Baby Dice Island
Modeling Exponential Growth

OBJECTIVE
Students will use dice rolling to model unrestricted exponential growth in an imaginary population of baby dice.

LEVEL
Middle Grades: Life Science

NATIONAL STANDARDS
UCP.2, C.3, F.2

TEKS
6.2(B), 6.2(C), 6.2(D), 6.2(E), 6.4(B)
7.2(B), 7.2(C), 7.2(D), 7.2(E), 7.4(B)
8.2(B), 8.2(C), 8.2(D), 8.2(E), 8.4(B)
IPC 2(B), 2(C), 2(D), 4(A)

CONNECTIONS TO AP
AP Biology:
    III. Organisms and Populations: C. Ecology 1. Population dynamics 3. Global issues
AP Environmental Science:
    I. Interdependence of Earth's Systems: Fundamental Principles and Concepts E. The Biosphere
    2. populations and communities: exponential growth, carrying capacity
    II. Human Population Dynamics A. History and Global Distribution 1. numbers 2. demographics, such as birth rates and death rates B. Carrying Capacity — Local, Regional, Global

TIME FRAME
150 minutes or 3 class periods

MATERIALS
(For a class of 28 working in groups of 4)

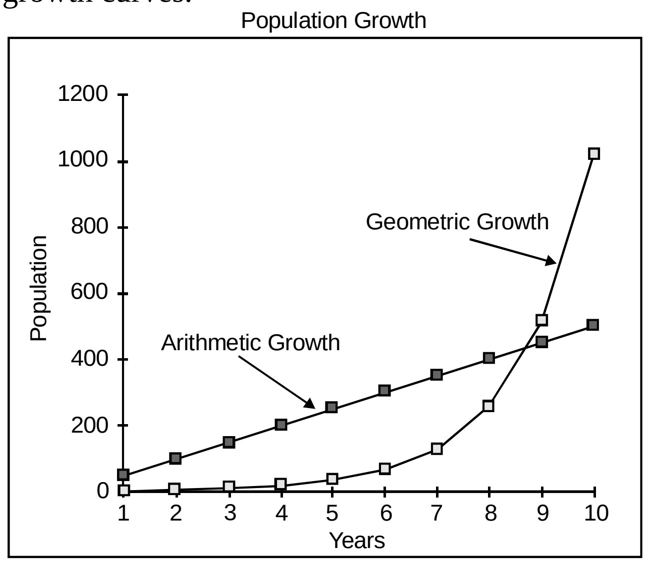
140 dice (20 dice per group of 4 students)
Plastic cup for rolling dice

TEACHER NOTES
Baby Dice Island: Modeling Exponential Growth is an excellent introduction to the concept of unrestricted exponential population growth. This lab can be used to introduce discussions about population growth, ecological issues, or biomes, and fits easily into an ecology unit.

Students model the growth of an imaginary population of individuals on a make believe island by using dice to model births and deaths through a series of years. Each group of students starts with an initial population of 20. If the students roll a "three" or a "six," this represents a birth within their population. If the students roll a "one," this represents a death within their population. After the 20 dice have been rolled, the total number of deaths (1's) rolled is subtracted from the total number of births (3's and 6's) rolled. This total is then added to the initial population for that year. The new sum is the final population for the current year and the initial population for the next year. For the next years data the students will need to roll the number of dice in the initial population for the new year. For example, if year one ended with a final population of 26, year two would have an initial population of 26. The students would roll the 20 dice, record the births and deaths, and then re-roll 6 dice. This will give them 26 dice rolled. They will again subtract the deaths from the births and add them to the initial population of 26. This new total is the final population for year two and the initial population for year three. The students will continue to roll the dice, modeling and recording the number of "births" and "deaths" of their population for each year until the final population reaches 500 individuals.

