

AP Physics C: Mechanics

Name: _____

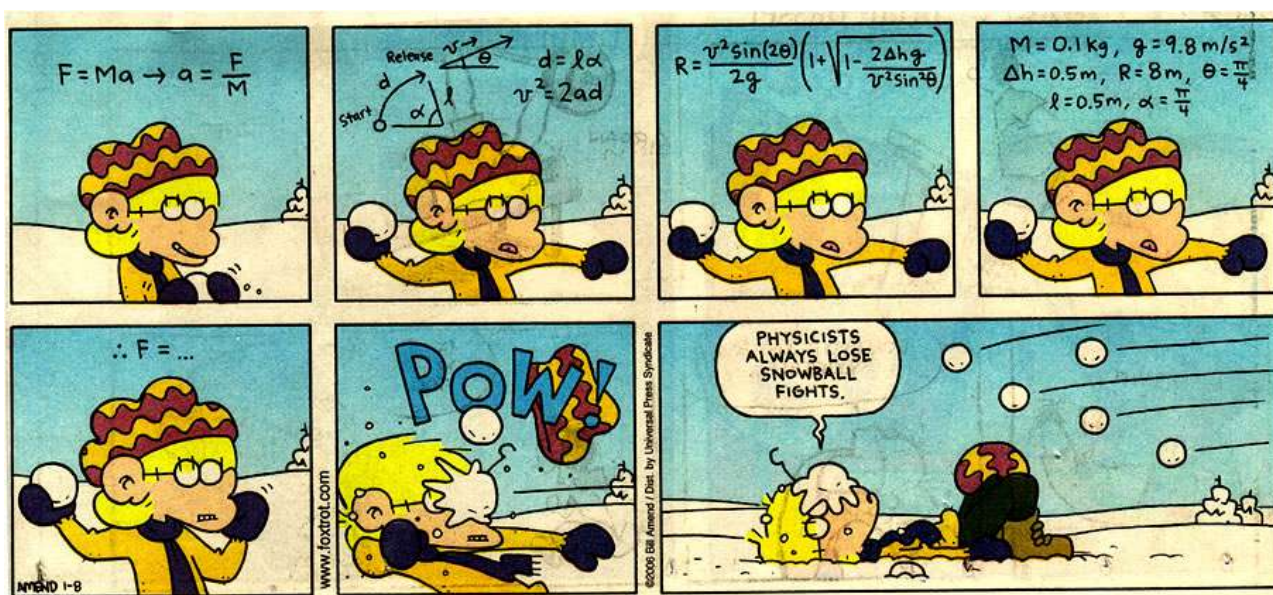
AP Physics C: Mechanics requires an exceptional proficiency in algebra, trigonometry, and geometry. In addition to the science concepts, AP Physics C: Mechanics often seems like a course in applied mathematics. The following assignment includes mathematical problems that are considered routine in AP Physics C: Mechanics. This includes knowing several key metric system conversion factors and how to employ them. Another key area in AP Physics C: Mechanics is understanding vectors. The purpose of this summer assignment is to fight your “couch inertia” and get you on the right path to success in the fall.

The attached pages contain a brief review, hints, and example problems. It is hoped that combined with your previous math knowledge this assignment is merely a review and a means to brush up before school begins in the fall. Please read the text and instructions throughout. If you use any extra paper for scratch work, please attach it to the packet before you turn it in.

The completed packet is due the first day of class in August.

Q: *What if I don't get all the problems or don't understand the instructions?*

A: *Simply do the best you can, but show some work or effort in order to receive credit.*



1. The following are ordinary physics problems. Place the answer in scientific notation when appropriate and simplify the units. Scientific notation is used when it takes less time to write than the ordinary number does. As an example 200 is easier to write than 2.00×10^2 , but 2.00×10^8 is easier to write than 200,000,000. Do your best to cancel units, and attempt to show the simplified units in the final answer.

