

SECTION

10.1

Exercises

1. **Toyota or Nissan?** Are Toyota or Nissan owners more satisfied with their vehicles? Let's design a study to find out. We'll select a random sample of 400 Toyota owners and a separate random sample of 400 Nissan owners. Then we'll ask each individual in the sample: "Would you say that you are generally satisfied with your (Toyota/Nissan) vehicle?" $\rightarrow$ % SATISFACTION

- (a) Is this a problem about comparing means or comparing proportions? Explain.
(b) What type of study design is being used to produce data?

3. **Computer gaming** Do experienced computer game players earn higher scores when they play with someone present to cheer them on or when they play alone? Fifty teenagers who are experienced at playing a particular computer game have volunteered for a study. We randomly assign 25 of them to play the game alone and the other 25 to play the game with a supporter present. Each player's score is recorded.

- (a) Is this a problem about comparing means or comparing proportions? Explain.
(b) What type of study design is being used to produce data?

5. **I want red!** A candy maker offers Child and Adult bags of jelly beans with different color mixes. The company claims that the Child mix has 30% red jelly beans while the Adult mix contains 15% red jelly beans. Assume that the candy maker's claim is true. Suppose we take a random sample of 50 jelly beans from the Child mix and a separate random sample of 100 jelly beans from the Adult mix.

- (a) Find the probability that the proportion of red jelly beans in the Child sample is less than or equal to the proportion of red jelly beans in the Adult sample. Show your work.

$$P(\hat{p}_C \leq \hat{p}_A) = .0213$$

- (b) Suppose that the Child and Adult samples contain an equal proportion of red jelly beans. Based on your result in part (a), would this give you reason to doubt the company's claim? Explain. **YES**

THERE IS ONLY A 2% CHANCE OF GETTING AS FEW OR FEWER RED JELLY BEANS IN THE CHILD SAMPLE THAN THE ADULT SAMPLE IF THE COMPANY'S CLAIM IS TRUE

1A Proportions (% satisfaction) based on categorical data.

1B This is an observational study since there are no treatments.

3A means (score) based on Quantitative data.

3B Randomized experiment - 50 experienced game players are randomly assigned to 2 Groups. Treatment: play alone vs play with a supporter

5A TWO RANDOM VARIABLE
Child's Mix: $p_C = .30$, $n = 50$
Adult Mix: $\hat{p}_A = .15$, $n = 100$

$$\mu_{\hat{p}_C} - \mu_{\hat{p}_A} = .30 - .15 = .15$$

$$\sigma_{\hat{p}_C - \hat{p}_A} = \sqrt{\frac{(.30)(.70)}{50} + \frac{(.15)(.85)}{100}} = .07399$$

$$P(\hat{p}_C \leq \hat{p}_A) = P(\hat{p}_C - \hat{p}_A \leq 0)$$

$$Z = \frac{0 - .15}{.07399} \quad P(Z \leq -2.027) = .0213$$

normalcdf(-E99, -2.027, 0, 1)

SECTION 10.1

Exercises

Explain why the conditions for using two-sample z procedures to perform inference about $p_1 - p_2$ are not met in the settings of Exercises 7 through 10.

7. **Don't drink the water!** The movie *A Civil Action* (Touchstone Pictures, 1998) tells the story of a major legal battle that took place in the small town of Woburn, Massachusetts. A town well that supplied water to eastern Woburn residents was contaminated by industrial chemicals. During the period that residents drank water from this well, 16 of the 414 babies born had birth defects. On the west side of Woburn, 3 of the 228 babies born during the same time period had birth defects.

9. **Shrubs and fire** Fire is a serious threat to shrubs in dry climates. Some shrubs can resprout from their roots after their tops are destroyed. One study of resprouting took place in a dry area of Mexico.⁷ The investigators randomly assigned shrubs to treatment and control groups. They clipped the tops of all the shrubs. They then applied a propane torch to the stumps of the treatment group to simulate a fire. All 12 of the shrubs in the treatment group resprouted. Only 8 of the 12 shrubs in the control group resprouted.

17 EAST SIDE:

$$\hat{p} = 16/414 = .0386$$

$$\# \text{ successes} = 16 > 10 \checkmark$$

$$\# \text{ failures} = 398 > 10 \checkmark$$

WEST SIDE:

$$\hat{p} = 3/228 = .0132$$

$$\# \text{ successes} = 3 > 10 \times$$

- ① NORMAL CONDITION IS NOT MET. WEST SIDE DOES NOT HAVE 10 OR MORE SUCCESSSES.

- ② Random condition is also not met.