Depending on the background of your students you may need to review proper graphing techniques outlined in *Foundation Lesson IV—Graphing Skills*. In a graph representing growth, time is displayed on the *x*-axis and population numbers are displayed on the *y*-axis. The *x*-axis has units of years, hours, or seconds. The *y*-axis represents the population numbers from zero to some maximum population number and typically has units of "number of individuals". At each point in time on the graph, there is a corresponding value of population. A line of best fit may be drawn in order to use the graph for extrapolation of population data. The most common trend seen in a growth graph typically starts low and generally increases over time.

Growth can occur according to different mathematical patterns, including arithmetic growth and exponential (also called geometric) growth. Arithmetic growth, represented by a straight line, increases at a constant rate represented by the equation $y = mx + b$. Exponential growth is represented by a much more complex mathematical equation, $y=an^{bx}$, and is beyond the scope of most middle school/junior high school students. This type of growth does not increase at a constant rate. In nature, almost any organism provided with ideal conditions for growth and reproduction will experience a rapid increase in its population, much more so than is represented by the arithmetic growth pattern. More importantly, the larger the population gets, the faster it grows. If nothing stops or limits the population from growing, it will continue to increase at a faster and faster rate. This kind of growth pattern creates a graph with an exponential growth curve. Below is a diagram (Fig. 1) showing the differences between arithmetic and exponential growth curves.



TEACHER PAGES

Fig. 1

The lab is for groups of four students and the student jobs are as follows:

- *dice roller*: rolls dice
- *recorder*: records all final data per year in the data table
- *death tracker*: keeps track of number of deaths each year on tally sheet
- *birth tracker*: keeps track of number of births each year on tally sheet

Some assumptions built into the lab that should be explained to the students:

- the initial population of year one is 20 individuals (the number of dice that you are using)
- one-year-old dice on the imaginary island can have babies
- each individual die can have an offspring (sex of the die does not matter)

The major logistical concern for you to perform this lab with a class of students is finding enough dice. The lab is designed for students working in groups of four. If you have a class of 28 students, then 7 lab groups of materials will be needed. Each lab group ideally needs 20 dice but 15 or even 10 will produce adequate results. Keep in mind that using smaller numbers of dice will increase the time needed to reach a final population of 500.

One suggestion for managing classroom chaos during this activity is to instruct the students that for every die that hits the floor, 1 point will be deducted from the 100 point final grade. Another suggestion is to use lids from copy paper boxes to contain the dice while they are being rolled.

Students will be faced with the logistical challenge of tallying all the births and deaths per roll until the final population reaches 500. You should suggest that the death tracker and the birth tracker use a temporary tally sheet to track their "hits" and then transfer their data to the final data table once the year's worth of rolls are completed.

To illustrate how the tally sheets may be used, the following example is presented. If the students have reached a final population of 60 at the end of year 8, the initial population for year 9 would also be 60. With only 20 dice the students will have to roll the dice three different times to model the population change for those 60 individuals during year 9. The group would start by rolling the 20 dice one time. The birth tracker would count how many "3's" and "6's" were rolled and record this on their tally sheet. The death tracker would simultaneously keep track of the number of "1's" that were rolled and record this on their tally sheet. This procedure would need to be repeated 2 more times to account for the total of 60 individuals. At the end of the 3 rolls representing year 9 the total number of births and deaths can be transferred to the official data table.

Examples of typical tally sheets are presented below, indicating how many deaths and births occurred, in total, for year 9, with an initial population of 60.

Tally sheet for birth tracker				
Year	New births—1st roll (of 20 dice)	New births—2nd roll (of 20 dice)	New births—3rd roll (of 20 dice)	Total births for year
9	6	7	5	18

Tally sheet for death tracker				
Year	New deaths—1st roll (of 20 dice)	New deaths—2nd roll (of 20 dice)	New deaths—3rd roll (of 20 dice)	Total deaths for year
9	2	4	3	9

As the birth tracker and the death tracker keep tally of the births and deaths for each year, the recorder only transfers the number of the total births and deaths in a year to the final data table.

