A.P. CALCULUS (BC) NAME _____

Outline – Chapters 2A (Basic Derivatives) Date _____

Previously in Chapter 1 we determined the slope of a tangent line to a curve at a **point** as the limit of the slopes of secant lines using that **point** as a {stationary}endpoint. Later we derived a general formula for the slope of the tangent line for a curve at any point on the curve by letting $x = a$ instead of a particular value. We have investigated the relationship between slopes of lines and rates of change, and have seen how the instantaneous rate of change at a particular point can be determined as the limit of the average rates of change using that particular point as an endpoint. Sound familiar?

The study of [instantaneous] rates of change is called ***differential calculus***, and the formula used to determine the instantaneous rate of change of a function, ***f***, at any point is called ***the derivative***.

PART I. BASIC DIFFERENTIATION (Sections 2.1 – 2.3)

Section 2.1 – Derivative of a Function (3 – 4 days)

VIDEO (KHAN ACADEMY): DERIVATIVE AS A CONCEPT

1) Algebraic Analysis

- Review definition of slope of a curve (i.e. slope of the tangent to a curve)
- Definition of derivative of a function (**DoD**):

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

VIDEO (KHAN ACADEMY): FORMAL DEFINITION OF THE DERIVATIVE AS A LIMIT

- Definition of derivative of a function at a point $x = a$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

VIDEO (KHAN ACADEMY): FORMAL AND ALTERNATE FORM OF THE DERIVATIVE

- Applying all three forms of the definition (Examples 1 and 2 on pages 105/106)
- Notation (page 107) – please refrain from using the "y prime" notation for now

VIDEO (KHAN ACADEMY): WORKED EXAMPLE: DERIVATIVE AS A LIMIT
VIDEO (KHAN ACADEMY): WORKED EXAMPLE: DERIVATIVE FROM LIMIT EXPRESSION
VIDEO (KHAN ACADEMY): THE DERIVATIVE OF X^2 AT ANY POINT USING THE FORMAL DEFINITION
VIDEO (McTAN UNIVERSITY): DEFINITION OF THE DERIVATIVE 1 (POLYNOMIAL FUNCTIONS)

{**See Page 579 for a review of the Binomial Theorem – How to expand expressions of**

the form : **$(x + h)^n$, where n is any integer ≥ 0**}

Homework 2.1a: page 111 # 1, 3, 4, 7, 9, 10, 12, 17, 19, 20
You must use the Definition of the Derivative for these problems – no shortcuts!

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2) Graphical Analysis

- Relationship between graphs of f and f ′ (Examples 3 & 4, pages 107/108)

VIDEO (KHAN ACADEMY): DERIVATIVE AS SLOPE OF CURVE
VIDEO (KHAN ACADEMY): THE DERIVATIVE AND TANGENT LINE EQUATIONS

Homework 2.1b: page 111 # 13 – 16, 21, 22, 26, 27

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3) Numerical Analysis

- Determining the derivative from data

VIDEO (KHAN ACADEMY): ESTIMATING DERIVATIVES

4) One-sided Derivatives (Example 6 on page 110)

Homework 2.1d: page 111 # 31, 36 – 41, 44, 45

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Section 2.2 – Differentiability (2 – 3 days)

1) How f ′ (a) Might Fail to Exist (i.e. when a function is not differentiable at a point)

   a. Connection Between <u>Graphical Analysis</u> and <u>Algebraic Analysis</u>

- Corner (f is continuous; LHD and RHD both exist, but are not equal)
- Cusp (f is continuous; LHD and RHD approach opposite infinities)
- Vertical tangent (f is continuous; LHD and RHD both approach the same infinity)
- *Discontinuity (automatic disqualification; continuity is a required and necessasry condition for differentiability)*

VIDEO (KHAN ACADEMY): DIFFERENTIABILITY AT A POINT: GRAPHICAL

Homework 2.2a: page 120 # 1 – 16, 31, 35

CONTINUE →

2) Symmetric Difference Quotient vs. One-Sided Difference Quotient

- $\mathbf{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}}$

- Show that SDQ yields same derivative formula as regular DQ for $f(x) = x^3$.