9 NORMAL CONDITION IS NOT MET

$$\text{TREATMENT GROUP} - \hat{p} = 12/12 = 1.00?$$

$$n\hat{p} = 12(1.0) = 12 \neq 10 \times \text{NOT MET}$$

$$\text{CONTROL GROUP} - \hat{p} = 8/12 = .667$$

$$n\hat{p} = 12(.667) = 8 \neq 10 \times \text{NOT MET}$$

$$n(1-\hat{p}) = 4 \neq 10 \times \text{NOT MET}$$

#15 +13 COMPLETE YELLOW TEST TEMPLATE

11. **Who uses instant messaging?** Do younger people use online instant messaging (IM) more often than older people? A random sample of IM users found that 73 of the 158 people in the sample aged 18 to 27 said they used IM more often than email. In the 28 to 39 age group, 26 of 143 people used IM more often than email.⁹ Construct and interpret a 90% confidence interval for the difference between the proportions of IM users in these age groups who use IM more often than email.

See attached

13. **Young adults living at home** A surprising number of young adults (ages 19 to 25) still live in their parents' homes. A random sample by the National Institutes of Health included 2253 men and 2629 women in this age group.¹¹ The survey found that 986 of the men and 923 of the women lived with their parents.

(a) Construct and interpret a 99% confidence interval for the difference in population proportions (men minus women).

(b) Does your interval from part (a) give convincing evidence of a difference between the population proportions? Explain.

See Attached

#11

USE "TEST OF SIGNIFICANCE TEMPLATE"

FOLLOW THE FORMATPARAMETERS: p_1 = true proportion of young (18-27) who use IM more than EMAIL p_2 = true proportion of older (28-39) people who use IM more than EMAILTEST Two SAMPLE Z INTERVAL FOR $p_1 - p_2$ SIGNIFICANCE 90% CI for $p_1 - p_2$ CONDITIONS

Random - BOTH SAMPLES WERE SELECTED RANDOMLY

Independent - There are more than 1,580 young people and 1,430 older people in U.S

Normal - Successes + Failures are all greater than 10

$$n_1 p_1 = 73 \checkmark \quad n_1 q_1 = 85 \checkmark \quad n_2 p_2 = 26 \checkmark \quad n_2 q_2 = 117 \checkmark$$

SAMPLING DISTRIBUTION

PLUGS

$$x_1 = 73$$

$$n_1 = 158$$

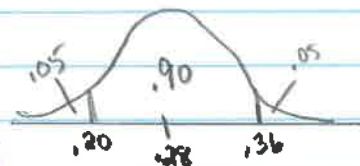
$$\hat{p}_1 = 73/158 = .462$$

$$x_2 = 26$$

$$n_2 = 143$$

$$\hat{p}_2 = 26/143 = .1818$$

$$Z^*_{.005} = 1.64 \text{ via Norm}(.05, 0, 1) = 1.64$$

TEST STATISTIC

$$\hat{p}_1 - \hat{p}_2 \pm Z^* \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$.462 - .182 \pm 1.64 \sqrt{\frac{(.462)(.538)}{158} + \frac{(.182)(.818)}{143}}$$

$$.28 \pm 1.64 (.084)$$

$$.28 \pm .084 \rightarrow (.196, .364)$$

Conclusion: We are 90% CONFIDENT THAT THE INTERVAL, 196 to .364 CAPTURES THE TRUE DIFFERENCE IN THE PROPORTIONS OF YOUNG PEOPLE WHO USE IM MORE THAN EMAIL.

CALC COMMAND
STAT TESTS
B: 2 PROP Z INTERVAL
DO FIRST TO check your work

TIP:

#13

11.18
HWPARAMETERS: P_m = actual proportion of men who live at home P_w = actual proportion of women who live at homeTEST: 2 SAMPLE Z INTERVAL FOR $P_m - P_w$
WITH A 99% Confidence interval

Sig level

CONDITIONS:RANDOM - BOTH SAMPLES WERE RANDOMLY SELECTEDINDEPENDENT - THERE ARE MORE THAN $2253(10) = 22,530$ young men and $2629(10) = 26,290$ young women in the USNORMAL Condition is met: The number of successes and failures are men: 986, 1267 women: 923, 1706Sampling DistributionTEST STATISTIC

SINCE I NAMED IT ABOVE THEN

I DO NOT NEED TO WRITE Formula

$$X_m = 986$$

$$X_w = 923$$

$$n_m = 2253$$

$$n_w = 2629$$

$$\hat{p}_m = .438 \checkmark$$

$$\hat{p}_w = .351 \checkmark$$

$$\hat{p}_m - \hat{p}_w = .087$$

$$Z^* = \text{invNorm}(.005, 0, 1)$$

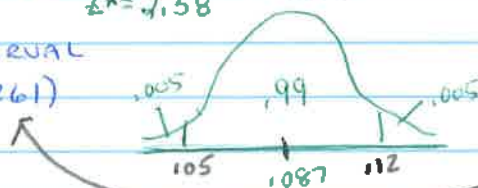
$$Z^* = 2.58$$

$$.087 \pm 2.58 \sqrt{\frac{(.438)(.562)}{2253} + \frac{(.351)(.649)}{2629}}$$

$$.087 \pm 2.58 (.0139)$$

$$.087 \pm .036$$

$$(.051, .123)$$

CALC: Z-PROP Z INTERVAL
(.0505, .12261)

Checks ☺

CONCLUDE: WE ARE 99% CONFIDENT THAT THE INTERVAL FROM .051 TO .123 CAPTURES THE TRUE DIFFERENCE IN PROPORTIONS OF YOUNG MEN AND WOMEN WHO LIVE AT HOME.