Once transferred to the final data table, the year 9 data will appear as follows:

Data Table for Modeling Exponential Growth Lab					
Year	Initial Population	Births	Deaths	Change	Final Population
9	60	18	9	+9	69

Students are instructed to continue rolling dice and adding years until a final population of at least 500 is reached. It typically takes anywhere from 16-23 years before a population of 500 is reached by the students.

The data table is to be written in ink to discourage students from changing/falsifying data. Students are to turn this into you each day after doing the lab. By doing this the recorder will not have a chance to lose the data table. You can then return the data table each day.

Calculating doubling time:
To find the doubling time of a population growing at a constant annual growth rate, divide the growth rate into 70. This represents number of years required for the population to double. For example, with a 10% annual growth rate, the doubling time of a population would be 70/10=7, or 7 years.

The math behind the rule of the "70" used in the doubling time equation is very complex and beyond the scope of this book. However, there is a detailed explanation at the "Exponential Growth and The Rule of 70" website found here:

http://www.ecofuture.org/pop/facts/exponential70.html

REFERENCES:

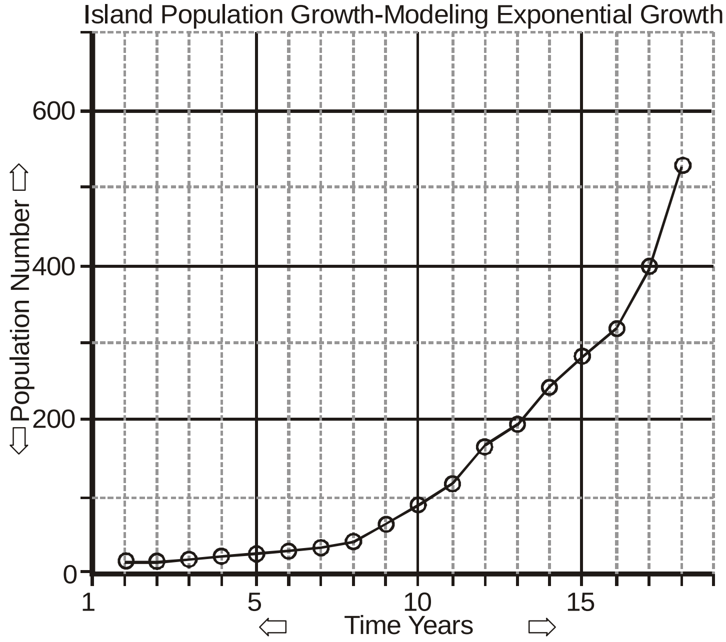
http://www.ecofuture.org/pop/facts/exponential70.html
http://www.balance.org/population.html
http://www.census.gov

SAMPLE DATA

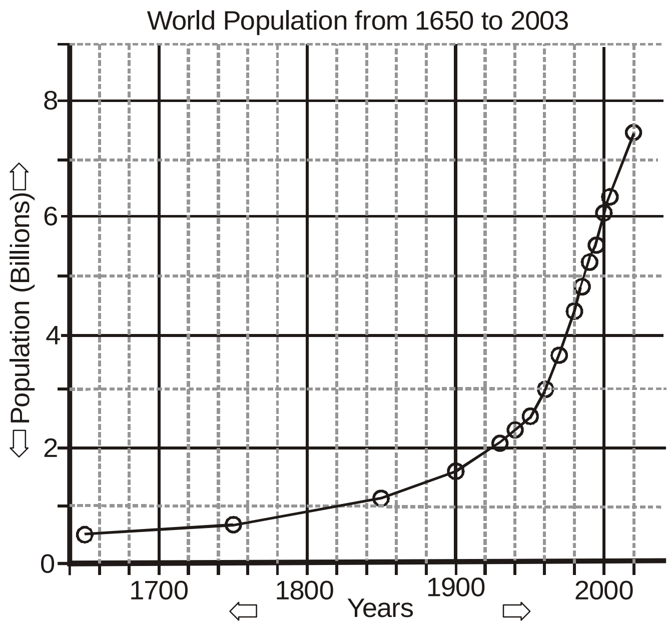
Data Table for Modeling Exponential Growth					
Year	Initial Population	Births	Death s	Change	Final Population
1	15	4	1	+3	18
2	18	4	4	+0	18
3	18	5	1	+4	22
4	22	4	7	-3	19
5	19	8	2	+6	25
6	25	6	7	-1	24
7	24	10	4	+6	30
8	30	9	3	+6	36
9	36	12	5	+7	43
10	43	25	4	+21	64
11	64	31	8	+23	87
12	87	26	15	+11	98
13	98	23	8	+15	113
14	113	38	24	+14	127
15	127	47	25	+22	149
16	149	45	24	+21	170
17	170	47	21	+26	196
18	196	76	26	+50	246
19	246	64	36	+28	274
20	274	110	27	+83	357
21	357	142	36	+106	463
22	463	156	55	+101	564

