Name ☆Key☆

AP Calculus
Worksheet 5-4 and 5-5

Find each derivative.

1.) $y = e^{7x}$

$y' = 7e^{7x}$

2.) $y = 8^{3x+2}$

$y' = 3(\ln 8)\, 8^{3x+2}$

3.) $y = (3x^2)(e^{-4x})$

$y' = 3x^2(-4e^{-4x}) + e^{-4x}(6x)$

$= -12x^2e^{-4x} + 6xe^{-4x}$

$= \boxed{6xe^{-4x}(1-2x)}$

4.) $y = (4^{3x})(x^3)$

$y' = (4^{3x})(3x^2) + x^3(3\ln 4)4^{3x}$

$= 3x^2 4^{3x} + 3x^3(\ln 4)4^{3x}$

$= \boxed{3x^2 4^{3x}(1 + x\ln 4)}$

5.) $y = \log_3(x^4 - 2x)$

$\boxed{y' = \dfrac{4x^3-2}{(\ln 3)(x^4-2x)}}$

6.) $y = \ln(6xe^{3x})$

$y = \ln 6 + \ln x + \ln e^{3x}$

$y = \ln 6 + \ln x + 3x$

$y' = \frac{1}{x} + 3$

$\boxed{y' = \dfrac{1+3x}{x}}$

7.) $y = e^{9x-2}$

$y' = 9e^{9x-2}$

8.) $y = (\ln x)(e^{4x}) + e^{-x}$

$y' = \ln x(4e^{4x}) + e^{4x}(\frac{1}{x}) - e^{-x}$

$\boxed{y' = 4\ln x\, e^{4x} + \dfrac{e^{4x}}{x} - e^{-x}}$

9.) $y = \ln(x + e^{-x})$

$y' = \dfrac{1 - e^{-x}}{x + e^{-x}}$

10.) $y = 3^{x/2} e^{x/2}$

$y' = 3^{x/2}\left(\dfrac{e^{x/2}}{2}\right) + e^{x/2}\left(\dfrac{(\ln 3)}{2}\right)3^{x/2}$

$= \dfrac{3^{x/2} e^{x/2}}{2} + \dfrac{(\ln 3) 3^{x/2} e^{x/2}}{2}$

$= \boxed{\dfrac{3^{x/2} e^{x/2}}{2}(1 + \ln 3)}$

11.) $y = \ln \frac{x^2}{x-1}$

$y = 2\ln x - \ln(x-1)$

$y' = \dfrac{2}{x} - \dfrac{1}{x-1}$

$y' = \dfrac{\overset{2x-2}{2(x-1)} - x}{x(x-1)}$

$\boxed{y' = \dfrac{x-2}{x(x-1)}}$

12.) $y = \log_2 \sqrt[3]{x-1}$

$y = \frac{1}{3} \log_2 (x-1)$

$y' = \frac{1}{3}\left(\frac{1}{x-1}\right)\frac{1}{\ln(2)}$

$\boxed{y' = \dfrac{1}{3(\ln 2)(x-1)}}$

13.) $y = e^{5/x^4}$

$\boxed{y = -\dfrac{20e^{5/x^4}}{x^5}}$

$5x^{-4}$

$-20x^{-5}$

14.) $y = (\tan \pi x)(3^x)$

$y' = \tan\pi x(\ln 3 \cdot 3^x) + 3^x(\sec^2\pi x$

$= (\ln 3)3^x \tan\pi x + \pi 3^x \sec^2\pi x$

$= \boxed{3^x((\ln 3)\tan\pi x + \pi\sec^2\pi x}$

Find each indefinite integral.

15.) $\int e^{10x} dx = \frac{1}{10}\int e^u du$

$u = 10x$

$du = 10dx$

$\frac{du}{10} = dx$

$= \frac{1}{10} e^u + C$

$= \boxed{\dfrac{e^{10x}}{10} + C}$

16.) $\int x3^{-x^2} dx = -\frac{1}{2}\int 3^u du$

$u = -x^2$

$du = -2x dx$

$\frac{du}{-2} = x dx$

$= -\frac{1}{2} \cdot \frac{1}{\ln 3} \cdot 3^u +$

$= \dfrac{-3^{-x^2}}{2(\ln 3)} + C$

$= \boxed{-\dfrac{1}{2(\ln 3)3^{x^2}} +}$

17.) $\int 6^x + 7^{-x} dx$

$\int 6^x dx + \int 7^{-x} dx$ $\quad u=-x$

$\int 6^x dx - \int 7^u du$ $\quad du=-dx$

$\frac{du}{-1}=dx$

$\frac{6^x}{\ln 6} - \frac{7^u}{\ln 7} + C$

$\boxed{\frac{6^x}{\ln 6} - \frac{7^{-x}}{\ln 7} + C}$

18.) $\int \frac{4e^{4x}-e^{-x}}{e^{4x}+e^{-x}} dx = \int \frac{1}{u} du$

$u=e^{4x}+e^{-x}$ $\quad = \ln|u|+C$

$du=(4e^{4x}-e^{-x})dx$

$= \boxed{\ln|e^{4x}+e^{-x}|+C}$

19.) $\int \frac{e^{4x}+4}{e^x} dx$

$= \int \frac{e^{4x}}{e^x} dx + \int \frac{4}{e^x} dx$

$= \int e^{3x} dx + \int 4e^{-x} dx$

$u=3x$ $\quad u=-x$

$du=3dx$ $\quad = \frac{1}{3}\int e^u du - 4\int e^u du$ $\quad du=-dx$

$\frac{du}{3}=dx$ $\quad = \boxed{\frac{e^{3x}}{3} - 4e^{-x} + C}$ $\quad \frac{du}{-1}=dx$

