Name_____Period_____Date_____

AP Calculus Mid-Term
MAT 190 Final (Practice)

Directions: You may write on this test and/or use scratch paper. Choose the letter of the best answer and fill it in on the scantron. When finished, staple any scratch paper to this test.

1. Determine whether the function is even, odd, or neither.

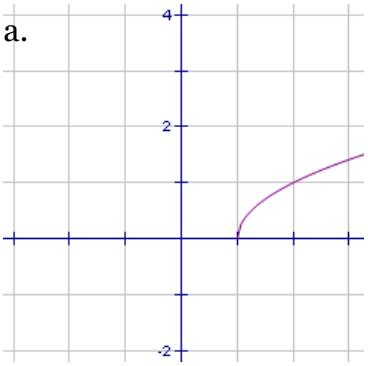
$$f(x) = x^3 - x$$

   a. Even
   b. Odd
   c. Neither

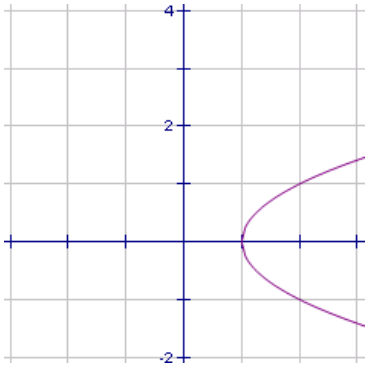
2. Find any intercepts of $y = x^2 - x - 12$
   a. x-intercept: (4, 0) ; y-intercept: (0, -12)
   b. x-intercept: (-3, 0), (4, 0); y-intercept: (0, -12)
   c. x-intercept: (12, 0) ; y-intercept: (0, 3), (0, -4)
   d. x-intercept: (-3, 0), (4, 0) ; y-intercept: (0, 12), (0, -12)
   e. x-intercept: (-3, 0), (4, 0) ; y-intercept: (0, 3), (0, -4)

3. Sketch the graph of the equation $x - y^2 = 1$.

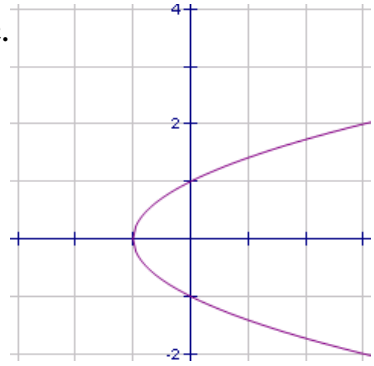
a.



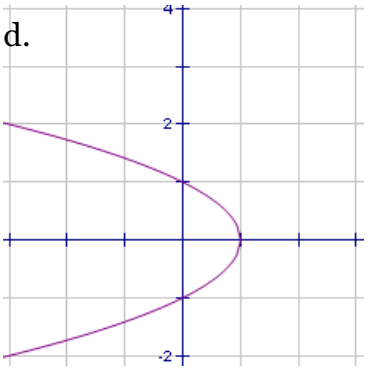
b.



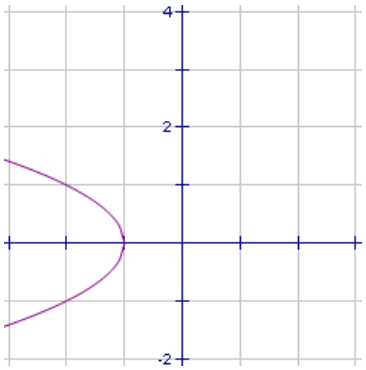
c.



d.



e.



4. Write an equation of the line that passes through the point parallel to the given line, and perpendicular to the given line.

Point (8, -7) Line $x = 7$

a. Parallel: $x = 8$
   Perpendicular: $y = -7$
b. Parallel: $y = -7$
   Perpendicular: $x = 8$
c. Parallel: $x = -7$
   Perpendicular: $y = 8$
d. Parallel: $x = -8$
   Perpendicular: $y = 7$
e. Parallel: $y = 7$
   Perpendicular: $x = 8$