- (B) IF THERE IS NO DIFFERENCE, THEN WE WOULD EXPECT IT TO BE ZERO. SINCE THE INTERVAL DOES NOT INCLUDE 0, THEN THERE IS CONVINCING EVIDENCE THAT THE TWO PROPORTIONS ARE NOT THE SAME.

Tip- I like to do these side by side. Follow a format that works for you

15

+

17

PARAMETERS: p_1 = actual proportion of teens with IPOD/MP3
 p_2 = actual proportion of young adults with IPOD or MP3

HYPOTHESIS:

$$H_0: p_1 = p_2$$

OR

$$H_0: p_1 - p_2 = 0$$

$$H_A: p_1 \neq p_2$$

$$H_A: p_1 - p_2 \neq 0$$

TEST: 2 SAMPLE Z TEST FOR p SIGNIFICANCE LEVEL: $\alpha = .05$ CONDITIONS:

- ① Random - Both samples were randomly selected
- ② Independent - There are more than $800(10) = 8,000$ teens and $400(10) = 4,000$ young adults that live in the U.S.
- ③ Normal condition was met. Success + Failure are at least 10
 teens = 632, 168 young adults = 268, 132

Sampling Distribution:STAT TEST 2 PROP ZTEST

$$x_1 = 632$$

$$n_1 = 800 \checkmark$$

$$\hat{p}_1 = .79 \checkmark$$

$$x_2 = 268$$

$$n_2 = 400 \checkmark$$

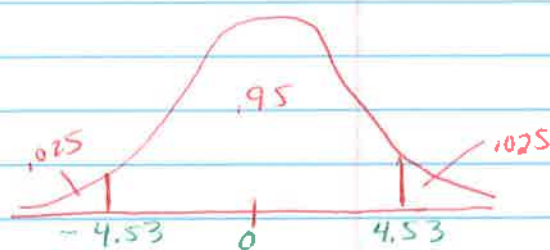
$$\hat{p}_2 = .67 \checkmark$$



$$z = 4.53$$

$$p \approx 0$$

$$\hat{p} = .75$$



$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2} = \frac{632 + 268}{800 + 400} = .75 \checkmark$$

10.1c

17 CONT

TEST STATISTIC :

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}_c \hat{q}_c \cdot \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\begin{aligned}\hat{p}_1 &= .79 \\ \hat{p}_2 &= .67 \\ n_1 &= 800 \\ n_2 &= 400\end{aligned}$$

$$\begin{aligned}\hat{p}_c &= .75 \\ \hat{q}_c &= .25\end{aligned}$$

$$Z = \frac{.79 - .67}{\sqrt{.75(.25) \cdot \left(\frac{1}{800} + \frac{1}{400} \right)}}$$

$$Z = \frac{.12}{(.4330)(.0612)} = \frac{.12}{.02651} = 4.53$$

Z = 4.53 ✓

PVALUE

$$P(Z \leq -4.53) \text{ or } P(Z \geq 4.53) \approx 0$$

normalcdf(-E99, -4.53, 0, 1) ≈ 0 ↑

Pvalue $\approx 0 < .05$ (alpha) Reject H_0

④ Conclude: Since the pvalue $< .05$, we reject H_0 and conclude that the actual proportions of teens and young adults who own iPods/MP3 players are different.

⑤ TEST 2 SAMPLE ZINTERVAL FOR $p_1 - p_2$ (95% CI)

$$\hat{p}_1 - \hat{p}_2 \pm Z^* \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = .79 - .67 \pm 1.96 \sqrt{\frac{(.79)(.21)}{800} + \frac{(.67)(.33)}{400}}$$
$$.12 \pm 1.96(.0276)$$
$$.12 \pm .054 \quad | (.066, .174) | \checkmark$$

CALC: STAT
TESTS
Z-PROZINT
(.066, .174)

CONCLUDE: we are 95%

confident that the interval .066 to .174 captures the difference in proportions of teens and young adults who own iPods or MP players. This is consistent with our decision to Reject H_0 . In both cases we ruled out the difference of proportion being 0 as a plausible value.