a. $T = 2\pi \sqrt{\frac{4.5 \times 10^{-2} \text{ kg}}{2.0 \times 10^3 \text{ kg/s}^2}}$ $T = \underline{\hspace{2cm}}$

b. $K = \frac{1}{2} (6.6 \times 10^2 \text{ kg}) (2.11 \times 10^4 \text{ m/s})^2$ $K = \underline{\hspace{2cm}}$

c. $F = (9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(3.2 \times 10^{-9} \text{ C})(9.6 \times 10^{-9} \text{ C})}{(0.32 \text{ m})^2}$ $F = \underline{\hspace{2cm}}$

d. $\frac{1}{R_p} = \frac{1}{4.5 \times 10^2 \Omega} + \frac{1}{9.4 \times 10^2 \Omega}$ $R_p = \underline{\hspace{2cm}}$

e. $\eta = \frac{1.7 \times 10^3 \text{ J} - 3.3 \times 10^2 \text{ J}}{1.7 \times 10^3 \text{ J}}$ $\eta = \underline{\hspace{2cm}}$

f. $1.33 \sin 25.0^\circ = 1.50 \sin \theta$ $\theta = \underline{\hspace{2cm}}$

g. $K_{\max} = (6.63 \times 10^{-34} \text{ J/s})(7.09 \times 10^{14} \text{ s}) - 2.17 \times 10^{-19} \text{ J}$ $K = \underline{\hspace{2cm}}$

2. Often problems on the AP exam are done with variables only. Solve for the variable indicated. Don't let the different letters confuse you. Manipulate them algebraically as though they were numbers.

a. $U = \frac{1}{2} kx^2$ $x =$ _____ d. $mgh = \frac{1}{2} mv^2$ $v =$ _____

b. $T = 2\pi \sqrt{\frac{L}{g}}$ $g =$ _____ e. $x = x_0 + v_0 t + \frac{1}{2} at^2$ $t =$ _____

c. $F_g = G \frac{m_1 m_2}{r^2}$ $r =$ _____ f. $\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{m}$ $v =$ _____

g. $T - F_f - m_1 g \sin \theta = m_1 a$
 $m_2 g - T = m_2 a$

T may not appear
in the answer for
 a , nor a in the
answer for T $a =$ _____
 $T =$ _____

h. $v = \frac{2\pi r}{T}$ Solve for T by eliminating v
 $\frac{GM}{r^2} = \frac{v^2}{r}$
 $T =$ _____

3. Sometimes it is easier to use your calculator to solve an equation rather than algebra. To do this, graph each side of the = sign as a different function. Then use your calculator to find the point(s) where the graphs intersect.

a. $\sin \theta + \cos^2 \theta = 2\theta$ $\theta =$ _____

b. $\sin \theta \cos \theta + \sin \theta = 1$ $\theta =$ _____

4. Physics uses the International System of Units (abbreviated **SI** from French: *Le Système International d'Unités*) with the **MKS** base unit system. MKS stands for meter, kilogram, and second. These are the units of choice of physics, and all other units are derived from these three. The equations in physics depend on unit agreement so you must convert to MKS in most problems to arrive at the correct answer. Common conversions are:

- kilometers (km) to meters (m) and meters to kilometers
- centimeters (cm) to meters (m) and meters to centimeters
- millimeters (mm) to meters (m) and meters to millimeters
- micrometers (μm) to meters (m) and meters to micrometers
- nanometers (nm) to meters (m) and meters to nanometers
- liters (L) to cubic meters (m^3) and cubic meters to liters
- grams (g) to kilograms (kg) and kilograms to grams
- Celsius ($^{\circ}\text{C}$) to Kelvin (K) and Kelvin to Celsius
- atmospheres (atm) to Pascals (Pa) and Pascals to atmospheres

Other conversions will be taught as they become necessary.

What if you don't know the conversion factors? Colleges want students who can find their own information. Try a good dictionary and look under "measure" or "measurement". Or the Internet? Enjoy.

a. 4008 g = _____ kg

h. 25.0 μm = _____ m

b. 1.2 km = _____ m

i. 2.65 mm = _____ m

c. 823 nm = _____ m

j. 8.23 m = _____ km

d. 298 K = _____ $^{\circ}\text{C}$

k. 5.4 L = _____ m^3

e. 0.77 m = _____ cm

l. 40.0 cm = _____ m

f. 8.8×10^{-8} m = _____ mm

m. 6×10^{-7} m = _____ nm

g. 1.2 atm = _____ Pa

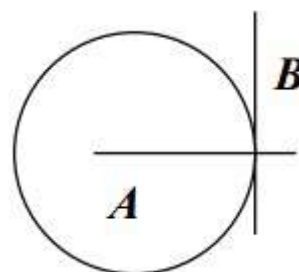
n. 1.5×10^{11} m = _____ km

5. Solve the following geometric problems.

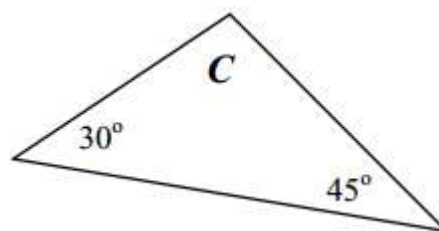
a. Line **B** touches the circle at a single point. Line **A** extends through the center of the circle.

i. What is line **B** in reference to the circle?