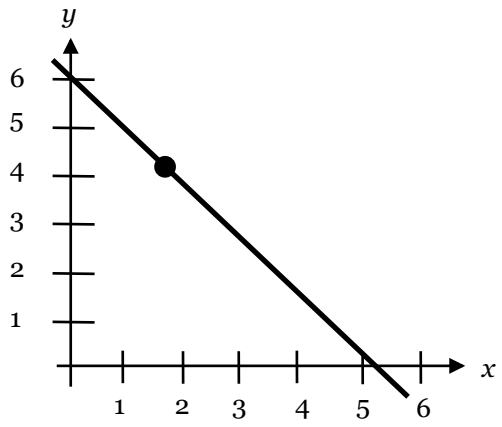
5. Find the domain and range of the function $h(x) = \dfrac{13}{x - 13}$

a. Domain: all reals, $x \neq 13$
   Range: all reals
b. Domain: all reals
   Range: all reals
c. Domain: all reals, $x \neq 13$
   Range: all reals, $y \neq 0$
d. Domain: $x > 13$
   Range: $y > 0$
e. Domain: $x < 13$
   Range: $y > 0$

6. Determine the following limit based on the graph. Use the closest whole number.

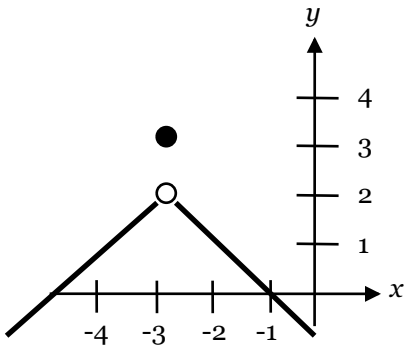
$\lim_{x \to 2} f(x)$

a. 8
b. 6
c. 2
d. 4
e. DNE



7. Use the graph as shown to determine the following limits, and discuss the continuity of the function at x = -3. Use the closest whole numbers.

$$\lim_{x\to -3^+} f(x),\ \lim_{x\to -3^-} f(x),\ \lim_{x\to -3} f(x)$$



a. 2, 2, 2, not continuous
b. 2, 2, 2, continuous
c. 3, 3, 3, not continuous
d. 3, 3, 3, continuous
e. -3, -3, -3, continuous

8. Find the limit: $\lim_{x\to 3} \cos\left|\frac{\pi x}{4}\right|$

a. $\frac{1}{4\sqrt{2}}$
b. $\frac{\sqrt{2}}{2}$
c. $-\frac{\sqrt{2}}{2}$
d. 0
e. $-\frac{1}{4\sqrt{2}}$

9. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$$

a. 7, $x+2$
b. 2, $x+2$
c. 10, $x+5$
d. 3, $x-2$
e. DNE

10. Find the limit: $\lim_{x \to -2} \frac{x^2 + 2x}{x^2 + 4x + 2}$

a. 4
b. $\frac{1}{4}$
c. -4
d. $-\frac{1}{4}$
e. -2

11. Find the *x*-values (if any) at which the function is not continuous. If any values exist, discuss whether discontinuity is removable or nonremovable.

$$f(x) = \frac{x+2}{x^2 - 6x - 16}$$

a. x = -2 (nonremovable), x = 8 (nonremovable)
b. x = -2 (nonremovable), x = 8 (removable)
c. x = -2 (removable), x = 8 (nonremovable)
d. no points of continuity
e. No points of discontinuity

12. Find the vertical asymptotes (if any) of the function $f(x) = \frac{x^2 + 7x + 10}{x^2 - 4}$.

a. x = 2
b. x = -2
c. x = 4
d. none
e. A and B

13. The radius, *r*, of a circle is increasing at a rate of 3 centimeters per minute. Find the rate of change of area when r = 2.

a. $\frac{dy}{dt} = -37.70$
b. $\frac{dy}{dt} = 12.57$
c. $\frac{dy}{dt} = 94.25$
d. $\frac{dy}{dt} = 37.70$
e. $\frac{dy}{dt} = -12.57$

14. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 16 feet above the ground. When he is 12 feet from the base of the light find the rate the tip of the shadow is moving.

a. -8 feet per second
b. 1.6 feet per second
c. 8 feet per second
d. -1.6 feet per minute
e. None of the above

15. Find the derivative of the following function using the limit definition.

$$f(x) = 4x^2 - 3x - 2$$

a. $f'(x) = 8x - 3$
b. $f'(x) = 8x - 3x - 2$
c. $f'(x) = 8x + 3$
d. $f'(x) = 4x - 3$
e. None of the above

