
2. Decay when $0 < b < 1$ (Decimal or fraction) use the formula $b = 1 - r$ (where r is the rate in decimal form)

To determine the rate of increase or decrease you must first know the common ratio.

Example: Given the following equation $y = 3(1.95)^x$ determine the rate

Step 1: determine if the function is Growth or decay

Step 2: use the formula for growth $b = 1 + r$ or Decay $b = 1 - r$

Step 3: Plug in for "b" into the correct formula and solve for "r"

Step 4: convert to a percent by multiplying by 100

$$y = 3(1.95)^x$$

Growth because $b > 1$
 $b = 1 + r$, $b = 1.95$

$$1.95 = 1 + r$$

$$-1 \quad -1$$

$$.95 = r$$

$$.95 \times 100 = 95\%$$

Rate of increase is 95%

Determining if a function is Linear, Quadratic, or Exponential:

You can use patterns between consecutive data pairs to

determine which type of function models the data. The differences of consecutive

y -values are called *first differences*. The differences of consecutive first differences are called *second differences*.

- Linear Function The first differences are constant.
- Exponential Function Consecutive y -values have a common *ratio*.
- Quadratic Function The second differences are constant.

In all cases, the differences of consecutive x -values need to be constant.

Linear Function

$$y = mx + b$$

Exponential Function

$$y = ab^x$$

Quadratic Function

$$y = (x - p)(x - q)$$

54. Tell whether each table represents a linear, quadratic, or exponential function. Then write an equation for each.

x	0	y	1
1		4	
2		16	
3		64	
4		256	
5		1024	

exponential
 $y = a \cdot b^x$

12
 36
 48
 144
 192
 576
 768

$$b = \frac{256}{64} = 4$$

$$y = 1(4)^x$$

x	0	y	4
1		6	
2		8	
3		10	
4		12	
5		14	

linear

$$y = 2x + 4$$

55.

(Lesson 32) Determine if each equation represents exponential growth or exponential decay.

Equation	Growth or Decay?
$y = 400(.99)^x$	Decay
$y = 0.5(1.001)^x$	Growth
$y = 12(4)^x$	Growth
$y = 100\left(\frac{1}{2}\right)^x$	Decay

56.

A city of 53,000 people has an annual increase in population.

Given the table below answer the following questions

Year, x	0	1	2	3	4
Population, y	53,000	54,325	55,683	57,075	58,502

a. Write an exponential function that represents the population.

$$y = a \cdot b^x$$

$$b = \frac{58502}{57075} = 1.025$$

$$y = 53,000(1.025)^x$$

b. What is the rate of increase?

$$b = 1.025$$

Growth

$$b = 1 + r$$

$$\begin{array}{r} 1.025 = 1 + r \\ -1 \quad -1 \\ \hline .025 = r \end{array}$$

$$\begin{array}{l} .025 \times 100 \\ r = 2.5\% \end{array}$$

c. Determine the population 10 years from now.

$$y = 53,000(1.025)^{10} = 67844$$

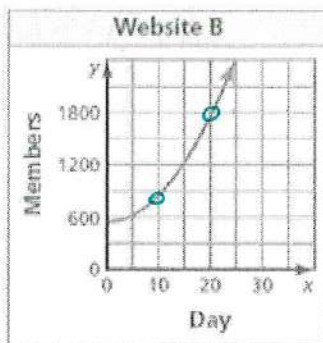
57.

hint: find / compare slopes.

Two social media websites open their memberships to the public.

(a) Compare the websites by calculating and interpreting the average rates of change from Day 10 to Day 20. (b) Predict which website will have more members after 50 days. Explain.

Website A	
Day, x	Members, y
0	650
5	1025
10	1400
15	1775
20	2150
25	2525



$$\text{rate of change} = \frac{f(b) - f(a)}{b - a}$$

$\begin{array}{r} \text{A} \\ 2150 - 1400 \\ \hline 20 - 10 \\ \hline 750 \\ \hline 10 \end{array} = 75$	$\begin{array}{r} \text{B} \\ 1800 - 860 \\ \hline 20 - 10 \\ \hline 940 \\ \hline 10 \end{array} = 94$
--	---

website B will have more members b/c they have a higher rate of change.

58. In 1900, Littleton had a population of 1000 people. Littleton's population increased by 50 people each year. In 1900, Tinyville had a population of 500 people. Tinyville's population increased by 5% each year.

a. In what year were the populations about equal?