21 PARAMETERS:

p_1 = the actual proportion of women with a family history of breast cancer who are assigned to the low fat diet

p_2 = the actual proportion of women ... assigned normal diet

Hypothesis: $H_0: p_1 = p_2$

$H_A: p_1 \neq p_2$

Significance: Calculate

TEST: 2 SAMPLE Z TEST FOR $p_1 - p_2$

Conditions: Random - This was a randomized study

Independent - Due to the random assignment, these 2 groups of women can be viewed as independent.

Normal - The samples are large and success and

failures are all above 10 - Low fat: $3396 + 16,145$
normal diet: $4,929 + 24,365$

Sampling Distribution

TEST STATISTICS

Z PROP Z TEST
↓

$$x_1 = 3396$$

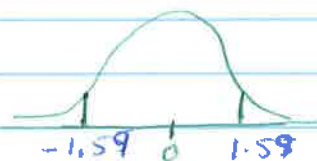
$$n_1 = 19,541$$

$$\hat{p}_1 = .1737$$

$$x_2 = 4929$$

$$n_2 = 29,294$$

$$\hat{p}_2 = .1683$$



$$\bar{z} = \hat{p}_1 - \hat{p}_2 - 0$$

$$\sqrt{\hat{p}_1 \hat{q}_1} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\bar{z} = .1737 - .1683 = .0054$$

$$\sqrt{.171(.829)} \sqrt{\frac{1}{19541} + \frac{1}{29294}}$$

$$\boxed{Z = 1.55}$$

$$\hat{p}_c = \frac{3396 + 4929}{19,541 + 29,294} = .1705$$

P-VALUE

$$P(Z \leq -1.59) \text{ OR } P(Z \geq 1.59) =$$

$$\text{normal cdf}(1.59, \infty, 0, 1) = .056 \times 2 = .111$$

$\boxed{P\text{VALUE} = .111}$ SINCE p is large "FAIL TO REJECT"

$$Z = 1.59$$

$$p = .111$$

LET ME KNOW IF
YOU GOT $Z = 1.73$
 $P\text{value} = .0836$

21 CONT

- (a) conclude: Since the p-value (.11) is large $> .05$, we fail to reject H_0 . We do not have enough evidence to conclude there is a statistically significant difference in the proportion of women assigned to the 2 groups (low-fat vs regular diet) who have a family history of breast cancer.

21b A TYPE I ERROR would be to say that the groups are significantly different when they are not.

A TYPE II ERROR would be to say that the 2 groups ARE NOT SIGNIFICANTLY DIFFERENT WHEN THEY ARE. A TYPE 2 ERROR WOULD BE MORE SERIOUS BECAUSE THE EXPERIMENT WOULD PROCEED ASSUMING THAT THE 2 GROUPS WERE SIMILAR TO BEGIN WITH. ANY CONCLUSIONS ABOUT THE DIFFERENCE BETWEEN THE 2 GROUPS AT THE END OF THE STUDY WOULD THEN BE SUSPECT.

23

PARAMETERS:

p_1 = actual proportion of women getting pregnant who do receive prayers

p_2 = actual proportion of women getting pregnant who did NOT receive prayers

HYPOTHESIS:

$H_0: p_1 = p_2$

$H_A: p_1 > p_2$ "Do prayers help?"

TEST: 2 sample Z TEST For $p_1 - p_2$

CONDITIONS: Random - this was a randomized experiment

Independent - due to the random assignment, these 2 groups of women can be viewed as independent.

Normal - Condition met since success & failures at least 10.

Prayer: 44, 44 non-prayer: 21, 60

Sampling Distribution



TEST STATISTIC:

2PROP Z TEST

$x_1 = 44$

$x_2 = 21$

$n_1 = 88$

$n_2 = 81$

$\hat{p}_1 = .5$

$\hat{p}_2 = .259$

$Z = \hat{p}_1 - \hat{p}_2 - 0$

$\sqrt{\hat{p}_c \hat{q}_c} \cdot \sqrt{1/n_1 + 1/n_2}$

$\hat{p}_c = \frac{44+21}{88+81} = .385 \checkmark$

$Z = .5 - .259 = .241$

$\sqrt{(.385)(.615)} \cdot \sqrt{1/88 + 1/81}$

$Z = 3.21$

$p \approx 0$

$Z = 3.21 \checkmark$

PVALUE

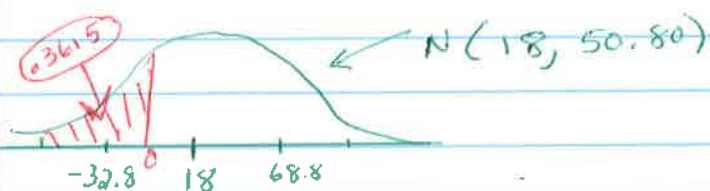
$P(Z \geq 3.21) = \text{normalcdf}(3.21, 999, 0, 1) \approx 0.0007$