16. Find the slope of the graph of the function at the given value.

$$f(x) = (4x - 9)^2 \text{ when } x = 5$$

a. $f'(5) = 22$
b. $f'(5) = 88$
c. $f'(5) = 2662$
d. $f'(5) = 44$
e. $f'(5) = 232$

17. Find the derivative of the function $f(x) = 8x^2 + 6\cos x$

a. $f'(x) = 8x - 6\sin x$
b. $f'(x) = 16x + 6\sin x$
c. $f'(x) = 16x - 6\sin x$
d. $f'(x) = 8x - 6\cos x$
e. None of the above

18. Use the product rule to differentiate $R(t) = t^{-5}\sin t$

a. $R'(t) = -t^{-5}\cos t - 5t^{-4}\sin t$
b. $R'(t) = t^{-5}\cos t - 5t^{-6}\sin t$
c. $R'(t) = t^{-5}\cos t + 5t^{-6}\sin t$
d. $R'(t) = t^{-5}\cos t - 5t^{-4}\sin t$
e. $R'(t) = -t^{-5}\cos t + 5t^{-6}\sin t$

19. Use the quotient rule to differentiate $H(v) = \dfrac{2v}{v^3 + 6}$ and evaluate $H'(2)$.

a. $H'(2) = 0.0703$
b. $H'(2) = -0.3878$
c. $H'(2) = -3.7143$
d. $H'(2) = -0.1020$
e. $H'(2) = 0.1020$

20. Evaluate the derivative of the function at the given point.

$$y = \frac{8x + 2}{2x - 1} \qquad \text{at } (1, 10)$$

a. $y'(1) = -12$
b. $y'(1) = 4$
c. $y'(1) = 12$
d. $y'(1) = -4$
e. $y'(1) = -0.4444$

21. Find the second derivative of $f(x) = x^6\cos x$.

a. $f''(x) = -x^6\cos x - 12x^5\sin x - 30x^4\cos x$
b. $f''(x) = -x^6\cos x - 12x^5\sin x + 30x^4\cos x$
c. $f''(x) = -x^6\cos x - 30x^4\cos x$
d. $f''(x) = x^6\cos x - 30x^4\cos x$
e. $f''(x) = x^6\cos x - 12x^5\sin x + 30x^4\cos x$

22. Find the derivative of $y = \frac{5}{7}\sec^2 x$.

a. $y' = -\frac{10}{7}\sec^2 x \tan x$

b. $y' = \frac{10}{7}\sec^2 x \tan^2 x$

c. $y' = \frac{5}{7}\sec^2 x \tan x$

d. $y' = \frac{10}{7}\sec^2 x \tan x$

e. $y' = \frac{10}{7}\sec x \tan x$

23. Find the derivative of $f(v) = 7v\sin v + 5\cos v$

a. $f'(v) = -7v\sin v + 2\cos v$
b. $f'(v) = 7v\sin v - 7\cos v$
c. $f'(v) = 7v\sin v - 2\cos v$
d. $f'(v) = 7v\cos v + 2\sin v$
e. $f'(v) = 7v\cos v + 5\sin v$

24. Find dy/dx by implicit differentiation.

$$x^5 + 9x + x^{11}y - y^3 = 9$$

a. $\frac{dy}{dx} = \frac{5x^4 + 9 + x^{11}y}{3y^2 - x^{11}}$

b. $\frac{dy}{dx} = \frac{5x^4 + 9 + 11x^{10}y}{3y^2 - x^{11}}$

c. $\frac{dy}{dx} = \frac{5x^4 + 9 + 11y}{2y^2 - 11x}$

d. $\frac{dy}{dx} = \frac{5x^4 - 9 + 11x^{10}y}{3y^2 - x^{11}}$

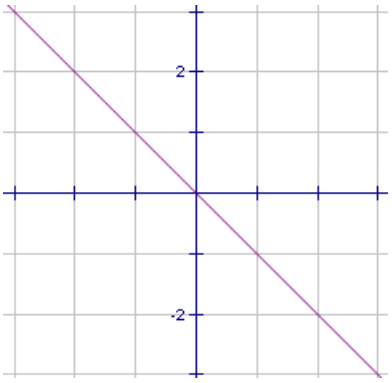
e. $\frac{dy}{dx} = \frac{5x^4 + 9 + 11x^{10}y}{2y^2 - x^{11}}$

