Name Akya Period____ Date________

AP Calculus
Practice 5.1 – 5.2

Find the derivative of each function.

1.) $f(x)=\ln[(1+x)(1+x^2)^2(1+x^3)^3]$

$$f(x)=\ln(1+x)+2\ln(1+x^2)+3\ln(1+x^3)$$

$$f'(x)=\frac{1}{1+x}+2\left(\frac{2x}{1+x^2}\right)+3\left(\frac{3x^2}{1+x^3}\right)$$

$$=\frac{1}{1+x}+\frac{4x}{1+x^2}+\frac{9x^2}{1+x^3}$$

$$=\frac{(1+x^2)(1+x^3)+\overset{4x+4x^2}{\cancel{4x(1+x)}}(1+x^3)+\overset{9x^2+9x^3}{\cancel{9x^2(1+x)}}(1+x^2)}{(1+x)(1+x^2)(1+x^3)}$$

$$=\frac{1+x^3+x^2+x^5+4x+4x^4+4x^2+4x^5+9x^2+9x^4+9x^3+9x^5}{(1+x)(1+x^2)(1+x^3)}$$

$$=\boxed{\frac{14x^5+13x^4+10x^3+14x^2+4x+1}{(1+x)(1+x^2)(1+x^3)}}$$

2.) $f(x)=\ln[(\sin 2x)(\sqrt{x^2+1})]$

$$f(x)=\ln\sin 2x+\tfrac{1}{2}\ln(x^2+1)$$

$$f'(x)=\frac{\cos 2x\,2}{\sin 2x}+\frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2+1}\right)$$

$$=\boxed{\frac{2\cos 2x(x^2+1)+x\sin 2x}{\sin 2x(x^2+1)}}$$

3.) $f(x)=\ln\left(\dfrac{(\sqrt[3]{4x^2+1})(6x-5)^4}{x^2}\right)$

$$f(x)=\tfrac{1}{3}\ln(4x^2+1)+4\ln(6x-5)-2\ln x$$

$$f'(x)=\frac{1}{3}\left(\frac{8x}{4x^2+1}\right)+4\left(\frac{6}{6x-5}\right)-2\left(\frac{1}{x}\right)$$

$$=\frac{8x}{3(4x^2+1)}+\frac{24}{6x-5}-\frac{2}{x}$$

$$=\frac{8x^2(6x-5)+\overset{72x}{24\cdot 3}(4x^2+1)x-\overset{-24x^2-6}{\cancel{2\cdot 3(4x^2+1)}}(6x-5)}{3x(4x^2+1)(6x-5)}$$

$$=\frac{48x^3-40x^2+288x^3+72x-144x^3+120x^2-36x+30}{3x(4x^2+1)(6x-5)}$$

$$=\frac{192x^3+80x^2+36x+30}{3x(4x^2+1)(6x-5)}=\boxed{\frac{2(96x^3+40x^2+18x+15)}{3x(4x^2+1)(6x-5)}}$$

4.) $f(x)=\ln\sqrt{\dfrac{x^2-3}{3+x^2}}=\ln\dfrac{\sqrt{x^2-3}}{\sqrt{3+x^2}}$

$$f(x)=\tfrac{1}{2}\ln(x^2-3)-\tfrac{1}{2}\ln(3+x^2)$$

$$f'(x)=\frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2-3}\right)-\frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{3+x^2}\right)$$

$$=\frac{x}{x^2-3}-\frac{x}{3+x^2}$$

$$=\frac{x(3+x^2)-x(x^2-3)}{(x^2-3)(3+x^2)}$$

$$=\frac{3x+x^3-x^3+3x}{(x^2-3)(3+x^2)}$$

$$=\boxed{\frac{6x}{(x^2-3)(3+x^2)}}$$

5.) $f(x) = \ln\left|\dfrac{2-\cos^2 x}{-3+\sin x}\right|$

$f(x) = \ln|2-\cos^2 x| - \ln|-3+\sin x|$

$f'(x) = \dfrac{-2\cos x(-\sin x)}{2-\cos^2 x} - \dfrac{\cos x}{-3+\sin x}$

$= \dfrac{2\sin x\cos x}{2-\cos^2 x} - \dfrac{\cos x}{\sin x - 3}$

$= \dfrac{2\sin x\cos x(\sin x-3) - \cos x(2-\cos^2 x)}{(2-\cos^2 x)(\sin x-3)}$

$= \boxed{\dfrac{2\sin^2 x\cos x - 6\sin x\cos x - 2\cos x + \cos^3 x}{(2-\cos^2 x)(\sin x - 3)}}$

