Course Title: Advanced Placement Calculus (BC) – Period 5

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Textbook: CALCULUS: Graphical, Numerical, Algebraic; Third Edition
Authored by: Finney, Demana, Waits, and Kennedy
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Course Objectives

- To expose students to the first two semesters of college calculus;
- To prepare students for advanced coursework in mathematics, science, technology, engineering, etc;
- To prepare students for the Advanced Placement Exam in May;
- To provide opportunities for students to integrate the use of modern technologies along with traditional methods of analysis in problem solving and discovery;
- To provide opportunities for students to work cooperatively in problem solving and discovery.

Course Goals

- Students should be able to work with functions represented in various ways: graphically, numerically, analytically, and/or verbally. They should also understand the, and be able to make, connections among these representations;
- Students should understand the meaning of the derivative both as a limit of a difference quotient, and as a rate of change; and should be able to use derivatives to solve problems;
- Students should understand the meaning of the definite integral both as a limit of Reimann sums, and as a net accumulation of change; and should be able to use definite integrals to solve problems;
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus;

- Students should be able to communicate mathematics both orally and in well-written sentences, and should be able to explain solutions to problems;

- Students should be able to model a written description of a physical situation with a function, a differential equation, or a definite integral;

- Students should be able to use technology to help: solve problems, explore situations, experiment with ideas, investigate phenomena, interpret results, and verify conclusions;

- Students should be able to use approximation methods to estimate solutions, and understand how approximation techniques are used to develop exact methods;

- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement;

- Students should develop an appreciation of calculus as a meaningful and coherent body of knowledge and as a human accomplishment.

Topical Outline

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. Hence the emphasis is on the interplay between geometric and analytic information and on the use of calculus both to predict and explain the observed local and global behavior of a function.

Limits of a function

* Calculating limits using algebra.
* Estimating limits from graphs or tables of data.
* Formal Definition of a Limit

Asymptotic and unbounded behavior.

* Understanding asymptotes in terms of graphical behavior.
* Describing asymptotic behavior in terms of limits involving infinity.
* Comparing relative magnitudes of functions and their rates of change.

Continuity as a property of functions. The central idea of continuity is that close values of the domain lead to close values of the range.

* Understanding continuity in terms of limits.
* Geometric understanding of graphs of continuous functions (IVT and EVT).

II. Derivatives

Concept of a derivative. The concept of the derivative is presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change.

* Derivative defined as a limit of the difference quotient.
* Relationship between differentiability and continuity.

Derivative at a point.

* Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
* Tangent line to a curve at a point and local linear approximation.
* Instantaneous rate of change as the limit of average rate of change.
* Approximate rate of change from graphs and tables of values.

Derivative of a function.

* Corresponding characteristics of graphs of **f** and **f'**.
* Relationship between the increasing /decreasing behavior of **f** and the sign of **f'**.
* The Mean Value Theorem (MVT) and its geometric consequences.
* Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

* Corresponding characteristics of the graphs of **f**, **f'**, and **f"**.
* Relationship between the concavity of f and the sign of **f"**.
* Points of inflection as places where the concavity changes.

Applications of derivatives.

* Analysis of curves, including the notions of monotonicity and concavity.
* Optimization, both absolute (global) and relative (local) extrema.
* Modeling rates of change, including related rates problems.
* Use of implicit differentiation to determine the derivative of an inverse function.
* Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
* Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives.

* Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.

* Basic rules for the derivative of sums, products, and quotients of functions.
* Chain rule and implicit differentiation.

III. Integrals

Reimann sums and properties of definite integrals.

* Concept of a Reimann sum over equal subdivisions.
* Computation of Reimann sums using left, right, and midpoint evaluations.
* Definite integral as a limit of Reimann sums.
* Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
  $f'(x)dx = f(b) - f(a)$.
* Basic properties of definite integrals.

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, social, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to yield accumulated change or using the method of setting up an approximating Reimann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross section, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus.