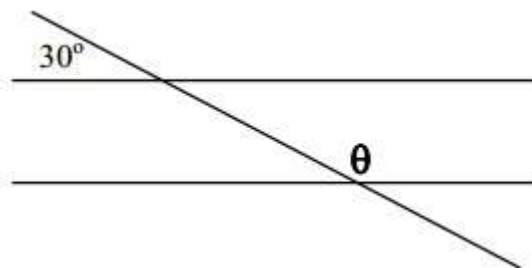
ii. How large is the angle between lines **A** and **B**?



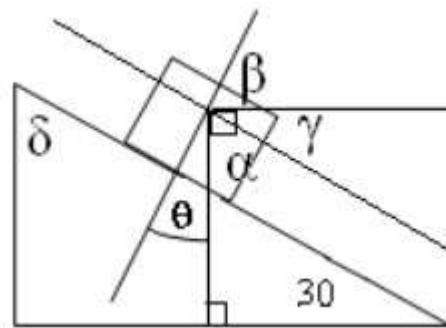
b. What is angle **C**?



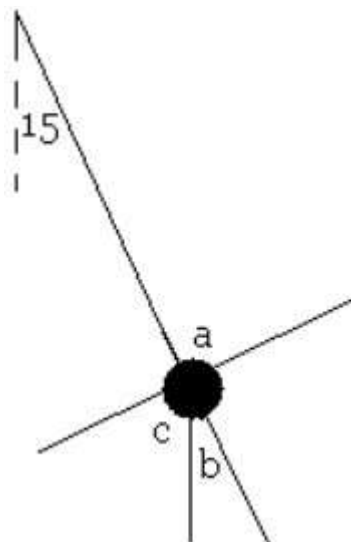
c. What is angle θ ?



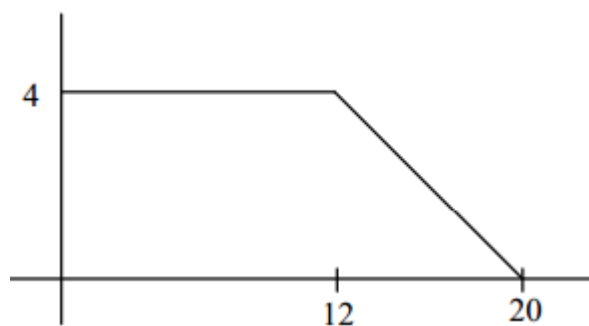
d. How large are angles α , β , γ , δ , and θ ?



e. How large are angles **a**, **b**, and **c**?



f. What is the area under the curve at right?

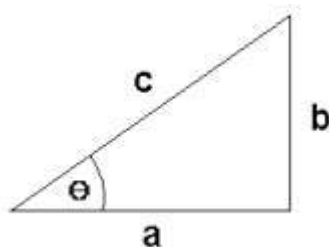


g. The radius of a circle is 5.5 cm

i. What is the circumference in meters?

ii. What is its area in square meters?

6. Use the generic triangle shown below, right-triangle trigonometry, and the Pythagorean theorem to solve the following problems. Your calculator must be set to degree mode.



a. $\theta = 55^\circ$ and $c = 32$ m, solve for a and b .

b. $\theta = 45^\circ$ and $a = 15$ m/s, solve for b and c .

c. $b = 17.8$ m and $\theta = 65^\circ$, solve for a and c .

d. $a = 250$ m and $b = 180$ m, solve for θ and c .

e. $a = 25$ cm and $c = 32$ cm, solve for b and θ .

VECTORS

Most of the quantities in physics are vectors. This makes proficiency in vectors extremely important.

Scalar

A physical quantity described by a single number and units. A quantity described by magnitude only. (Examples: time, mass, and temperature)

Vector

A physical quantity with both a magnitude and a direction. A directional quantity. (Examples: velocity, acceleration, force)

Magnitude

Size or extent. The numerical value of the physical quantity represented by the vector.

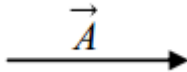
Direction

The alignment or orientation of the vector with respect to a coordinate system.