6.) $f(x) = \ln\sqrt{4+\tan^2 2x}$

$f(x) = \ln(4+\tan^2 2x)^{1/2}$

$f(x) = \frac{1}{2}\ln(4+\tan^2 2x)$

$f'(x) = \frac{1}{\cancel{2}}\left(\dfrac{2\tan 2x\sec^2 2x \cdot \cancel{2}}{4+\tan^2 2x}\right)$

$= \boxed{\dfrac{2\tan 2x\sec^2 2x}{4+\tan^2 2x}}$

7.) $y = \dfrac{x^3\sqrt{2x+3}}{(x-2)^2}$

$\ln y = \ln \dfrac{x^3(2x+3)^{1/2}}{(x-2)^2}$

$\ln y = 3\ln x + \frac{1}{2}\ln(2x+3) - 2\ln(x-2)$

$\frac{y'}{y} = 3\left(\frac{1}{x}\right) + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{2x+3}\right) - 2\left(\frac{1}{x-2}\right)$

$\frac{y'}{y} = \frac{3}{x} + \frac{1}{2x+3} - \frac{2}{x-2}$

$\frac{y'}{y} = \dfrac{\overset{6x+9}{3(2x+3)}(x-2) + x(x-2) - 2x(2x+3)}{x(2x+3)(x-2)}$

$\frac{y'}{y} = \dfrac{6x^2-12x+9x-18+x^2-2x-4x^2-6x}{x(2x+3)(x-2)}$

$y \cdot \frac{y'}{y} = \dfrac{3x^2-11x-18}{x(2x+3)(x-2)} \cdot \dfrac{x^3(2x+3)^{1/2}}{(x-2)^2}$

$\boxed{y' = \dfrac{x^2(3x^2-11x-18)}{(2x+3)^{1/2}(x-2)^3}}$

8.) $y = \dfrac{(x+2)\sqrt{1-x^2}}{4x^3}$

$\ln y = \ln \dfrac{(x+2)(1-x^2)^{1/2}}{4x^3}$

$\ln y = \ln(x+2) + \frac{1}{2}\ln(1-x^2) - \ln 4 - 3\ln x$

$\frac{y'}{y} = \frac{1}{x+2} + \frac{1}{\cancel{2}}\left(\frac{-\cancel{2}x}{1-x^2}\right) - 3\left(\frac{1}{x}\right)$

$\frac{y'}{y} = \frac{1}{x+2} - \frac{x}{1-x^2} - \frac{3}{x}$

$\frac{y'}{y} = \dfrac{x(1-x^2) - x^2(x+2) - \overset{-3x-6}{3(x+2)}(1-x^2)}{x(x+2)(1-x^2)}$

$\frac{y'}{y} = \dfrac{x-x^3-x^3-2x^2-3x+3x^3-6+6x^2}{x(x+2)(1-x^2)}$

$y \cdot \frac{y'}{y} = \dfrac{x^3+4x^2-2x-6}{x(x+2)(1-x^2)} \cdot \dfrac{(x+2)(1-x^2)^{1/2}}{4x^3}$

$\boxed{y' = \dfrac{(x^3+4x^2-2x-6)}{4x^4(1-x^2)^{1/2}}}$

Find each integral.