ANALYSIS: SAMPLE GRAPHS

Part I: Unrestricted Exponential Growth of the Baby Dice Island Population



Part II: World Population Trends



Use your graph to predict world population for the year 2020 (Hint: Use a dotted line to extend your graph into the future.)

- Answers will vary here according to each student's hand drawn graph. The current estimate, according to the United States Census Bureau (http://www.census.gov) website is that the population will be approximately 7.5 billion.

POSSIBLE ANSWERS TO THE CONCLUSION QUESTIONS

Part I: Unrestricted Exponential Growth of the Baby Dice Island Population

1. What is the ratio of births to deaths in this model population?
   - 2 to 1. When rolling the dice, there are two different ways to model a birth, rolling a 3 or a 6. To model a death, there is only one way, by rolling a 1.

2. How many "years" did it take you to reach a population of 100?
   - Answers will vary but typically it takes between 9-13 years for students to reach a population of 100.

3. After you reached a population of 100, how many more "years" did it take to reach a population of 200? How many **more** years to reach 300? 400? 500?
   - Again, answers will vary but because of the exponential growth, the amount of time it takes to increase by each increment of 100 individuals decreases. So its takes less years to get to 200 and even less time to get to 300 and so on.

4. Using this experiment, define exponential (or geometric) growth.
   - Answers will vary but exponential population growth occurs with slow growth at the start turning into faster and faster growth reaching an almost vertical spike.

5. In what way do exponential (or geometric growth rates) differ from arithmetic growth rates?
   - Arithmetic growth is represented by a straight line increasing to the right and exponential growth increases at a much faster rate and is represented by a curved line showing rapid increase.

Part III: Doubling Time of a Population

6. Calculate the growth rate for the U.S. (Show your work.)
   As of 2002, the population of the US was approximately 292,000,000. The increase in the US population is approximately 3,300,000 a year, from immigration and new births.
   - Growth rate=amount of change in a population/total population x 100
   - Growth rate(USA)=3,300,000/292,000,000 x 100
   - Growth rate (USA)= 1.13%

7. Calculate the doubling rate for the population of the U.S. (Show your work.)
   - To determine doubling time, divide 70 by the populations' growth rate.
   - Doubling time (USA)=70/growth rate
   - Doubling time (USA)=70/1.13%
   - Doubling time (USA)=61.95 years

TEACHER PAGES

Baby Dice Island
Modeling Exponential Growth

In this experiment you will roll dice to model population growth of the individuals on *Baby Dice Island*. Each die represents a living organism, capable of reproducing. You will start out with an initial population of 20. Every time you roll a “three” or a “six”, this represents the birth of an offspring, adding an individual to your initial population. Each time a “one” is rolled, a death has occurred, decreasing your initial population by one. After all the dice for the initial population have been rolled (representing one year) you will determine your final population for that year, by adding the numbers of births and subtracting the number of deaths from your initial population. You will be rolling dice over a series of “years”, adding births, and subtracting deaths from your initial population, until you finally reach a final population of 500 individuals.