Notation

In your textbook, vector quantities are always indicated by bold type, like this: **A**
In your notes simply put an arrow over the letter representing the vector, as shown below.

Vectors are drawn as arrows:



The length of the arrow is proportional to the vectors magnitude.

The direction the arrow points is the direction of the vector.

Negative Vectors

Negative vectors have the same magnitude as their positive counterpart. They are just pointing in the opposite direction.



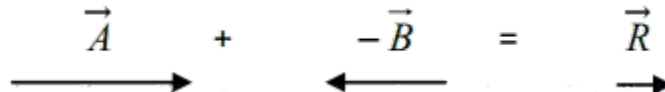
Vector Addition and Subtraction

Think of it as vector addition only. The result of adding vectors is called the resultant **R**.

$$\vec{A} + \vec{B} = \vec{R}$$


If **A** has a magnitude of 3 and **B** has a magnitude of 2, then **R** has a magnitude of $3 + 2 = 5$.

When you need to subtract one vector from another, think of the one being subtracted as being a negative vector. Then add them : **A - B = R** is really **A + -B = R**

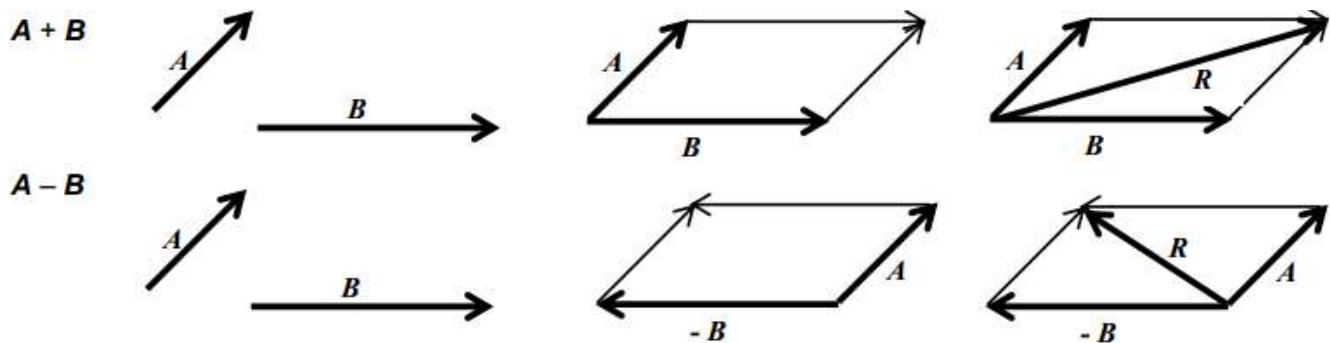
$$\vec{A} + (-\vec{B}) = \vec{R}$$


A negative vector has the same length as its positive counterpart, but its direction is reversed. If **A** has a magnitude of 3 and **B** has a magnitude of 2, then **R** has a magnitude of $3 + (-2) = 1$.

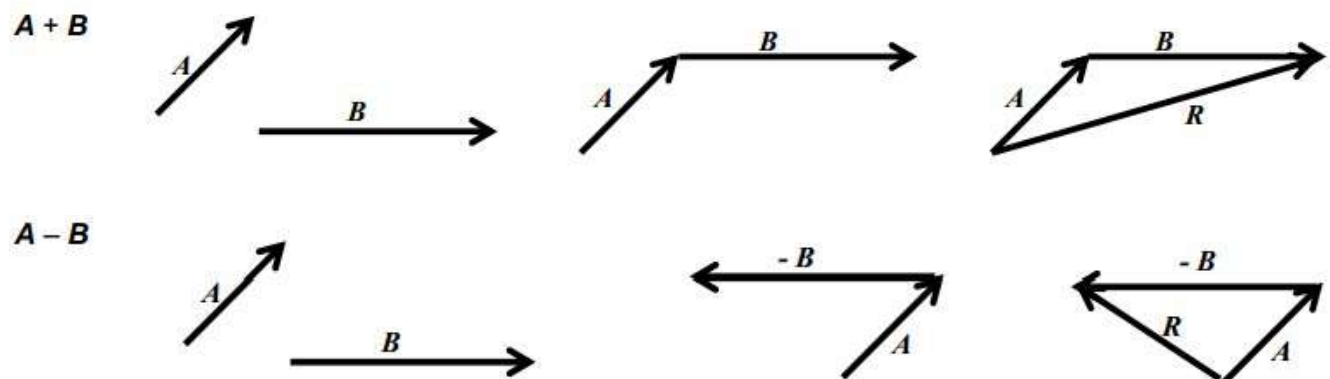
This is VERY important: in physics a negative number does *not* always mean a smaller number. In math class, -2 is smaller than +2, but in physics these numbers have the exact same magnitude, they just point in different directions (180° apart).