Littleton $y = 50x + 1000$
 Tinyville $y = 500(1.05)^x$

approximately 35 years later in 1935

Use desmos:

b. Suppose Littleton's initial population doubled to 2000 and maintained a constant rate of increase of 50 people each year. Did Tinyville's population still catch up to Littleton's population? If so, in which year?

$$y = 50x + 2000$$

$$y = 500(1.05)^x$$

yes they still caught up in approximately 43 years in 1943

c. Suppose Littleton's rate of increase doubled to 100 people each year, in addition to doubling the initial population. Did Tinyville's population still catch up to Littleton's population? Explain.

$$y = 100x + 2000$$

$$y = 500(1.05)^x$$

yes they still caught up in approximately 56 years in 1956

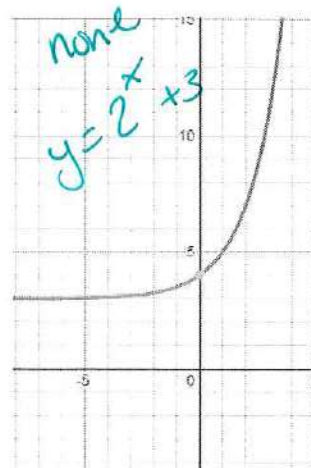
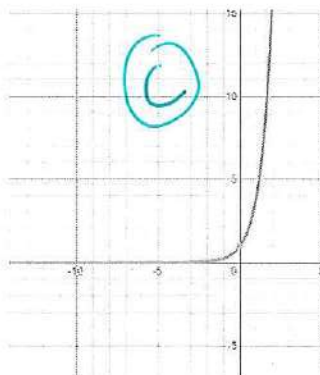
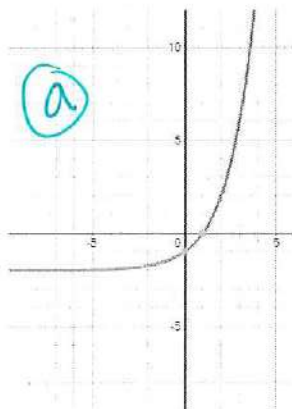
59. Match each equation to the correct transformation of an exponential function

a) $y = 2^x - 2$

b) $y = 2^{x+3}$

c) $y = 4^x$

d) $y = 3(2^{x-1}) + 1$



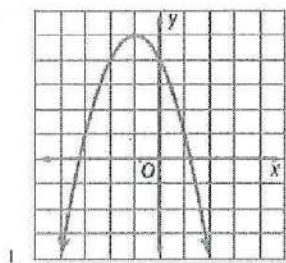
Part XXII: Transformations of a Quadratic Function

60. What is the transformation of the quadratic function based on the equation?

$y = 2(x + 4)^2 - 8$ *remember vertex form is $y = a(x - h)^2 + k$

vertical stretch
left 4
down 8

61. For both graphs identify the following



Find the vertex:

$(-1, 5)$

Find the Axis of Symmetry: $x = -1$

The domain: All real #'s

The range:

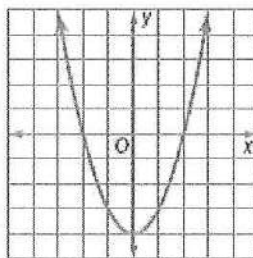
$y \leq 5$

When is the function increasing:

$x < -1$

When is the function decreasing:

$x > -1$



vertex: $(0, -4)$

A.O.S $x = 0$

All real #'s

$y \geq -4$

increasing: $x > 0$

decreasing: $x < 0$

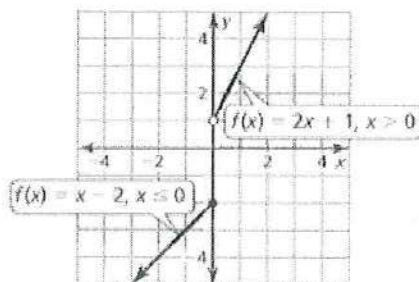
Part XXIII: Writing Piecewise Functions

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

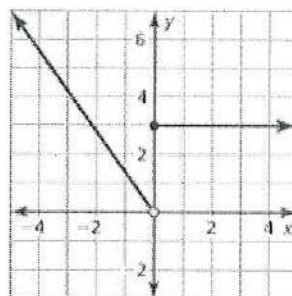
$$f(x) = \begin{cases} x - 2, & \text{if } x \leq 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.