* Use of the Fundamental Theorem to evaluate definite integrals.
* Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

* Antiderivatives following directly from derivatives of basic functions.
* Antiderivatives by substitution of variables.

Applications of antidifferentiation.

* Determining specific antiderivatives using initial conditions.
* Solving separable differential equations and using them in modeling.

Numerical approximation to definite integrals. Use of Reimann sums and the Trapezoidal rule to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

Additional Topics in AP Calculus (BC)

Parametric, Vector, and Polar Functions

- Analysis of graphs
- Derivatives
- Definite Integrals and Applications
  - Area of a region bounded by a polar curve
  - Length of a curve

Techniques of antidifferentiation

- By parts
- Simple partial fractions
- Improper Integrals as limits of Definite Integrals

Applications of antidifferentiation

- Modeling/Solving separable differential equations
  - Exponential growth
  - Logistic growth

Numerical Solutions of Differential Equations – Euler's Method

Determining Limits for Indeterminate Forms – L'Hopital's Rule

Polynomial Approximations and Series

- Concept of Series
  - Sequence of partial sums
  - Convergence vs. Divergence

- Types of Series
  - Geometric Series
  - p-Series
  - Harmonics
  - Alternating Series

- Tests for Convergence of Series
  - Integral Test
  - Ratio Test
  - Limit Comparison Test

- Taylor Series centered at x = a
  - Polynomial Approximations (including sine and cosine)
  - Maclaurin Series (Taylor Series centered at x = 0)
  - Important Maclaurin Series: e^x, sin x, cos x, 1/(1 – x)
  - Calculus of Taylor Series
  - Functions defined by a Power Series
  - Radius and interval of Convergence
  - Lagrange error bound for Taylor Polynomials

Homework and Grading Policy

Homework is an integral part of any learning experience and it is necessary for students to complete the assignments on a regular basis in order to be successful. In addition to appropriate attention to homework, proper effort and attitude are essential aspects, as well. AP Calculus is a college level course and in an attempt to prepare students for a college level grading system, their quarter and final grades are based solely on the grades achieved on tests and quizzes. However, in order to acknowledge these aspects of a successful approach to calculus, the scale used reflects and incorporates credit for effort, attitude, and homework.

AVERAGE	GRADE
88 – 100	A
82 – 87	A-
76 – 81	B+
68 – 75	B
62 – 67	B-
56 – 61	C+
48 – 55	C
42 – 47	C-
36 – 41	D+
32 – 35	D
0 – 31	F

Advanced Placement Calculus (BC)

Welcome! I trust you made the most of your summer vacation. Whether your time was spent being productive and active or relaxing and catching up on some needed rest I hope you enjoyed the summer.

We are about to embark on a journey of exploration into the world of **higher mathematics**. And we will begin this journey with the calculus; in much the same manner that many mathematicians (including the **Great Ones** and yours truly) began their journey before you.

These are exciting times for the learning and teaching of calculus, a branch of mathematics that has remained virtually unchanged since its discovery/creation in the mid 17th Century through its first 100 years. “What could be exciting about something that hasn’t changed much in over two hundred years?”, you ask. Excellent question! Although the body of knowledge known as the calculus is still in tact, the way we learn and teach it has undergone a major change. With the advent of modern technologies and the forward looking/reaching educators around the world, we have an opportunity to blaze a new trail into the world of calculus and higher mathematics.

This course has been redesigned, reorganized, and rededicated in order to make calculus more accessible to and meaningful for high school students. And I will do my very best to challenge you in an environment that makes you look forward to each and every class the same way that I will.

So I invite you not to sit back, relax and enjoy the ride, but to sit up, roll up your sleeves, and jump right in. This will not be a spectator sport that you admiringly watch from the sidelines or the comfort of your living room. This is an adventure into a new world with a far different mind set that will require hard work, dedication, and most of all an appreciation for that which is not yet known or clear and an understanding of the power the unknown can unleash.

Best wishes for a Most Excellent Journey.

Your Guide,

Mr. “C”