$p\text{value} = .0007 < .05 (\alpha)$ Reject H_0

23 Cont

- (a) answered on prior page
- (b) $p = .0007$ $\rightarrow$ IF THERE IS NO DIFFERENCE IN PREGNANCY RATES OF WOMEN WHO ARE BEING PRAYED FOR AND THOSE WHO ARE NOT, THERE IS A 0.07% CHANCE OF SEEING AS MANY OR MORE PREGNANCIES WHILE BEING PRAYED FOR AS WE DID.
- (c) SINCE $p\text{-value} < .05$, WE REJECT H_0 . WE HAVE ENOUGH EVIDENCE TO CONCLUDE THAT THE PROPORTION OF PREGNANCIES AMONG WOMEN LIKE THESE WHO ARE PRAYED FOR IS HIGHER THAN THAT AMONG WOMEN WHO ARE NOT PRAYED FOR.
- (d) IF THE WOMEN HAD KNOWN WHETHER THEY WERE BEING PRAYED FOR, THIS MIGHT HAVE AFFECTED THEIR BEHAVIOUR IN SOME WAY (EVEN UNCONSCIOUSLY) THAT WOULD HAVE AFFECTED WHETHER THEY BECAME PREGNANT OR NOT.

35A

 $M = 20-34 \text{ men} \quad - \quad N(188, 41)$ $B = 14 \text{ yr boy} \quad - \quad N(170, 30)$ DISTRIBUTION OF $M-B$ SHAPE: M and B are Normal, so $M-B$ is NormalCENTER: $\mu_{M-B} = 188 - 170 = 18 \text{ mg/dL}$ SPREAD: $\sigma_{M-B} = \sqrt{\sigma_M^2 + \sigma_B^2} = \sqrt{41^2 + 30^2} = 50.80 \text{ mg/dL}$ 

35B $P(B > M) = P(0 > M-B) = P(M-B < 0) = 0.3615$
 normalcdf(-E99, 0, 18, 50.8)

Method 2
find Z

$Z = \frac{0-18}{50.8} = -0.35 \quad P(Z \leq -0.35) = \text{normalcdf}(-E99, -0.35, 0, 1) = 0.3632$

37A

DISTRIBUTION $\bar{X}_M - \bar{X}_B$ $n_M = 25 \quad n_B = 36$

SHAPE: NORMAL

CENTER: $\mu_{\bar{X}_M - \bar{X}_B} = M - B = 18 \text{ mg/dL}$ SPREAD: $\sigma_{\bar{X}_M - \bar{X}_B} = \sqrt{\frac{\sigma_M^2}{n_M} + \frac{\sigma_B^2}{n_B}} = \sqrt{\frac{41^2}{25} + \frac{30^2}{36}} = 9.60 \text{ mg/dL}$

37B $P(\bar{X}_M - \bar{X}_B < 0) = \text{normalcdf}(-E99, 0, 18, 9.6) = 0.0304$

OR FIND $Z = \frac{0-18}{9.6} = -1.88 \quad P(Z < -1.88) = 0.0301$

37C

IT SOMETIMES MAKES SENSE TO REWRITE $P(\bar{X}_M < \bar{X}_B) = 0.03$

* YES IT WOULD BE SURPRISING TO HAVE THE SAMPLE MEAN OF THE BOYS GREATER THAN THE MEN SINCE THE PROBABILITY IS ONLY 3%.

39] THE NORMAL CONDITION IS NOT MET. ^{$N \leq 30$} THE GRAPH OF MALES SHOWS A SMALL SAMPLE ($n=20$) AND 2 OUTLIERS (35+38 pairs of shoes)

41] NO INDEPENDENT CONDITION NOT MET. More than 1 person from a household violates independence. This is an obscure question but interesting thought to keep in mind.

43]

(a) THE CENTERS OF THE TWO GROUPS SEEM TO BE QUITE DIFFERENT, WITH PEOPLE DRINKING RED WINE GENERALLY HAVING MORE POLYPHENOL IN THEIR BLOOD

THE SPREAD, HOWEVER ARE APPROXIMATELY THE SAME.

(b) PARAMETERS

μ_R = actual mean change in polyphenol drinking red wine

μ_W = actual mean change in polyphenol drinking white wine

TEST: A 2 Sample t interval for $\mu_1 - \mu_2$

We want to estimate the difference $\mu_R - \mu_W$ at a 90% confidence level

CONDITIONS:

RANDOM: THIS WAS A RANDOMIZED EXPERIMENT

NORMAL: BOTH SAMPLE SIZES ARE LESS THAN 30. THE GIVEN DOT PLOT DOES NOT INDICATE AND OUTLIERS OR NO SERIOUS SKEWNESS.

INDEPENDENT:

① DUE TO RANDOM ASSIGNMENT, THESE 2 GROUPS OF MEN CAN BE VIEWED AS INDEPENDENT

② INDIVIDUALS CHANGE IN POLYPHENOL LEVEL GIVES NO INFORMATION ABOUT ANOTHER INDIVIDUAL.