There are two methods of adding vectors.

Parallelogram Method



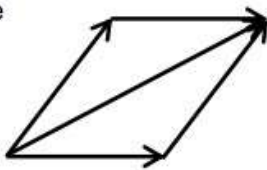
Tip-To-Tail Method



It should be readily apparent that both methods arrive at the exact same solution since each method is essentially a parallelogram. It is useful to understand both systems. In some problems one method is advantageous, while in other problems the alternative method is superior.

7. Draw the resultant vector using the parallelogram method of vector addition

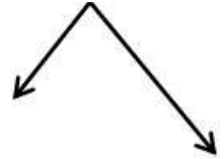
Example



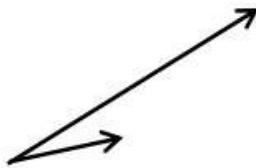
b.



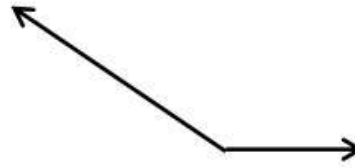
d.



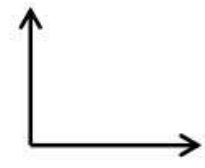
a.



c.

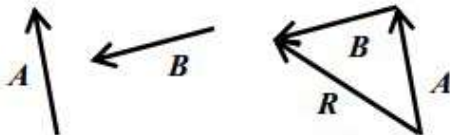


e.

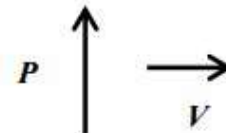


8. Draw the resultant vector using the tip to tail method of vector addition. Label the resultant as vector **R**.

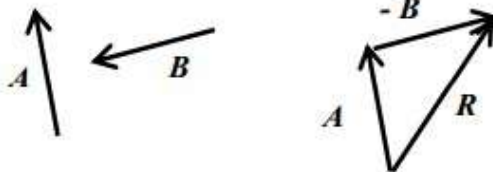
Example 1: $A + B$



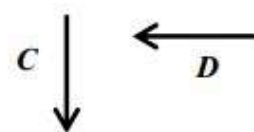
c. $P + V$



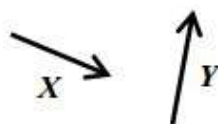
Example 2: $A - B$



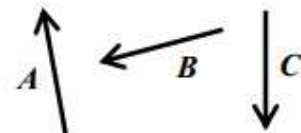
d. $C - D$



a. $X + Y$



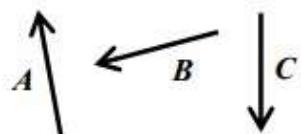
e. $A + B + C$



b. $T - S$

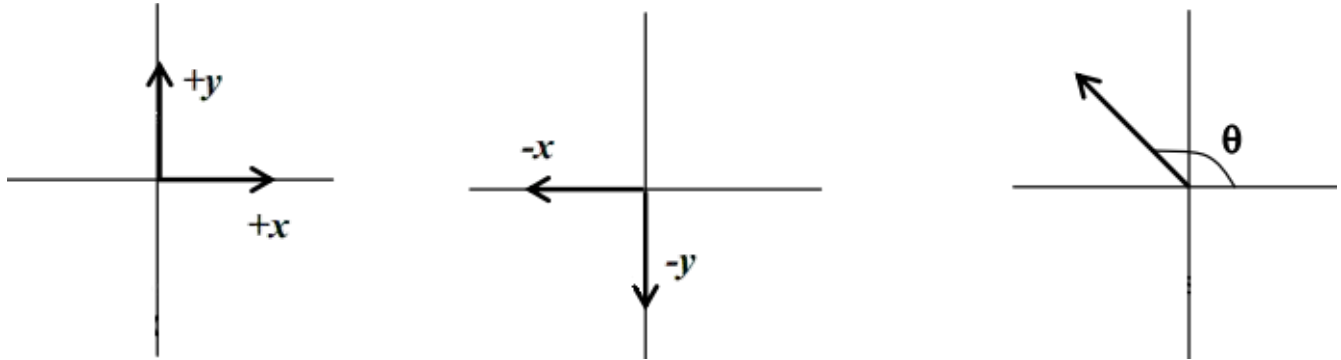


f. $A - B - C$



Direction

What does positive or negative direction mean? How is it referenced? The answer is the coordinate axis system. In physics a coordinate axis system is used to give a problem a frame of reference. Positive direction is a vector moving in the positive x, positive y, or positive z direction, while a negative vector moves in the negative x, negative y, or negative z direction.

