Use calculator to evaluate integrals in #2.

Finding Arc Length
AP Calculus BC

Name: Answers

1) Find the arc length of each curve below on the given interval of x.

a) $y = 9 - 3x$ on the interval $[1, 3]$ $\quad \text{arc} = \int_a^b \sqrt{1-\left(\frac{dy}{dx}\right)^2}\,dx$

$$\text{arc length} = \int_1^3 \sqrt{1+(-3)^2}\,dx$$

$$= \int_1^3 \sqrt{10}\,dx = \left[\sqrt{10}\,x\right]_1^3 = 3\sqrt{10} - \sqrt{10} = \boxed{2\sqrt{10}}$$

b) $f(x) = x^{\frac{3}{2}}$ on $[1, 2]$

$$\text{arc} = \int_1^2 \sqrt{1+\left(\tfrac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx$$

$$= \int_1^2 \left(1+\tfrac{9}{4}x\right)^{1/2} dx \approx \boxed{2.086}$$

$$= \frac{4}{9}\cdot\frac{2}{3}\left[\left(1+\tfrac{9}{4}x\right)^{3/2}\right]_1^2 = \frac{8}{27}\left(\left(1+\tfrac{18}{4}\right)^{3/2} - \left(1+\tfrac{9}{4}\right)^{3/2}\right)$$

$$= \boxed{\frac{1}{27}\left(22\sqrt{22} - 13\sqrt{13}\right)}$$

2) Find the arc length of each curve below (each described by parametric equations) on the given t-interval.

a) $x = t - t^2$, $y = 2t^{\frac{3}{2}}$ on $[0, 2]$

$$\text{arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\text{arc length} = \int_0^2 \sqrt{(1-2t)^2 + \left(3t^{\frac{1}{2}}\right)^2}\,dt$$

$$\approx 6.292$$

b) $x = lnt,\ y = e^t$ on $[1, e]$

$$\text{arc} = \int_1^e \sqrt{\left(\frac{1}{t}\right)^2 + \left(e^t\right)^2}\,dt$$

$$= \int_1^e \sqrt{\frac{1}{t^2} + e^{2t}}\,dt$$

$$\approx \boxed{12.507}$$

c) $x = 2t^{\frac{3}{2}}$, $y = 1 + (8-t)^{\frac{3}{2}}$ on $[0, 4]$

$$\text{arc} = \int_0^4 \sqrt{\left(3t^{\frac{1}{2}}\right)^2 + \left(\tfrac{3}{2}(8-t)^{\frac{1}{2}}\cdot -1\right)^2}\,dt$$

$$= \int_0^4 \sqrt{9t + \tfrac{9}{4}(8-t)}\,dt = \int_0^4 \sqrt{\tfrac{27}{4}t + 18}\,dt$$

$$= \frac{4}{27}\cdot\frac{2}{3}\left(\tfrac{27}{4}t+18\right)^{\frac{3}{2}}\Big]_0^4 = \boxed{\frac{1}{3}\left(20^{3/2} - 8^{3/2}\right) \approx 22.272}$$

d) $x = 2tsint,\ y = 3cost$ on $[0, 2\pi]$
(Calculator is needed.)

$$\int_0^{2\pi} \sqrt{\left(2\sin t + 2t\cos t\right)^2 + \left(-3\sin t\right)^2}\,dt$$

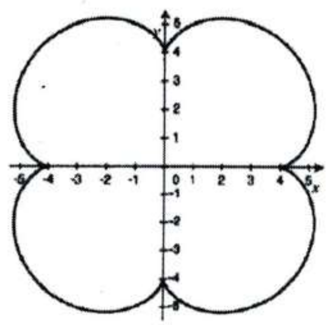
product rule!

$$\approx \boxed{31.362}$$

3) Show that the parametric equation $x = r\cos\theta, y = r\sin\theta$ is a circle. Then demonstrate that the circumference of a circle of radius r is $2\pi r$

This is fun! You'll know if you did it correctly. This may be a daily checkeroo question!

4) A circle of radius 1 rolls around the circumference of a larger circle of radius 5. The epicycloid traced by a point on the circumference of the smaller circle is given by $x = 5\sin t - \sin 5t$, $y = 5\cos t - \cos 5t$ as shown by the figure below. Find the arc length of the epicycloid.



5) A bicycle race-course is in the shape of a spiral whose parametric equations are given by $x = \frac{t}{\pi}\cos t$, $y = \frac{t}{\pi}\sin t$, where x and y are measured in miles, as shown below. The race starts at the origin, does 3 spiral revolutions, and then goes back to the start. What is the distance the bikers ride?