6 UNKNOWN ($\pm$ inference)

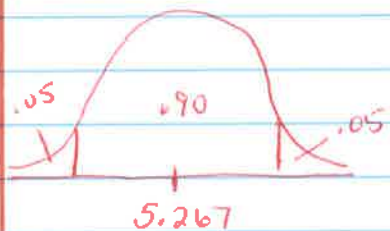
43 CONT

enter Red Wine in L1 and White Wine in L2

ZVAR STATS

Red Wine : $\bar{x} = 5.5$ $s_x = 2.5169$ $n = 9$ White Wine : $\bar{y} = .233$ $s_y = 3.292$ $n = 9$

$$90\% \text{ CI: } (5.5 - .233) \pm t^* \cdot \sqrt{\frac{(2.517)^2}{9} + \frac{(3.292)^2}{9}}$$



$$5.267 \pm (1.860)(1.381)$$

$$5.267 \pm 2.569$$

$$(2.698, 7.836)$$

Conservative $df = 8$

$$t^* = \text{INV}T(.05, 8) = 1.860$$

Check w/ CALC STAT TESTS O: 2-SAMP T INTERVAL

> DATA, L1, L2, 1, 1, .9, (NO) TECHNOLOGY TIP
ALWAYS RECOMMEND
"NO" TO POOLING

$$(2.845, 7.689) \quad df = 14.97$$

$$\bar{x}_1 = 5.5$$

$$\bar{x}_2 = .23$$

$$s_{x_1} = 2.517$$

$$s_{x_2} = 3.292$$

$$n_1 = 9$$

$$n_2 = 9$$

CONCLUDE: We are 90% Confident that the interval 2.70 to 7.84 CAPTURES THE TRUE DIFFERENCE IN THE ACTUAL MEAN CHANGE IN POLYPHENOL LEVEL IN MEN WHO DRINK RED WINE AND MEN WHO DRINK WHITE WINE.

Ⓛ SINCE THIS INTERVAL DOES NOT CONTAIN 0, IT DOES SUPPORT THE RESEARCHER'S BELIEF THAT THE CHANGE IN POLYPHENOL LEVEL IS DIFFERENT FOR MEN WHO DRINK RED WINE THAN THOSE WHO DRANK WHITE WINE.

45 [A] THE DISTRIBUTIONS ARE SKEWED TO THE RIGHT BECAUSE THE EARNINGS AMOUNTS CANNOT BE NEGATIVE, YET THE STANDARD DEVIATIONS ARE ALMOST AS LARGE AS THE DISTANCE BETWEEN THE MEANS AND ZERO.

THE USE OF THE TWO-SAMPLE T PROCEDURE IS JUSTIFIED BECAUSE OF THE LARGE SAMPLE SIZES.

[B] PARAMETERS: μ_m = actual mean summer earnings of male students
 μ_f = actual mean earnings, female students

CONDITIONS: UNKNOWN (+ inference)

Random - both samples were randomly selected

Normal - both samples are at least 30

Independent - reasonable - there are more than 6,750 males and 6,210 females with summer-jobs

TEST: 2 SAMPLE T INTERVAL FOR $\mu_1 - \mu_2$ 90%

CALCULATOR

2-Samp TINT

STATS

$$\bar{x}_1 = 1884.52$$

$$s_{x_1} = 1368.37$$

$$n_1 = 675$$

$$\bar{x}_2 = 1360.39$$

$$s_{x_2} = 1037.46$$

$$n_2 = 621$$

C-LEVEL = 90

POOLED = NO

$$\rightarrow (413.62, 634.64)$$

$$ds = 1249.2$$

$$\bar{x}_m - \bar{x}_f \pm t^* \sqrt{\frac{s_m^2}{n_m} + \frac{s_f^2}{n_f}}$$

$$1884.52 - 1360.39 \pm 1.647 \sqrt{\frac{1368.37^2}{675} + \frac{1037.46^2}{621}}$$

$$t^* = \text{invT}(0.05, 620) = 1.647$$

$$df_m = 674$$

$$df_f = 620$$

$$524.13 \pm 1.647(67.136)$$

$$524.13 \pm 110.572$$

$$(413.56, 634.64)$$

CONCLUDE: WE ARE 90% CONFIDENT THAT THE TRUE DIFFERENCE IN MEAN SUMMER EARNINGS OF MALE AND FEMALE STUDENTS IS BETWEEN 413.56 and 634.64.