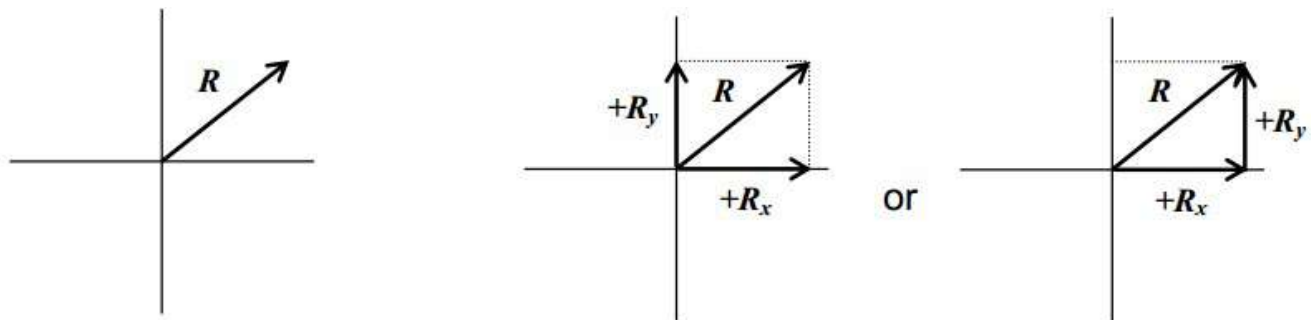


What about vectors that don't fall on the axis? You must specify their direction using degrees measured from East (+x).

Component Vectors

A resultant vector is a vector resulting from the sum of two or more other vectors. The resultant has the same magnitude and direction as the total of the vectors that compose the resultant. Could a vector be described by two or more other vectors? Would they have the same total result?

This is the reverse of finding the resultant. You are given the resultant and must find the component vectors on the coordinate axis that describe the resultant.



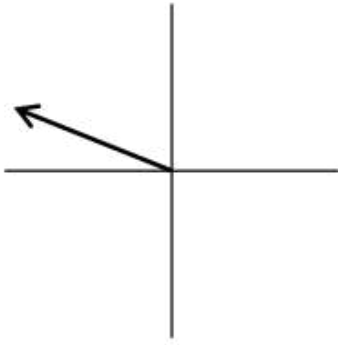
Any vector can be described by an x-axis vector and a y-axis vector which, when summed together, have a resultant that equals the original vector. The advantage is you can then use plus and minus signs for direction instead of the angle.

You should also Watch the following two videos:

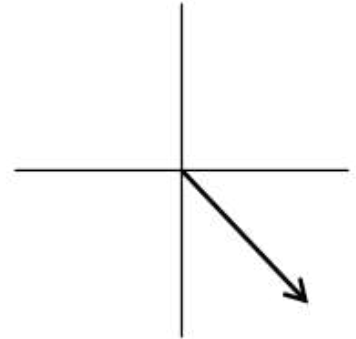
- <http://www.khanacademy.org/science/physics/v/introduction-to-vectors-and-scalars>
- <http://www.khanacademy.org/science/physics/v/visualizing-vectors-in-2-dimensions>

9. For the following vectors draw the component vectors along the x- and y-axes.

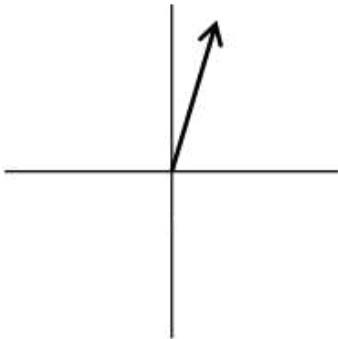
a.