20.) $\int \frac{e^{4x}}{e^{4x}+4} dx = \frac{1}{4}\int \frac{1}{u} du$

$u=e^{4x}+4$

$du=4e^{4x}dx$ $\quad = \frac{1}{4}\ln|u|+C$

$\frac{du}{4}=e^{4x}dx$ $\quad = \boxed{\frac{1}{4}\ln|e^{4x}+4|+C}$

21.) $\int \frac{6^{1/x}}{x^2(1-6^{1/x})} dx = -\frac{1}{\ln 6}\int \frac{1}{u} du$

$u=1-6^{1/x}$

$u=1-6^{x^{-1}}$ $\quad = \boxed{-\frac{1}{\ln 6}\cdot \ln|1-6^{1/x}|+C}$

$du=(\ln 6)6^{x^{-1}}(-x^{-2})dx$

$\frac{du}{-\ln 6} = \frac{6^{1/x}}{x^2} dx$

22.) $\int (x+2)^3 2^{(x+2)^4} dx = \frac{1}{4}\int 2^u du$

$u=(x+2)^4$ $\quad = \frac{1}{4(\ln 2)} 2^u + C$

$du=4(x+2)^3(1)dx$

$\frac{du}{4}=(x+2)^3 dx$ $\quad = \boxed{\frac{2^{(x+2)^4}}{4(\ln 2)}+C}$

23.) $\int e^{3x-1}(6-e^{3x-1})^5 dx = -\frac{1}{3}\int u^5 du$

$u=6-e^{3x-1}$ $\quad = -\frac{1}{3}\cdot\frac{u^6}{6}+C$

$du=-3e^{3x-1}dx$ $\quad = \boxed{-\frac{(6-e^{3x-1})^6}{18}+C}$

$\frac{du}{-3}=e^{3x-1}dx$

24.) $\int \frac{e^{2/x^3}}{x^4} dx = -\frac{1}{6}\int e^u du$

$u=2/x^3=2x^{-3}$ $\quad = -\frac{1}{6}e^u+C$

$du=-6x^{-4}dx$

$\frac{du}{-6}=\frac{1}{x^4}dx$ $\quad = \boxed{-\frac{e^{2/x^3}}{6}+C}$

25.) $\int 3^{\cos 2x} \sin 2x \, dx = -\frac{1}{2}\int 3^u du$

$u = \cos 2x$

$du = -\sin 2x \, 2 dx$

$\frac{du}{-2} = \sin 2x \, dx$

$= \boxed{-\frac{3^{\cos 2x}}{2(\ln 3)} + C}$

26.) $\int \frac{e^{1/x} 5^{e^{1/x}}}{x^2} dx = -\int 5^u du$

$u = e^{1/x} = e^{x^{-1}}$

$du = e^{x^{-1}}(-x^{-2})dx$

$\frac{du}{-1} = \frac{e^{1/x}}{x^2} dx$

$= \boxed{-\frac{5^{e^{1/x}}}{\ln 5} + C}$

27.) $\int e^{\tan 2x} \sec^2 2x \, dx = \frac{1}{2}\int e^u du$

$u = \tan 2x$

$du = \sec^2 2x \, 2 dx$

$\frac{du}{2} = \sec^2 2x \, dx$

$= \boxed{\frac{1}{2} e^{\tan 2x} + C}$

28.) $\int \cancel{\ln}(e^{x^2-2x+4}) \, dx$

$\int (x^2 - 2x + 4) dx$

$= \frac{x^3}{3} - \frac{2x^2}{2} + 4x + C$

$= \boxed{\frac{x^3}{3} - x^2 + 4x + C}$

29.) $\int e^{3x}\sqrt[3]{9 - e^{3x}} dx = -\frac{1}{3}\int u^{1/3} du$

$u = 9 - e^{3x}$

$du = -3e^{3x} dx$

$\frac{du}{-3} = e^{3x} dx$

$= -\frac{1}{3} \frac{u^{4/3}}{4} \cdot 3 + C$

$= \boxed{-\frac{(9 - e^{3x})^{4/3}}{4} + C}$

30.) $\int \frac{4^x + 3}{4^{6x}} dx$

$= \int \left(\frac{4^x}{4^{6x}} + \frac{3}{4^{6x}}\right) dx$

$u = -6x$

$du = -6dx$

$\frac{du}{-6} = dx$

$= \int 4^{-5x} dx + 3\int 4^{-6x} dx$

$u = -5x$

$du = -5dx$

$\frac{du}{-5} = dx$

$= -\frac{1}{5}\int 4^u du + \frac{3}{-6}\int 4^u du$

$= -\frac{4^u}{5(\ln 4)} - \frac{4^u}{2(\ln 4)} + C$

$= \boxed{\frac{-4^{-5x}}{5(\ln 4)} - \frac{4^{-6x}}{2(\ln 4)} + C}$