45 cont

C IF WE REPEATEDLY TOOK RANDOM SAMPLES OF 675 males and 621 females from the same university and each time constructed a 90% confidence interval in this same way about 90% of the resulting intervals would capture the actual difference in mean summer earnings

[5]

GROUPS

WITH ACTIVITIES	$n_1 = 21$	$\bar{x}_1 = 51.5$	$S_1 = 11.6$
CONTROL	$n_2 = 23$	$\bar{x}_2 = 41.5$	$S_2 = 17.1$

(A) Remember Cuss and BS

The activity groups center (51.5) is higher than the control (41.5)

The Control group has a larger spread (17.1) compared to the activity (11.6) spread.

The shape of the activity group appears to be slightly skewed to the left while the shape of the control group appears more symmetric. Neither group appears to have any outliers.

(B)

Parameters:

μ_1 = actual mean DRP for 3rd graders doing the activity
 μ_2 = actual mean DRP for 3rd graders Not doing activity

$$H_0: \mu_1 = \mu_2 \quad (\text{the same})$$

$$H_A: \mu_1 > \mu_2 \quad (\text{activity better})$$

$$\text{OR} \quad H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 > 0$$

Conditions

σ is unknown (+inference)

Random - this was a randomized comparative study

Normal - The box plot shows neither outliers or strong skewness

INDEPENDENT - Due to random assignment these 2 groups can be viewed as independent

510 CONT

TEST - 2 SAMPLE T TEST FOR $\mu_1 - \mu_2$

$$t^*: df_1 = 21 - 1 = 20 \quad \text{Select the conservative} \quad \alpha = 0.05$$

$$df_2 = 23 - 1 = 22$$

$$df = 20$$

Calc

2-Samp TTest
Pooled: NO

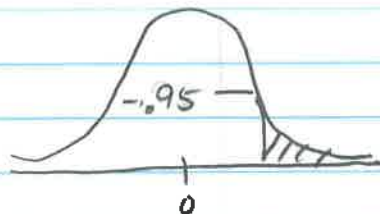
$$t = 2.33$$

$$p = .0127$$

$$df = 37.9$$

$$t_{20} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_1^2/n_1 + S_2^2/n_2}} = \frac{51.5 - 41.5}{\sqrt{\frac{11.0^2}{21} + \frac{17.1^2}{23}}}$$

$$t_{20} = 2.33$$



P-value: $P(t_{20} > 2.33) = t\text{-cdf}(2.33, E99, 20) = .0152$

Conclude: Since $p\text{-value} (.0152) < .05$, we Reject H_0 .
We have sufficient evidence to conclude there is a difference in the actual mean DRP scores for 3rd graders doing the activity and those not doing the activity.

510

Since this was a randomized controlled experiment, we can conclude that the activities caused an increase in the mean DRP Score.

510

95% 2 SAMPLE T INTERVAL FOR $\mu_1 - \mu_2$

CALC

2SAMP TINT

Pooled: NO

USING FULL Decimals

$$(1.233, 18.68)$$

$$df = 37.9$$

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$51.5 - 41.5 \pm 2.086 \sqrt{\frac{11^2}{21} + \frac{17.1^2}{23}}$$

$$10 \pm 8.966$$

$$(1.034, 18.966)$$

$$t_{20}^* = \text{INV T}(.025, 20) = 2.086$$

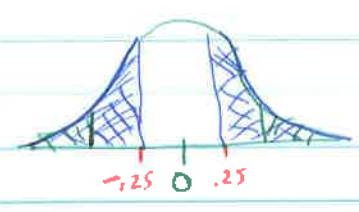
CONCLUDE: WE ARE 95% Confident the interval 1.03 to 18.97 captures the difference in actual mean DRP Scores for 3RD Grades that do the activity and those that do not.

53A

(μ_1) Women: $n_1 = 56$ $\bar{x}_1 = 16,177$ $s_1^2 = 7520$
 (μ_2) Men: $n_2 = 56$ $\bar{x}_2 = 16,569$ $s_2^2 = 9108$

μ_1 = actual mean number of words spoken by Women per day
 μ_2 = " " " " " " by men per day

$H_0: \mu_1 = \mu_2$
 $H_A: \mu_1 \neq \mu_2$



Conservative
 $df = 55$

CONDITIONS:

- Random: both samples randomly selected
- Normal: both samples are over 30 ($n = 56$)
- Independent: the groups (men + women) are independent.
- Reasonable to assume there are more than 560 men and 560 women at the school
- σ is UNKNOWN ($\pm$ inference)

TEST: 2 SAMPLE T-test for $\mu_1 - \mu_2$

$$t_{55} = \frac{16177 - 16569}{\sqrt{\frac{7520^2}{56} + \frac{9108^2}{56}}} = \frac{-392}{-} = -.248 \checkmark$$

CALC
 2SAMP TTEST
 Pooled (No)
 ↓

PVALUE: $P(t \leq -.248)$ or $P(t \geq .248) = .8050$
 $t.cdf(-.248, .248, 55) = .4025 \times 2$

$\mu_1 \neq \mu_2$
 $t = -.248 \checkmark$
 $p = .80$
 $df = 106.2$

CONCLUDE: Since the pvalue (.8050) $> .05$, WE FAIL TO REJECT H_0 . We do not have enough evidence to conclude that male and female students speak a different number of words per day on average.

