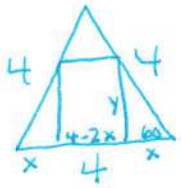
**AP Calculus**

*Optimization Review Problems* ★key★

1.) Mr. Alexander wants to build a window in the shape of an equilateral triangle of sides 4 meters. He wants to inscribe a rectangular piece of stained glass in the triangle, so that two of the vertices of the rectangle lie on one of the sides of the triangle. Find the dimensions of the rectangle of maximum area that can be inscribed in the given triangle.

1.) 


2.) $A = (4-2x)(y)$

3.) $\tan 60 = \frac{y}{x}$

$x\tan 60 = y$

$x\sqrt{3} = y$

4.) $A = (4-2x)(x\sqrt{3})$

$= 4\sqrt{3}x - 2\sqrt{3}x^2$

5.) $A' = 4\sqrt{3} - 4\sqrt{3}x$

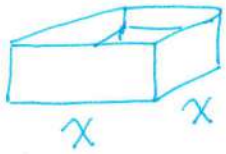
$0 = 4\sqrt{3} - 4\sqrt{3}x$

$1 = x$

check: $A'' = -4\sqrt{3}$ $<0$ max

$2 \times \sqrt{3}$ m

2.) An open oak wood box with a square base is to be constructed using 192 cm$^2$ of oak. If the volume of the box is to be maximized, find its dimensions.

1.) 


2.) $V = x^2y$

3.) $SA = x^2 + 4xy$

$192 = x^2 + 4xy$

$\frac{192 - x^2}{4x} = y$

4.) $V = x^2\left(\frac{192-x^2}{4x}\right)$

$V = 48x - \frac{1}{4}x^3$

$V' = 48 - \frac{3}{4}x^2$

5.) $0 = 48 - \frac{3}{4}x^2$

$\left(-\frac{4}{3}\right)(-48) = -\frac{3}{4}x^2 \cdot \left(-\frac{4}{3}\right)$

$64 = x^2$

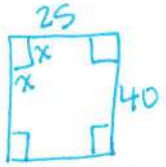
$8 = x$

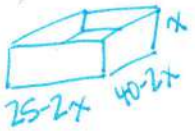
check: $V'' = -1.5x$, $V''(8) < 0$ max

$y = \frac{192-(8)^2}{4(8)} = 4$

8cm x 8cm x 4cm

3.) The newly acclaimed Sergio restaurant in Atlanta is designing a container for soup in the shape of an open box with a rectangular base. The box is to be made from a rectangular piece of yellow cardboard 25 cm wide by 40 cm long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square that will produce a container that will hold the most soup.

1.) 


2.) $V = (25-2x)(40-2x)(x)$

$= (1000 - 50x - 80x + 4x^2)(x)$

$= 4x^3 - 130x^2 + 1000x$

3.) $V' = 12x^2 - 260x + 1000$

$0 = 12x^2 - 260x + 1000$

$0 = 3x^2 - 65x + 250$ * or use quad. formula

$0 = (3x-50)(x-5)$

$x = \frac{50}{3}$ or $x = 5$

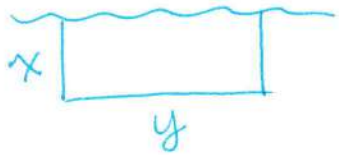
↓ not in domain

ck: $V'' = 24x - 260$, $V''(5) < 0$ ma

5cm x 5cm

4.) Yancy Creek High School's soccer field will be fenced in the near future. No fence will be required on the side lying along Yancy Creek. If the new wood fence costs $12.00 per meter for the side parallel to the creek and $4.00 per meter for the other two sides, find the dimensions of the soccer field of maximum area that could be enclosed with a budget of $3,600.00

1.)




2.) $A = xy$

3.) $12y + 8x = 3600$

$8x = 3600 - 12y$

$x = \frac{3600-12y}{8} = 450 - \frac{3}{2}y$

4.) $A = (450 - \frac{3}{2}y)y$

$A = 450y - \frac{3}{2}y^2$

$A' = 450 - 3y$

$0 = 450 - 3y$

$y = \frac{450}{3}$

ck $A' = -3$ < 0 max

$y = 150$

$x = 450 - \frac{3}{2}(150) = 225$

150m x 225m

5.) Mrs. Paul is interested in employing TAHS students to harvest the 9,000,000 vegetables in her summer garden. It is well known that each student can harvest 900 vegetables per hour and is paid the reasonable wage of $3.00 per hour. Mrs. Paul will also have to hire Mr. Armbruster to supervise the harvest. Mr. Armbruster would be paid $20.00 per hour and TAHS will receive a commission equivalent to $5.00 for each student employed. How many students should Mrs. Paul employ to minimize the cost of harvesting her garden?

1.) $C = 3xy + 20x + 5y$

let x = hours

let y = # students

2.) $x \cdot y \cdot 900 = 9000000$

$x = \frac{10,000}{y}$

3.) $C = 3(\frac{10000}{y})y + 20(\frac{10000}{y}) + 5y$

$C = 30000 + \frac{200000}{y} + 5y$

$C' = -\frac{200000}{y^2} + 5$

$0 = -\frac{200000}{y^2} + 5$

$-5 = -\frac{200000}{y^2}$

$y^2 = 40,000 \rightarrow y = 200$

double ck 200 is a min using 2nd der. Test

$C'' = 400,000x^{-3}$

$C''(200) = >0 \rightarrow$ min

200 students