Robbinsville High School
Mathematics Department
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Dear Students,

Welcome to AP Calculus AB! Attached you will find a summer packet for math reinforcement for the upcoming school year. This packet should be completed and returned to school on the ***first full day of school***. I will give you the answer key on the first day of school. The packet will be **collected** and **graded** as **a 20-point homework grade** based on **completion** and **effort**. Work is required for many of these problems, so unsupported answers will not receive credit.

The packet itself is only a sampling of concepts and questions that are prerequisite for entering AP Calc. In addition to the packet, Khan Academy is a great place to review limits and assess your understanding. Any of the videos/lessons under the **"Limits and Continuity"** section will be helpful.

Y**our first test will be sometime during the second week of school** and will cover material from this packet, including important Precalc topics and limits.

If you have any questions while completing the packet, please feel free to email me over the summer at sawin@rvilleschools.org. I check my email every couple of weeks.

Have a great summer!
Ms. Sawin

PreCalc Review (Ch. 1)

NON-CALCULATOR PORTION

Composite Functions: use the given information to determine the missing function

1. $f(x) = (x+3)^2 \qquad g(x) = \frac{1}{x}$

a) $f(g(x)) =$ _____

b) $g(g(3)) =$ _____

2. $f(g(x)) = \sqrt{x^3 - 6} \qquad g(x) = x^2$

$f(x) =$ _____

3. Let y = 3 cos (4x - π) – 5. Fill in the information below.

Period _____ Amplitude _____

Horizontal Shifts _____

Vertical Shifts _____

4. Evaluate the following trig values without a calculator:

a) $\cos\frac{\pi}{2} =$ _____ b) $\tan\frac{3\pi}{4} =$ _____ c) $\sin\left(-\frac{\pi}{3}\right) =$ _____

d) $\sin\frac{3\pi}{2} =$ _____ e) $\cot\pi =$ _____ f) $\csc\frac{11\pi}{6} =$ _____

g) $\tan\frac{5\pi}{3} =$ _____ h) $\sec\frac{\pi}{4} =$ _____ i) $\tan\left(-\frac{\pi}{2}\right) =$ _____

5. Evaluate the following without a calculator. Give all answers in radians.

a) arccos(1) $\arccos(1)$ _____ b) $\arcsin(-1)$ arcsin(−1) _____ c) $\arctan\left(\frac{\sqrt{3}}{3}\right)$ $\arctan\left(\frac{\sqrt{3}}{3}\right)$ _____

d) $\arctan(-1)$ $\arctan(-1)$ _____ e) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ _____ f) $\arcsin\left(-\frac{1}{2}\right)$ $\arcsin\left(-\frac{1}{2}\right)$ _____

g) $\arctan(0)\arctan(0)$ _____ h) $\arccos\left(\frac{1}{2}\right)\arccos\left(\frac{1}{2}\right)$ _____ i) $\arccos\left(-\frac{\sqrt{3}}{2}\right)\arccos\left(-\frac{\sqrt{3}}{2}\right)$ _____

j) $\arcsin\left(\sin\frac{2\pi}{3}\right)$ _____ k) $\sin\left(\arcsin\left(-.25\right)\right)$ _____ l) $\arctan\left(\tan\left(-\frac{\pi}{6}\right)\right)$ _____

m) $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$ _____ n) $\cos\left(\arccos\left(-5\right)\right)$ _____ o) $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right)$ _____

CALCULATOR PORTION

7. Money is deposited into an account that earns 7% interest, compounded annually. You will not earn the interest until the calendar year is over. How long will it take for the balance to double? Show the work that leads to your answer.

8. Use the functions below to answer the following questions. Show all work or give an explanation. **You may use your calculator to check yourself; however, "I used the calculator" is not proper justification for an answer.**

$F(x) = 2x^2 + 1$ $g(x) = 3^x - 4$ $h(x) = -1 + \sqrt{1-x^2}$

a) Is h(x) even, odd, or neither? Justify your answer.

b) Find g(f(1))

c) Find the zero(s) of g(x).

d) Find the inverse of g(x).

e) Determine the domain and range of f(x)

f) Determine the domain and range of g(x)

g) Determine the domain and range of h(x)

Section 2.1: Rates of Change and Limits

Find the limits below. Be sure to show all work and give exact answers.