25. Find an equation of the tangent line to the graph of the function at the given point.

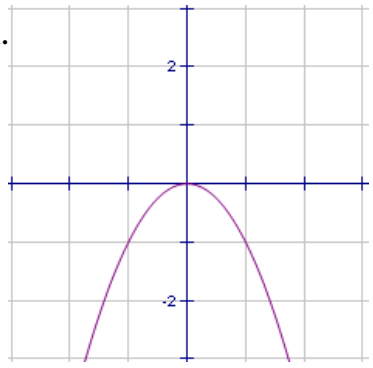
$$x^3 + y^3 = 4xy + 1 \text{ at } (2, 1)$$

a. $y = 1.6x + 2.2$
b. $y = 0.6x + 0.4$
c. $y = 0.63x - 0.25$
d. $y = -1.6x - 2.2$
e. $y = 1.6x - 2.2$

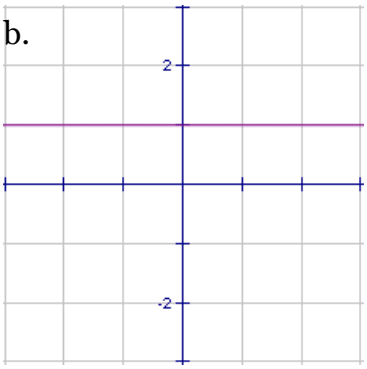
26. The graph of f is shown in the figure. Sketch the graph of the derivative of f.



a.



b.



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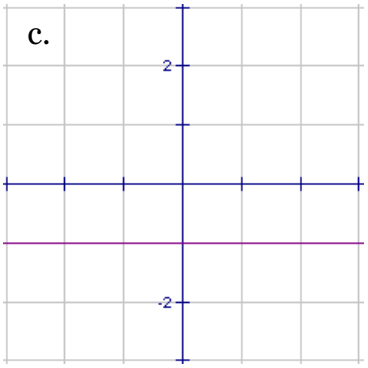
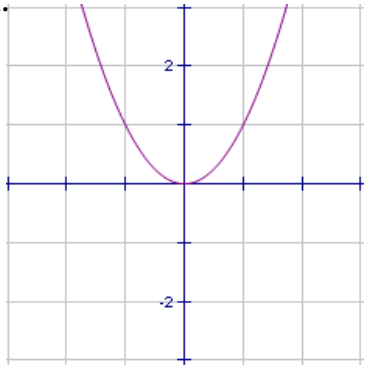
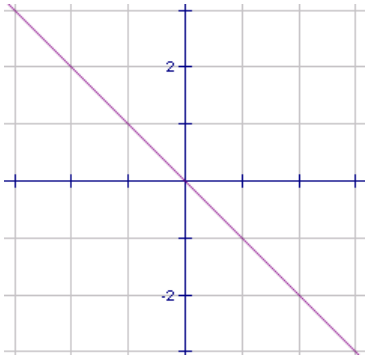


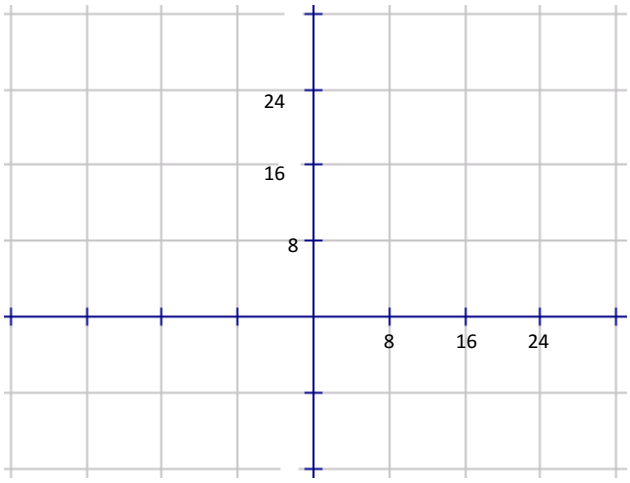
d.