3) Derivatives on a Calculator

a. Numerical Derivatives – numeric values of a derivative for a function at a specific point.
b. **NDeriv (MATH 8)** on the Graphing Calculator – cannot find symbolic derivatives
c. Parameters for: **nDeriv(f(x), x, a, h)**, h is the tolerance (default h = .001)
d. Example: Use **nDeriv** to determine f'(2) for $f(x) = x^3$; discuss value
e. Example: Use **nDeriv** for f(x) = abs(x) at x = 0; why does nDeriv = 0?
f. Using **nDeriv** to Graph a Derivative!!! **nDeriv(f(x), x, x)**

Homework 2.2b: page 120 # 21 (do #21 numerically and algebraically), 27

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4) Differentiability → Continuity

- Proof
- Discussion – use a piecewise function here

$$g(x) = \begin{cases} x^3 + 2, & x \leq 1 \\ x^2 + x, & x > 1 \end{cases}$$

VIDEO (KHAN ACADEMY): PROOF: DIFFERENTIABILITY IMPLIES CONTINUITY
VIDEO (KHAN ACADEMY): DIFFERENTIABILITY AT A POINT: ALGEBRAIC (FUNCTION IS DIFFERENTIABLE)
VIDEO (KHAN ACADEMY): DIFFERENTIABILITY AT A POINT: ALGEBRIAC (FUNCTION IS NOT DIFFERENTIABLE)

Homework 2.2c: page 120 # 39

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5) Intermediate Value Theorem for Derivatives

Homework 2.2d: page 114 # 40 – 42

Using the DoD, determine the derivatives for: x, x^2, x^3, x^4. {It helps if you know and use the Binomial Theorem}

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Section 2.3 – Rules for Differentiation – "The Shortcuts" (4 – 5 days)

1) Derivative of a Constant Function (y = k); Algebraically and Graphically
2) Power Rule (**for positive integer powers ONLY!!!**)
3) Constant Multiple Rule (proof)
4) Sum/Difference Rule

VIDEO (KHAN ACADEMY): PROOF OF THE POWER RULE FOR POSITIVE INTEGER POWERS (ONLY!!!)
VIDEO (KHAN ACADEMY): JUSTIFYING THE BASIC DERIVATIVE RULES
VIDEO (KHAN ACADEMY): BASIC DERIVATIVE RULES: TABLE

Homework 2.3a: page 129 # 1 – 6

Applications of the Rules for Derivatives

- Differentiating a Polynomial (Example 1 on page 124)
- Finding Horizontal Tangents (Example 2 on page 124)
- Using the GC and Calculus (Example 3 on page 124)

VIDEO (KHAN ACADEMY): DIFFERENTIATING POLYNOMIALS
VIDEO (KHAN ACADEMY): TANGENTS OF POLYNOMIALS

Homework 2.3b: page 129 # 7, 8, 10, 25, 37, 39, 40, 43a

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5) Product Rule

a. Have students come up with their own product rule
b. Now Consider an Example: $f(x) = 2x + 3$ and $g(x) = x - 2$
c. Proof of product rule
d. Example 4 (page 125)
e. Numeric Problem – Example 6 (page 127)

VIDEO (KHAN ACADEMY): PROVING THE PRODUCT RULE
VIDEO (KHAN ACADEMY): WORKED EXAMPLE: PRODUCT RULE with TABLE

Homework 2.3c: page 129 # 13, 16

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6) Quotient Rule

- State and Practice (leave denominator factored)
- Support results graphically (Example 5 on page 126)

VIDEO (KHAN ACADEMY): DIFFERENTIATING RATIONAL FUNCTIONS
VIDEO (KHAN ACADEMY): WORKED EXAMPLE: QUOTIENT RULE with TABLE

Homework 2.3d: page 129 # 17, 21, 23, 27

CONTINUE →

7) Power Rule for Negative Exponents – see Example 7 (page 127)

- Proof and Practice

VIDEO (KHAN ACADEMY): DIFFERENTIATING INTEGER POWERS (MIXED POSITIVE and NEGATIVE)

Homework 2.3e: page 129 # 29

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8) Higher Order Derivatives – see Example 8 (page 128)

Homework 2.3f: page 129 # 33, 47, 53 – 58
Quick Quiz for AP Preparation: page 132 # 2 – 4

Review Exercises for Test # 3 – Page 154: # 1, 2, 4, 5, 6, 25, 34, 43, 45, 51, 53, 57, 58, 59, 67