9.) $\int \frac{x(x-2)}{x^3}dx = \int \left(\frac{x-2}{x^2}\right)dx$

$= \int \left(\frac{x}{x^2} - \frac{2}{x^2}\right)dx$

$= \int \left(\frac{1}{x} - 2x^{-2}\right)dx$

$= \ln|x| - 2\left(\frac{x^{-1}}{-1}\right) + C$

$= \boxed{\ln|x| + \frac{2}{x} + C}$

10.) $\int \frac{4}{x\ln(x^2)}dx = \int \frac{4}{x(2\ln x)}dx$

$= \int \frac{2}{x\ln x}dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$= 2\int \frac{1}{u}du$

$= 2\ln|u| + C$

$= \boxed{2\ln|\ln x| + C}$

11.) $\int \frac{x^3}{4-x^4}dx = -\frac{1}{4}\int \frac{1}{u}du$

$u = 4 - x^4$

$du = -4x^3 dx$

$\frac{du}{-4} = x^3 dx$

$= -\frac{1}{4}\ln|u| + C$

$= \boxed{-\frac{1}{4}\ln|4-x^4| + C}$

12.) $\int \frac{x^3-4x+2}{-1+x}dx = \int \frac{x^3-4x+2}{x-1}dx$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -4 & 2 \\ & & 1 & 1 & -3 \\ \hline & 1 & 1 & -3 & -1 \end{array}$$

$= \int \left(x^2 + x - 3 - \frac{1}{x-1}\right)dx$

$= \boxed{\frac{x^3}{3} + \frac{x^2}{2} - 3x - \ln|x-1| + C}$

or

$$\begin{array}{r|l} & x^2 + x - 3 - \frac{1}{x-1} \\ x-1 & x^3 + 0x^2 - 4x + 2 \\ & -\cancel{x^3} + x^2 \\ \hline & \cancel{x^2} - 4x \\ & -\cancel{x^2} + x \\ \hline & -\cancel{3x} + 2 \\ & +\cancel{3x} - 3 \\ \hline & -1 \end{array}$$

13.) $\int \frac{\sin 3x - 2}{\cos 3x} dx = \int \left(\frac{\sin 3x}{\cos 3x} - \frac{2}{\cos 3x}\right) dx$

$= \int (\tan 3x - 2\sec 3x)\, dx$

$= \int (\tan 3x)\, dx - \int (2\sec 3x)\, dx$

$= \frac{1}{3}\int \tan u \frac{du}{} \; \frac{2}{3}\int \sec u\, du$

$u = 3x$

$du = 3dx$

$\frac{du}{3} = dx$

$= -\frac{1}{3}\ln|\cos u| - \frac{2}{3}\ln|\sec u + \tan u| + c$

$= \boxed{-\frac{1}{3}\ln|\cos 3x| - \frac{2}{3}\ln|\sec 3x + \tan 3x| + c}$

14.) $\int \frac{\csc 3\pi x}{2} dx = \frac{1}{2}\cdot\frac{1}{3\pi}\int \csc u\, du$

$u = 3\pi x$

$du = 3\pi\, dx$

$\frac{du}{3\pi} = dx$

$= -\frac{1}{6\pi}\ln|\csc u + \cot u|$

$= \boxed{-\frac{1}{6\pi}\ln|\csc 3\pi x + \cot 3\pi x|}$

15.) $\int_0^{\pi/4} \frac{\sin 2x - 3}{\cos 2x + 6x} dx \quad -\frac{1}{2}\int \frac{1}{u}\, du$

$u = \cos 2x + 6x$

$\frac{du}{-2} = \frac{-\sin 2x \, 2 + 6}{-2} dx$

$\frac{du}{-2} = (\sin 2x - 3)\, dx$

$= -\frac{1}{2}\ln|u|$

$= -\frac{1}{2}\ln|\cos 2x + 6x| \Big]_0^{\pi/4}$

$= -\frac{1}{2}\ln|\cos 2(\frac{\pi}{4}) + 6(\frac{\pi}{4})|$

$- \frac{1}{2}\ln|\cos 2(0) + 6(0)|$

$= -\frac{1}{2}\ln 4.71 + \frac{1}{2}\ln 1$

$= -\frac{1}{2}\ln 4.71$

$\approx \boxed{-0.77}$

16.) $\int_2^3 \frac{(\ln x)^2}{x} dx = \int_{\ln 2}^{\ln 3} u^2\, du$

$u = \ln x$

$du = \frac{1}{x} dx$

when $x = 3$,

$u = \ln 3$

when $x = 2$,

$u = \ln 2$

$= \frac{u^3}{3}\Big]_{\ln 2}^{\ln 3}$

$= \frac{(\ln 3)^3}{3} - \frac{(\ln 2)^3}{3}$

$\approx \boxed{0.33}$