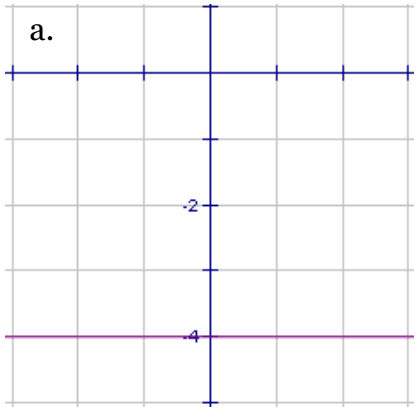
e.



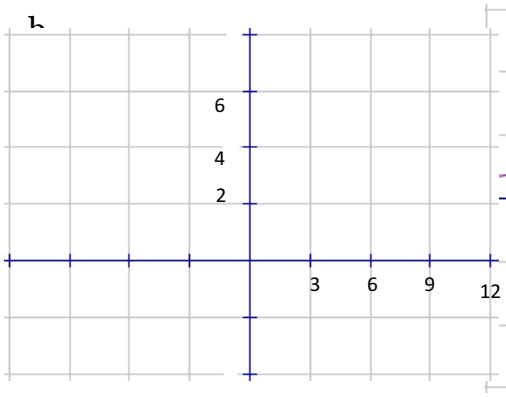
27. The graph of a function *f* is shown below. Which of the following graphs is the graph of its derivative?



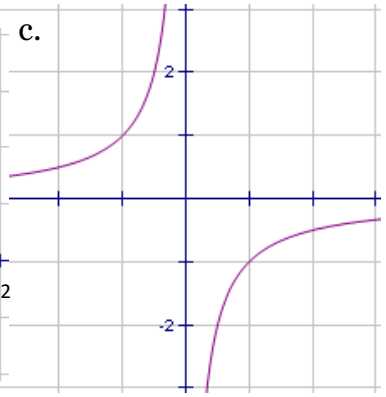
a.



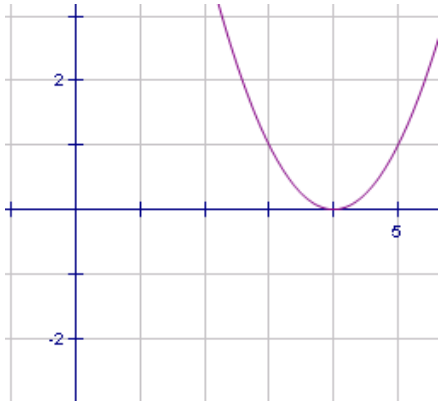
b.



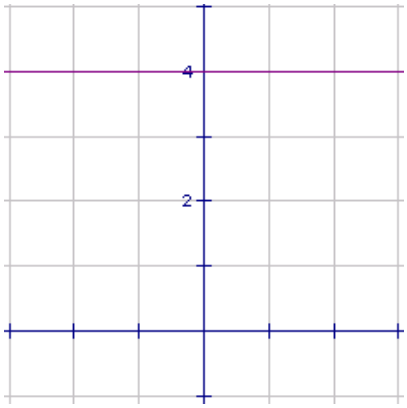
c.



d.



e.



28. Find the points of inflection and discuss the concavity of the function.

$$f(x) = -x^3 + 4x^2 + 5x + 4$$

a. Inflection point at $x = 4/3$; concave upward on $\left(-\infty, \frac{4}{3}\right)$; concave downward on $\left(\frac{4}{3}, \infty\right)$

b. Inflection point at $x = 4/3$; concave downward on $\left(-\infty, \frac{4}{3}\right)$; concave upward on $\left(\frac{4}{3}, \infty\right)$

c. Inflection point at $x = -4/3$; concave upward on $\left(-\infty, -\frac{4}{3}\right)$; concave downward on $\left(-\frac{4}{3}, \infty\right)$

d. Inflection point at $x = -4/3$; concave downward on $\left(-\infty, -\frac{4}{3}\right)$; concave upward on $\left(-\frac{4}{3}, \infty\right)$

e. None of the above

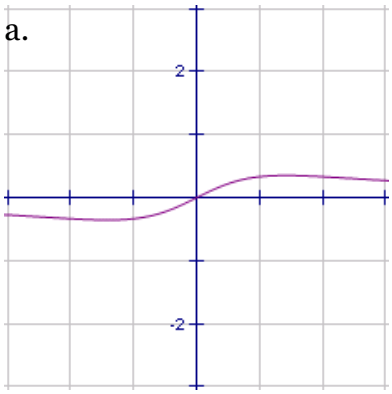
29. Find all relative extrema of the function. Use the Second Derivative Test where applicable.