538

Interpretation of P-value in context:

IF MALES AND FEMALES AT THIS UNIVERSITY SPEAK THE SAME NUMBER OF WORDS PER DAY ON AVERAGE, THEN WE HAVE ABOUT AN 80% CHANCE OF SELECTING A SAMPLE WHERE THE DIFFERENCE BETWEEN THE AVERAGE NUMBER OF WORDS SPOKEN PER DAY BY MALES AND FEMALES IS AS LARGE AS OR LARGER THAN THE DIFFERENCE WE ACTUALLY SAW.

59

PAIRED T OR 2 SAMPLE T

(a) TEST 2 DIFFERENT TIRES - 2 SAMPLE T TEST

(b) TEST EFFECT OF BACKGROUND MUSIC.
THE SAME SUBJECTS WORKED A
MONTH WITHOUT MUSIC. THEN A
MONTH WITH MUSIC - PAIRED T TEST

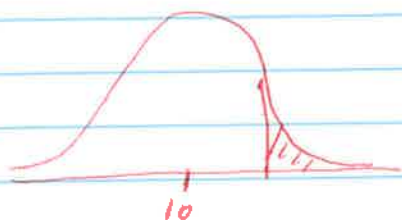
(c) SUBJECT WERE RANDOMLY ASSIGNED TO 2
GROUPS, ONE GROUP USED DIET A AND
THE OTHER GROUP DIET B - 2 SAMPLE
T test.

[65] μ_1 = actual mean cholesterol reduction using new drug
 μ_2 = actual " " " using current drug

$$H_0: \mu_1 - \mu_2 = 10$$

$$H_A: \mu_1 - \mu_2 > 10$$

mean cholesterol reduction with new drug is more than 10 mg/dl than the current drug



Conditions: μ is unknown (t inference)

Random: This was a randomized controlled experiment

Normal: Both samples has fewer than 30 observations, but we were told that no strong skewness or outliers were detected.

Independent: Due to random assignment, these 2 groups are ^{viewed as} independent. Also, knowing one patient's reduction in cholesterol gives no information about another patient's reduction in cholesterol.

TEST: 2 SAMPLE TTEST FOR $\mu_1 - \mu_2$

μ_1 : new drug - $n_1 = 15$ $\bar{x}_1 = 68.7 \text{ mg/dl}$ $s_1 = 13.3 \text{ mg/dl}$
 μ_2 : current drug - $n_2 = 14$ $\bar{x}_2 = 54.1$ $s_2 = 11.93$

$$df_1 = 14$$

$$df_2 = 13 \leftarrow \text{Conservative } df$$

$$df = 13$$

$$t = \frac{68.7 - 54.1 - (10)}{\sqrt{\frac{(13.3)^2}{15} + \frac{(11.93)^2}{14}}} = \frac{4.6}{4.686} = 0.982$$

6.5 CONTINUED

NOTE - Cannot use Calc for this ques.

$$t = .982 \quad df = 13$$

$$P\text{-value} = P(t \geq .982) = tcd f(.982, E99, 13) = .1720$$

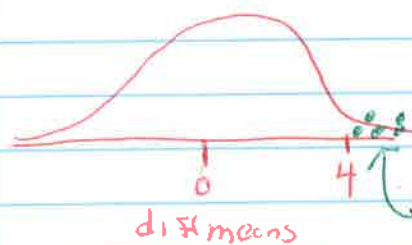
Conclude: Since the $p\text{-value}(.1720) > .05$, We fail to reject H_0 . We do not have enough evidence to conclude that the mean cholesterol reduction is more than 10 mg/dl more for the new drug than for the current drug.

6.5B

Since we Fail to Reject H_0 based on $\alpha = .05$ which means we did not find a Type I error; we could only have the possibility of a Type 2 error.

57A Researchers randomly assign the subjects to the 2 treatment groups to help balance out the effects of external variables.

57B THE ACTUAL EXPERIMENT $\bar{X}_I - \bar{X}_E = 4.15$
THE GRAPH OF THE SAMPLING DISTRIBUTION SHOWS ABOUT 5 OF THE 1,000 differences were 4.15 or bigger



$$H_0: \mu_I - \mu_E = 0$$

$$H_A: \mu_I - \mu_E > 0$$

$$5/1000 = .005 \rightarrow P(\bar{X}_I - \bar{X}_E \geq 4.15) = .005$$

$$\text{Since } P\text{-value} = .005 < .05$$

then we Reject H_0

Conclude: We would conclude that the mean rating for those with internal reasons is significantly higher than those with external reasons.

57C This is a type I error, since we rejected $H_0: \mu_I - \mu_E = 0$