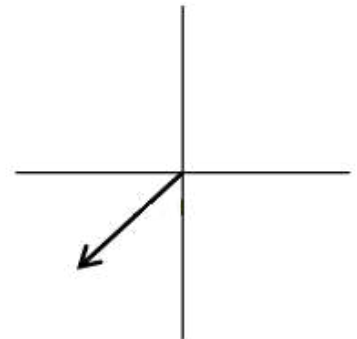
c.



b.



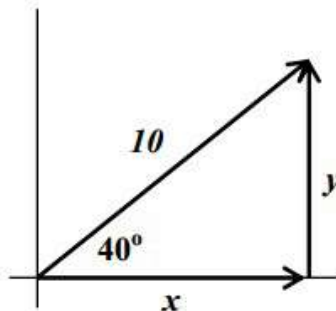
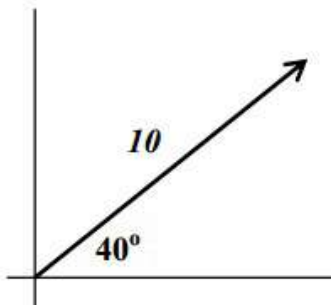
d.



Obviously the quadrant that a vector is in determines the sign of the x- and y-component vectors.

Trigonometry and Vectors

Given a vector, you can now draw the x- and y-component vectors. The sum of vectors **x** and **y** describe the vector exactly. Again, any math done with the component vectors will be as valid as with the original vector. The advantage is that math on the x- and/or y-axis is greatly simplified since direction can be specified with plus and minus signs instead of degrees. But, how do you mathematically find the length of the component vectors? Use trigonometry.



$$\cos \theta = \frac{adj}{hyp}$$

$$adj = hyp \cos \theta$$

$$x = hyp \cos \theta$$

$$x = 10 \cos 40^\circ$$

$$x = 7.66$$

$$\sin \theta = \frac{opp}{hyp}$$

$$opp = hyp \sin \theta$$

$$y = hyp \sin \theta$$

$$y = 10 \sin 40^\circ$$

$$y = 6.43$$

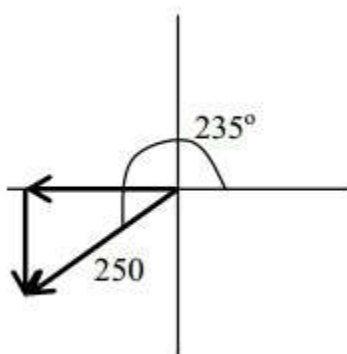
10. Solve the following problems. You will be converting from a polar vector, where direction is specified in degrees measured counterclockwise from east, to component vectors along the x- and y-axes. Remember the plus and minus signs on you answers. They correspond with the quadrant the original vector is in.

Draw the vector first to help you see the quadrant. Anticipate the sign on the x- and y-components. Do not bother to change the angle to less than 90° , using the value given will result in the correct $\pm$ signs.

The first number given will be the vector's magnitude, and the second the degrees from east. You calculator must be in degree mode.

Example

250 at 235°



$$x = \text{hyp} \cos \theta$$

$$x = 250 \cos 235^\circ$$

$$x = -143$$

$$y = \text{hyp} \sin \theta$$

$$y = 250 \sin 235^\circ$$

$$y = -205$$

a. 89 at 150°

d. 7.5×10^4 at 180°

b. 6.50 at 345°

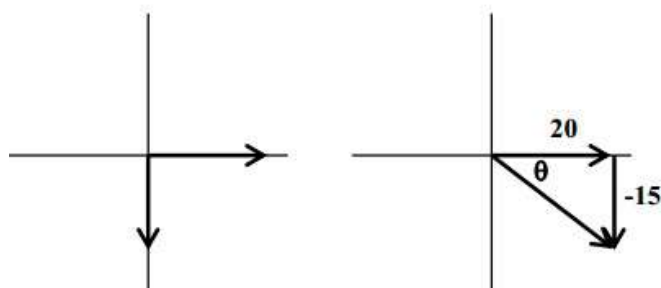
e. 12 at 265°

c. 0.00556 at 60°

f. 990 at 320°

11. Given two component vectors solve for the resultant vector. This is the opposite of number 10 above. Use the Pythagorean theorem to find the hypotenuse, then use inverse (arc) tangent to solve for the angle.