Each member of the four person lab group should perform one of the following roles. The roles must be changed each day that this lab is performed.

- *dice roller*: rolls dice
- *recorder*: records all final data per year in the data table
- *death tracker*: keeps track of number of deaths each year on tally sheet
- *birth tracker*: keeps track of number of births each year on tally sheet

PURPOSE

In this activity you will use dice to model exponential growth of an imaginary population.

MATERIALS

20 dice
Plastic cup

PROCEDURE

1. *Dice roller*: Put all dice into the cup, shake the cup and carefully pour the dice onto the table.

   Death tracker: Remove and count all the “ones” that appear. A “one” represents a death and will be subtracted from your initial population. Record the number of deaths on a tally sheet, according to the instructions provided by your teacher.

   Birth tracker: Determine the number of “threes” and “sixes” that appear. This number corresponds to births and will be added to your initial population. Record the number of births on a tally sheet, according to the instructions provided by your teacher.

   Recorder: Use a ruler to carefully draw, in ink, a data table similar to the one below that can be used to keep track of all the accumulated deaths and births that occur. Draw the table in the space provided on your student answer page.

Data Table for Modeling Exponential Growth Lab					
Year	Initial Population	Births	Deaths	Change	Final Population
1	20				

2. Continue rolling and tallying the births and deaths until the total population exceeds 500 individuals.

3. Complete the analysis section and the conclusion questions on the student answer page.

Name ___________________________________

Period __________________________________

Baby Dice Island
Modeling Exponential Growth

DATA

Using a PEN and RULER draw a data table as described in step 5 of the procedure. The table should contain space for at least 25 years of data.

ANALYSIS

Part I: Unrestricted Exponential Growth of the Baby Dice Island Population

Use your data table to plot a graph demonstrating the unrestricted exponential growth of the dice population. Use "Years" for the *x*-axis and "Total Population" for the *y*-axis. Be sure to give your graph an appropriate title and to label the axes of the graph.

Part II: World Population Trends

1650 = 0.5 billion	1940 = 2.3 billion	1985 = 4.8 billion
1750 = 0.7 billion	1950 = 2.5 billion	1990 = 5.3 billion
1850 = 1.1 billion	1960 = 3.0 billion	1995 = 5.5 billion
1900 = 1.6 billion	1970 = 3.6 billion	2000 = 6.1 billion
1930 = 2.1 billion	1980 = 4.4 billion	2003 = 6.3 billion

Using the above information, plot a graph of world population versus time from 1650 to 2003 in the space below. Use your graph to predict world population for the year 2020. (Hint: Use dotted lines to extend your graph into the future.)

CONCLUSION QUESTIONS

Part I: Unrestricted Exponential Growth of the Baby Dice Island Population

1. What is the ratio of births to deaths in this model population?

2. How many "years" did it take you to reach a population of 100?

3. After you reached a population of 100, how many more "years" did it take to reach a population of 200? How many **more** years to reach 300? 400? 500?

4. Using this experiment, define exponential (or geometric) growth.

5. In what way do exponential (or geometric growth rates) differ from arithmetic growth rates?

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6. Calculate the growth rate for the U.S. (Show your work.)
   - As of 2002, the population of the U.S. was approximately 292,000,000. The increase in the U.S. population is approximately 3,300,000 a year, from immigration and new births. Growth rate=amount of change in a population/total population x 100.

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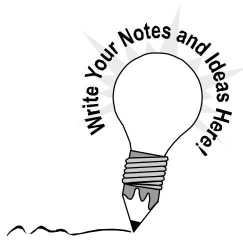
   - To determine doubling time, divide 70 by the populations' growth rate.


Write Your Notes and Ideas Here!