$$f(x) = x^3 - 3x^2 + 3$$

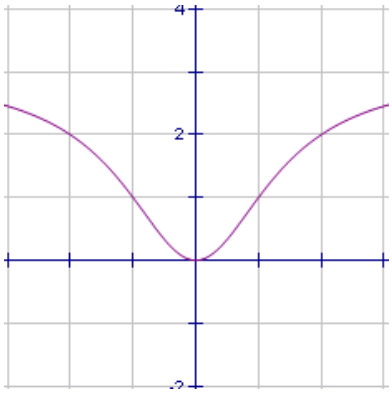
a. Relative min: (2, -1)
b. Relative max: (0, 3)
c. Relative min: (0, 3)
d. Test fails
e. Both A and B

30. Match the function $f(x) = \dfrac{3x^2}{x^2+2}$ with one of the following graphs.

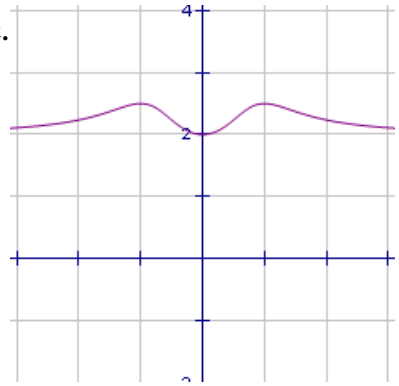
a.



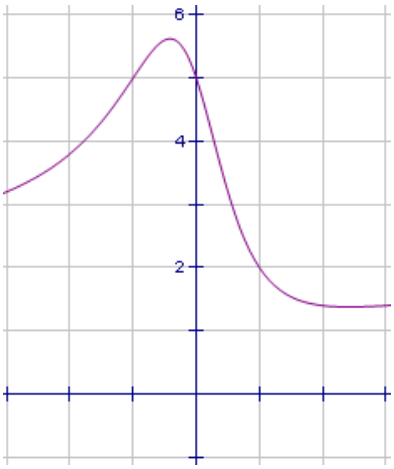
b.



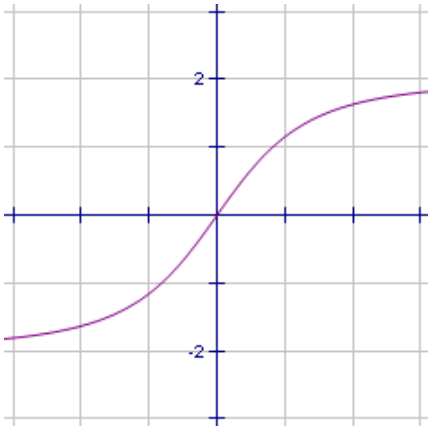
c.



d.



e.

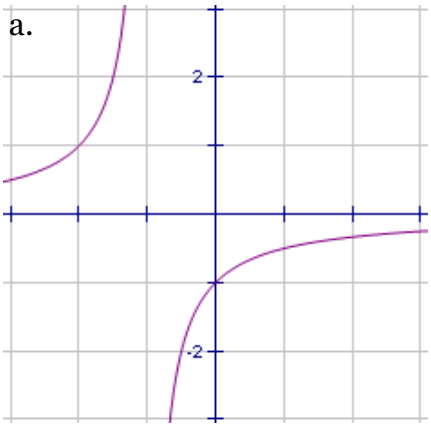


31. Find the limit: $\lim_{x\to\infty}\left(2-\frac{5}{x^4}\right)$.

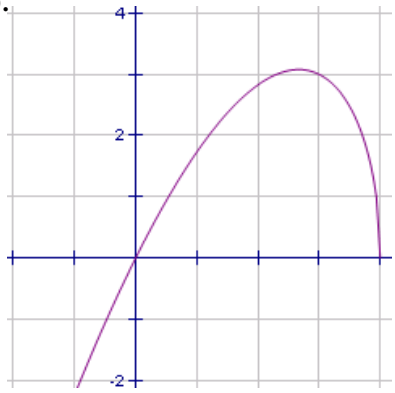
a. -5
b. ∞
c. $-\infty$
d. 2
e. 5

32. Analyze and sketch a graph of the function $y = x\sqrt{4 - x^2}$.

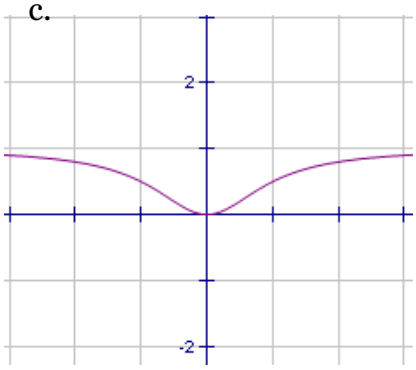
a.