Example: $x = 20$, $y = -15$



$$R^2 = x^2 + y^2$$

$$\tan \theta = \frac{opp}{adj}$$

$$R = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$$

$$R = \sqrt{20^2 + 15^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = 25$$

$$360^\circ - 36.9^\circ = 323.1^\circ$$

a. $x = 600$, $y = 400$

d. $x = 0.0065$, $y = -0.0090$

b. $x = -0.75$, $y = -1.25$

e. $x = 20,000$, $y = 14,000$

c. $x = -32$, $y = 16$

f. $x = 325$, $y = 998$

HOW ARE VECTORS USED IN PHYSICS?

They are used everywhere!

Speed

Speed is a scalar. It only has magnitude (numerical value). $v = 10 \text{ m/s}$ means that an object is going 10 meters every second. But we do not know where it is going.

Velocity

Velocity is a vector. It is composed of both magnitude and direction. Speed is the magnitude of velocity. $\mathbf{v} = 10 \text{ m/s north}$, or $\mathbf{v} = 10 \text{ m/s in the } +x \text{ direction}$, etc.

There are three types of speed and three types of velocity

- Instantaneous speed / velocity: The speed or velocity at an instant in time. You look down at your speedometer and it says 20 m/s. You are traveling at 20 m/s at that instant. Your speed or velocity could be changing, but at that moment it is 20 m/s.
- Average speed / velocity: If you take a trip you might go slow part of the way and fast at other times. If you take the total distance traveled divided by the time traveled you get the average speed over the whole trip. If you looked at your speedometer from time to time you would have recorded a variety of instantaneous speeds. You could go 0 m/s in a gas station, or at a light. You could go 30 m/s on the highway, but only go 10 m/s on neighborhood streets. But, while there are many instantaneous speeds, there is only one average speed for the whole trip.
- Constant speed / velocity: If you have cruise control you might travel the whole time at one constant speed. If this is the case then your average speed will equal this constant speed.

A trick question

Will an object traveling at a constant speed of 10 m/s also always have constant velocity? Not always. If the object is turning around a curve or moving in a circle it can have a constant speed of 10 m/s, but since it is turning, its direction is changing. And if direction is changing then velocity must change, since velocity is made up of speed and direction.

Constant velocity must have both constant magnitude and constant direction.

Rate

Speed and velocity are rates. A rate is a way to quantify anything that takes place during a time interval. Rates are easily recognized. They always have time in the denominator.

Examples: 10 m/s, 10 meters / second

THE VERY FIRST PHYSICS EQUATION

Velocity and speed both share the same equation. Remember speed is the numerical (magnitude) part of velocity. Velocity only differs from speed in that it specifies a direction.

$$v = \frac{x}{t}$$

v stands for velocity, x stands for displacement, and t stands for time

Displacement is a vector for distance traveled in a straight line. It goes with velocity. Distance is a scalar and goes with speed.

Displacement is measured from the origin. It is a value of how far away from the origin you are at the end of the problem. The direction of a displacement is the shortest straight line from the location at the beginning of the problem to the location at the end of the problem.

How do distance and displacement differ? Suppose you walk 20 meters down in the $+x$ direction and turn around and walk 10 meters in the $-x$ direction. The distance traveled does not depend on direction, so you walked $20 + 10 = 30$ meters. Displacement only cares about your distance from the origin at the end of the problem, so your displacement is $+20 - 10 = 10$ meters.

12. Solve the following problems. Take heed of the following.

- Always use the **MKS** system: units must be in meters, kilograms, and seconds.
- On the all tests, including the AP exam you must:
 1. List the original equation used
 2. Show correct substitution
 3. Arrive at the correct answer with correct units
- Distance and displacement are measured in meters (m)
- Speed and velocity are measured in meters per second (m/s)
- Time is measured in seconds (s)

Example: A car travels 1000 meters in 10 seconds. What is its velocity?

$$v = \frac{x}{t} \qquad v = \frac{1000\text{ m}}{10\text{ s}} \qquad v = 100\text{ m/s}$$

a. A car travels 35 km west and 75 km east. What distance did it travel?

b. A car travels 35 km west and 75 km east. What is its displacement?

c. A car travels 35 km west, 90 km north. What distance did it travel?

d. A car travels 35 km west, 90 km north. What is its displacement?

e. A bicyclist pedals at 10 m/s in 20 s. What distance was traveled?

f. An airplane flies 250.0 km at 300 m/s. How long does this take?

g. A skydiver falls 3 km in 15 s. How fast are they going?

h. A car travels 35 km west, 90 km north in two hours. What is its average speed?

i. A car travels 35 km west, 90 km north in two hours. What is its average velocity?

Congratulations, you're done!