1) $\lim_{x->0} \frac{x^2}{x+5} =$

2) $\lim_{x->5} 6 =$

3) $\lim_{x->3} \frac{(x-4)^2}{x+3} =$

4) $\lim_{x->0} 3x\cos x =$

5) $\lim_{x->0} \frac{\sin 3x}{6x} =$

6) $\lim_{x->-2} \frac{x^2+x-2}{x^2+5x+6} =$

7) $\lim_{x->7} \frac{\sqrt{2x-5}-3}{x-7} =$

8) $\lim_{x->0} \frac{1/(x+5) - 1/5}{x} =$

9) $\lim_{x\to 0}$ (ln (cos(x))) =

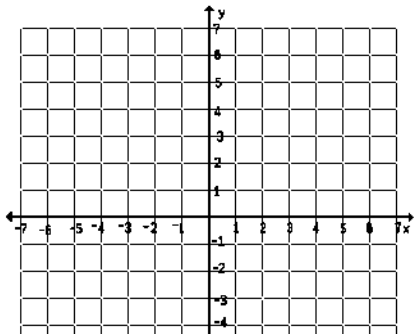
10) $\lim_{x\to \pi/2}$ (e^x sin(x)) =

11) $\lim_{x\to 1^-}$ int(x)

12) $\lim_{x\to 5} \frac{x^2-7x+10}{x-5} =$

13) Using the piecewise function, graph and then find the limits:

$$f(x) = \begin{cases} x-2 & x<1 \\ 3 & x=1 \\ 5-x & x>1 \end{cases}$$



a) $\lim_{x->1^+} f(x) =$ b) $\lim_{x->1^-} f(x) =$

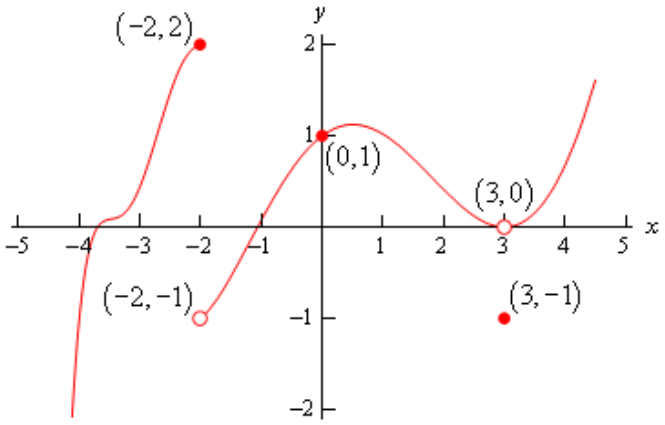
c) $\lim_{x->1} f(x) =$ d) $f(1) =$

14) Use the limits $\lim_{x->3} f(x) = 5$ *and* $\lim_{x->3} g(x) = -2$ to answer the following:

a) $\lim_{x->3} f(x) + g(x) =$ b) $\lim_{x->3} f(x) \cdot g(x) =$

c) $\lim_{x->3} 3f(x) - g(x) =$ d) $\lim_{x->3} \frac{f(x) - 5}{g(x)} =$

15) Use the following diagram to answer the questions:



a) f(3) =

b) $\lim_{x\to 3^-} f(x) =$

c) $\lim_{x\to 3^+} f(x) =$

d) $\lim_{x\to 3} f(x) =$

e) f(-2) =

f) $\lim_{x\to -2^-} f(x) =$ g) $\lim_{x\to -2^+} f(x) =$

h) $\lim_{x\to -2} f(x) =$ i) f(0) =

j) $\lim_{x\to 0^-} f(x) =$ k) $\lim_{x\to 0^+} f(x) =$

l) $\lim_{x\to 0} f(x) =$ m) $\lim_{x\to \infty} f(x) =$

n) $\lim_{x\to -\infty} f(x) =$

16) Given the information below, sketch a possible graph of *f(x)*.

a) $f(x)=0 \ \ at \ \ x=2$
$\lim_{x\to 4} f(x) = DNE$
$crosses\ y-axis\ at\ y=-1$
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} = -2$
$\lim_{x\to 4^+} = -\infty$
$f(x)\,\text{DNE}\ at\ x=-1\ and\ x=4$

b) $\lim_{x\to 2} f(x) = -1$
$\lim_{x\to 4^+} f(x) = -\infty$
$\lim_{x\to 4^-} f(x) = \infty$
$\lim_{x\to\infty} f(x) = \infty$
$\lim_{x\to-\infty} f(x) = 2$

Section 2.2: Limits Involving Infinity

Find the limits below.

1) $\lim_{x\to\infty} \ln x =$

2) $\lim_{x\to-\infty} e^{-x} =$

3) $\lim_{x\to\infty} \dfrac{4x^4-5x^3}{7x^4+9x^3} =$

4) $\lim_{x\to\infty} \dfrac{3x^3-x+1}{x+3} =$

5) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2-2}}{x^2+6} =$

6) $\lim_{x\to-\infty} \dfrac{1-7x^2}{x+5} =$

7) $\lim_{x\to 0} \dfrac{\sin x}{5x}$

8) $\lim_{x\to 0} \dfrac{3(1-\cos x)}{x}$

9) $\lim_{x\to 0} \dfrac{\cos x \tan x}{x}$

10) $\lim_{x\to 2} \dfrac{3x^2-7x+2}{x^2+5x-14}$

11) $\lim_{x\to 0} \dfrac{x^2-4}{x+2}$

12) $\lim_{x\to\infty} \dfrac{2x+\sin x}{x}$

13) $\lim_{x\to\infty} \dfrac{x-6}{x^2+2x-48}$

14) $\lim_{x\to-\infty} \dfrac{x^3+6x}{\sqrt{x^2+5}}$

15) $\lim_{x\to-4^+} \dfrac{1}{x+4}$

16) $\lim_{x\to-\infty} \sqrt[3]{\dfrac{8+x^2}{8x(x+1)}}$

17) $\lim_{x\to4} \dfrac{\sqrt{x^2+9}+-5}{x-4}$

18) $\lim_{x\to0} \dfrac{\tan x}{x}$

Section 2.3: Continuity

1) Find all points of discontinuity of the functions below and state the type of discontinuity. If the function has no points of discontinuity, then specify over what intervals it is continuous.