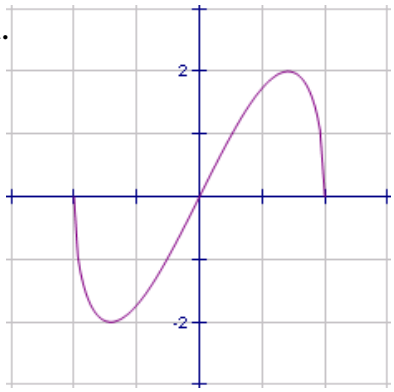
b.



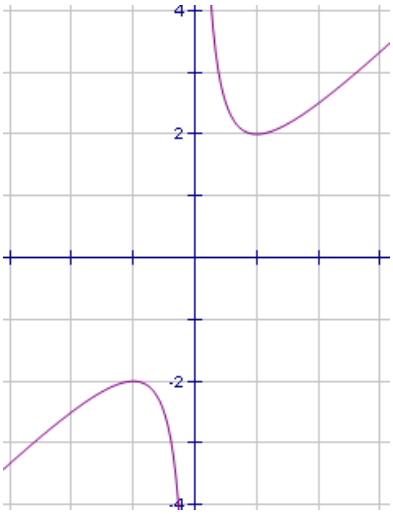
c.



d.



e.



33. A rectangular page is to contain 100 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.
    a. 12 by 12
    b. 10 by 10
    c. 8 by 8
    d. 11 by 11
    e. 9 by 9

34. Find the differential *dy* of the function $y = -2x^{2/5}$

a. $dy = \frac{4}{5}x^{-3/5}dx$

b. $dy = -2x^{-3/5}dx$

c. $\frac{dy}{dx} = -\frac{4}{5}x^{3/5}$

d. $dy = -\frac{4}{5}x^{-3/5}dx$

e. $dy = -\frac{2}{5}x^{-3/5}dx$

35. Locate the absolute extrema of the given function on the close interval [-40, 40].

$$f(x) = \frac{40x}{x^2 + 25}$$

a. Absolute max at x = 5
b. Absolute min at x = -5
c. Absolute min at x = -40
d. Both A and C
e. Both A and B

36. Find $\int \frac{1}{x^2} dx$.

a. $-\frac{1}{x} + C$

b. $\frac{x^3}{3} + C$

c. $-\frac{1}{x} + C$

d. $-\frac{x^3}{3} + C$

e. $\ln|x^2| + C$

37. Find $\int (4x^3 + 6x^2 - 1)dx$.

a. $4x^4 + 6x^3 - x + C$

b. $x^4 + 2x^3 + x + C$

c. $x^4 + 2x^3 - 1 + C$

d. $x^4 + 2x^3 - x + C$

e. $12x^2 + 12x + C$

38. Find $\int \frac{x^2+2x-3}{x^4}\,dx$

a. $-x-x^2+x^3+C$

b. $-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$

c. $\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+C$

d. $\frac{1}{x}+\frac{2}{x^2}-\frac{3}{x^3}+C$

e. $\frac{x^3+2x^2-3x}{x^5}+C$

39. Find $\int (2t^2-1)^2\,dt$.

a. $\frac{(2t^2-1)^3}{3}+C$

b. $\frac{4t^5}{5}-\frac{4t^3}{3}+t+C$

c. $4t^4-4t^2+1+C$

d. $\frac{4t^5}{5}+\frac{4t^3}{3}+t+C$

e. $\frac{2(2t^2-1)^3}{3}+C$

40. Find $\int (\csc^2 x-\cos x)\,dx$.

a. $-\csc x\cot x+\sin x+C$

b. $\frac{\csc^3 x}{3}-\sin x+C$

c. $\cot x+\sin x+C$

d. $\cot x-\sin x+C$

e. $-\cot x-\sin x+C$