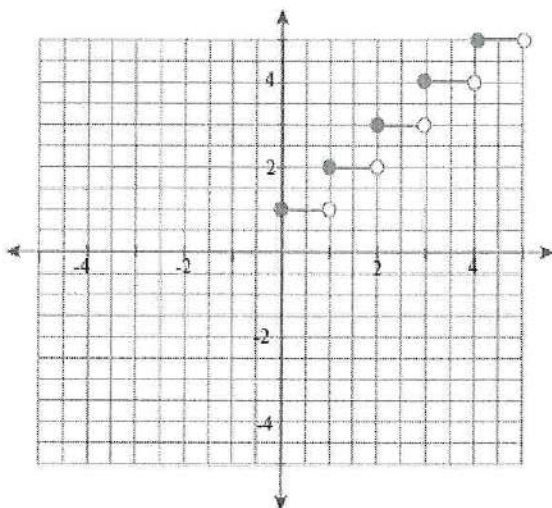


62. Write a Piecewise function for the following graph

$$f(x) = \begin{cases} 3, & x \geq 3 \\ -\frac{3}{2}x, & x < 0 \end{cases}$$

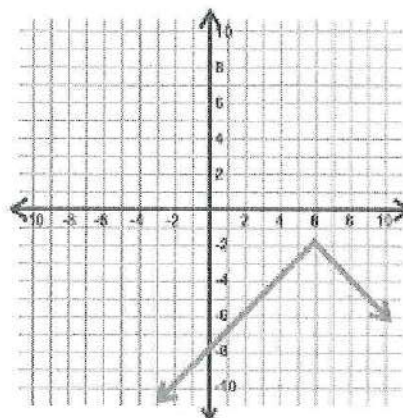


63.



$$f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 2, & 1 \leq x < 2 \\ 3, & 2 \leq x < 3 \\ 4, & 3 \leq x < 4 \\ 5, & 4 \leq x < 5 \end{cases}$$

64.



$$f(x) = \begin{cases} x - 8, & x \geq 6 \\ -x + 4, & x \leq 6 \end{cases}$$

Part XXIII: Solving Nonlinear Systems

Step 1: Eliminate a variable by multiplying one or both equations by a number that enables you to cancel out a variable when adding the two equations

Step 2: Combine any like-terms and get equation set equal to zero

Step 3: use the quadratic formula to solve

65. Solve the system by elimination.

$$y = x^2 - 3x - 2$$

Equation 1

$$-1(y = -3x - 8)$$

Equation 2

$$y = x^2 - 3x - 2$$

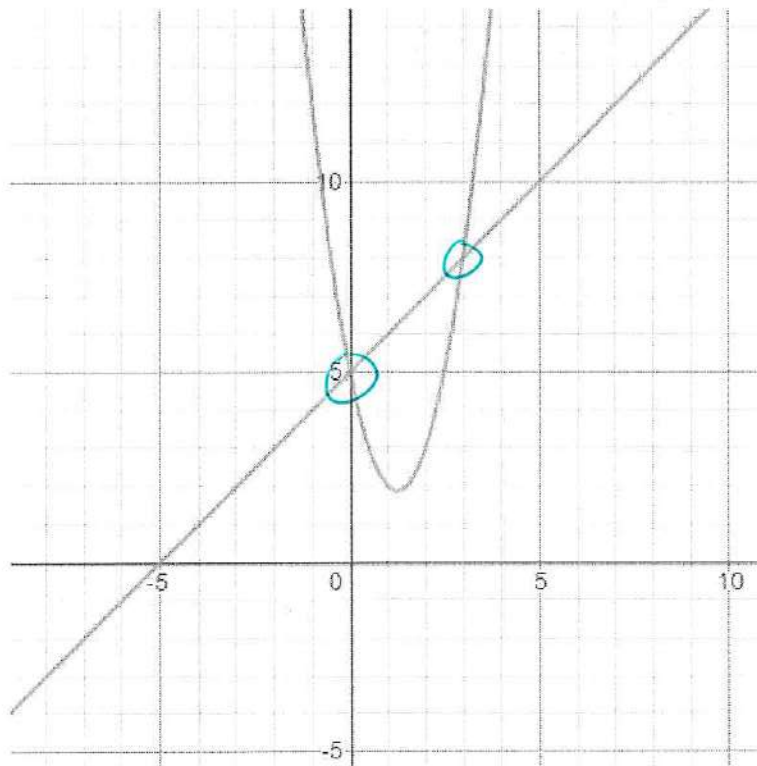
$$-y = -x^2 + 3x + 2$$

$$0 = x^2 + 6$$

$$x^2 = -6$$

No Solution.

66.



(0, 5) and (3, 8)