a) $f(x)=\dfrac{x+1}{x^2-4}$

b) $f(x)=\dfrac{x^2-8x+15}{x^2-25}$

c) $f(x)=3x+9$

d) $f(x)=\sqrt{2x-7}$

e) $f(x)=\dfrac{8-2x}{x^2-16}$

f) $f(x)=\dfrac{x}{|x|-3}$

2) At what x-coordinate on $f(x)=\dfrac{x^2-x-6}{x^2-9}$ is there a removable discontinuity?

3) Find a value for *a* so that function is continuous.

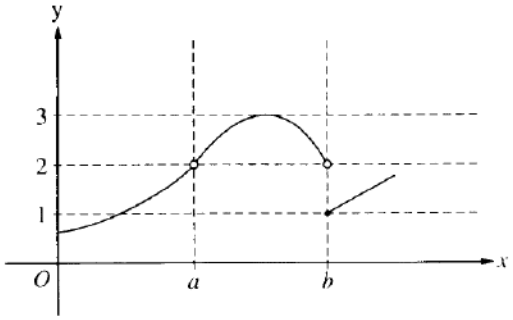
a) $f(x)=\begin{cases}4-x^2, & x<-1\\ ax^2-1, & x\geq -1\end{cases}$

b) $f(x)=\begin{cases}x^2+x+a, & x<1\\ x^3, & x\geq 1\end{cases}$

Mixed Review

_____ 1. Given the function $f(x)=\begin{cases}2x^2+3, & x\le 3\\ 3x+B, & x>3\end{cases}$, find a value of B such that $f(x)$ is continuous for all x.

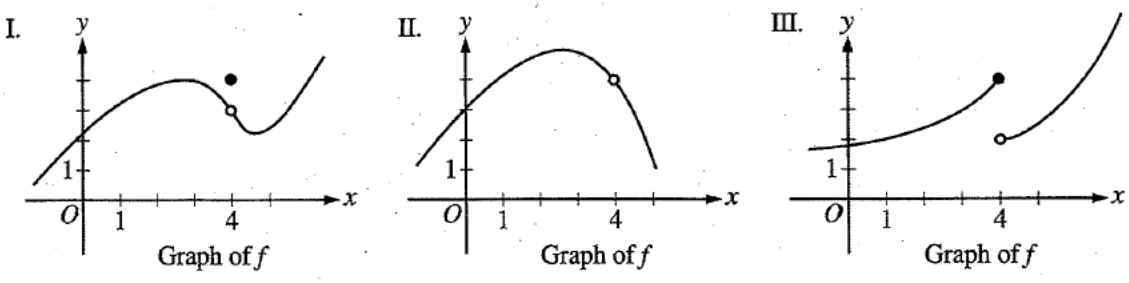
(A) 6 (B) 9 (C) 12 (D) 15 (E) There is no such value of B



_____ 2. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x)=\lim_{x\to b} f(x)$ (B) $\lim_{x\to a} f(x)=2$ (C) $\lim_{x\to b} f(x)=2$

(D) $\lim_{x\to b} f(x)=1$ (E) $\lim_{x\to a} f(x)=$ does not exist.



_____ 3. Using the graphs above, for which of the following does $\lim_{x\to 4} f(x)$ exist?

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

_____ 4. If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote at $y=2$ and an infinite discontinuity at $x=-3$, then $a+c=$

(A) -5
(B) -1

(C) 0
(D) 1
(E) 5

5. Evaluate each limit:

a) $\lim_{x\to\infty}\frac{4x^2+50}{x^3-85}$ a) _____

b) $\lim_{x\to 2}\frac{x^2-4x+4}{x^2+x-6}$ b) _____

c) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$ c) _____

d) $\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}$ d) _____

e) $\lim_{x\to 5}\sin\left(\frac{\pi x}{6}\right)$ e) _____

f) $\lim_{x\to\infty}\frac{x+5x^4}{-2x^4+3^{-x}}$ f) _____

g) $\lim_{x\to-\infty}\frac{x}{|x|}$ g) _____

h) $\lim_{x\to\infty}\frac{x}{|x|}$ h) _____

i) $\lim_{x\to\infty}\frac{\sqrt{3x^4+x}}{x^2-8}$ i) _____

j) $\lim_{x\to\infty}\left(\frac{1}{x}-\frac{x}{x-1}\right)$ j) _____