6.2a HW

37 Get on the boat! A small ferry runs every half hour from one side of a large river to the other. The number of cars X on a randomly chosen ferry trip has the probability distribution shown below. You can check that $\mu_X = 3.87$ and $\sigma_X = 1.29$.

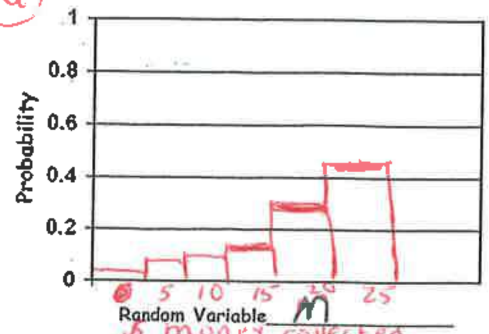
	\$0	\$5	\$10	\$15	\$20	\$25
Cars:	0	1	2	3	4	5
Probability:	0.02	0.05	0.08	0.16	0.27	0.42

(a) The cost for the ferry trip is \$5. Make a graph of the probability distribution for the random variable M = money collected on a randomly selected ferry trip. Describe its shape.

(b) Find and interpret μ_M.

(c) Compute and interpret σ_M. $\mu_X = 3.87$

(a)



The graph is skewed to the left. MOST OF THE TIME THE FERRY MAKES \$20 or \$25

(b) $\mu_M = 5 \cdot \mu_X = 5(3.87) = \$19.35$

The ferry makes about \$19.35 per trip, on AVERAGE

(c) $\sigma_X = 1.29$ $\sigma_M = 5\sigma_X = 5(1.29) = \$6.45$

The individual amount made on the ferry trips will vary by about \$6.45 from the mean (\$19.35), on average

39 LET G = grade of a randomly chosen student

(a) $\mu_G = 10(7.6) = 76$

(b) $\sigma_G = 10(1.32) = 13.2$

(c) $\sigma_X^2 = (1.32)^2$

$\sigma_G^2 = (10 \cdot 1.32)^2 = 100 \cdot \sigma_X^2$

The variance of G is a 100 times that of X.

40 (a) $\text{Median}_G = 8.5(10) = 85$

(b) $IQR_G = Q3 - Q1 = 9(10) - 8(10) = 10$

(c) The shape of the G distribution would be the same as the shape of the X distribution. Since the mean is less than the median, the distribution is skewed left.

6.2

41 **Get on the boat!** Refer to Exercise 37. The ferry company's expenses are \$20 per trip. Define the random variable Y to be the amount of profit (money collected minus expenses) made by the ferry company on a randomly selected trip. That is, $Y = M - 20$.

(a) How does the mean of Y relate to the mean of M? Justify your answer. What is the practical importance of μ_Y?

(b) How does the standard deviation of Y relate to the standard deviation of M? Justify your answer. What is the practical importance of σ_Y?

(A) The mean of Y is \$20 less than The mean of M. The mean of M is \$19.35. Thus the ferry company loses on average \$0.65 per trip.

(B) Since we are subtracting a constant, the variance and standard deviation are the same for both M and Y. Hence, the individual profits made on the ferry trips will vary by about \$6.45 from the mean (−\$.65), on average.

42. **The Tri-State Pick 3** Most states and Canadian provinces have government-sponsored lotteries. Here is a simple lottery wager, from the Tri-State Pick 3 game that New Hampshire shares with Maine and Vermont. You choose a number with 3 digits from 0 to 9; the state chooses a three-digit winning number at random and pays you \$500 if your number is chosen. Because there are 1000 numbers with three digits, you have probability 1/1000 of winning. Taking X to be the amount your ticket pays you, the probability distribution of X is

Payoff X:	\$0	\$500
Probability:	0.999	0.001

(a) Show that the mean and standard deviation of X are $\mu_X = \$0.50$ and $\sigma_X = \$15.80$.

(b) If you buy a Pick 3 ticket, your winnings are $W = X - 1$, because it costs \$1 to play. Find the mean and standard deviation of W. Interpret each of these values in context.

42 (a) $\mu_X = 0(.999) + 500(.001) = \boxed{\$.50}$

$\sigma_X^2 = (0-.5)^2(.999) + (500-.5)^2(.001) = 249.75$

$\sigma_X = \sqrt{249.75} = \boxed{\$15.80}$

(b) $W = X - 1$ $\quad \mu_W = .5 - 1 = \boxed{-\$.5}$

σ_W is the same $= \boxed{\$15.80}$

on average, when playing this game, people will lose \$.50. Individual outcomes will vary from this amount by \$15.80, on average.

6.2

43, 45, 47
48

43 From #37: $\mu_x = 3.87$ $\sigma_x = 1.29$
From #41 $\mu_y = \mu_M - 20$ where M = money collected
Y = PROFIT

$E(Y) = \mu_Y = 6\mu_X - 20$
$= 6(3.87) - 20 = \$3.22$

$\sigma_Y = 6\sigma_X = 6(1.29) = \$7.74$

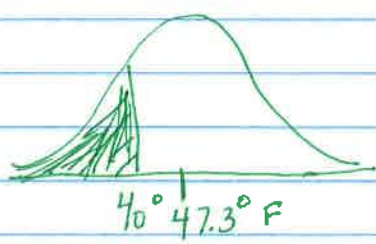
45 T = Cabin temp at midnight $N(8.5°C, 2.25°C)$

(a) Y = Cabin temp at midNIGHT IN °F

$F = (9/5)C + 32$

$E(Y) = \mu_Y = (9/5)(8.5) + 32 = 47.3°F$
$SD(Y) = \sigma_Y = (9/5)(2.25) = 4.05°F$

(b)



$P(Y < 40) = P\left(Z < \frac{40 - 47.3}{4.05}\right)$
$= P(Z < -1.80) = .0359$
normalcdf(-E99, -1.8, 0, 1)

There is about a 3.6% chance the cabin temp.
at midnight will be below 40°F

$\boxed{6.2}$

(47)

X	1	2	5
$P(x)$	.2	.5	.3

Y	2	4
$P(Y)$	.7	.3

$\mu_x = 2.7$ $\quad$ $\mu_y = 2.6$

$\sigma_x = 1.55$ $\quad$ $\sigma_y = .917$

(a) $T = X + Y$ (X and Y are independent)

possible values of T

$1+2=3$
$1+4=5$
$2+2=4$
$2+4=6$
$5+2=7$
$5+4=9$

T	Probability	
3	.14	1+2 (.2)(.7)
4	.35	2+2 (.5)(.7)
5	.06	1+4 (.2)(.3)
6	.15	2+4 (.5)(.3)
7	.21	5+2 (.3)(.7)
9	.09	5+4 (.3)(.3)
	1.00	

CALCULATE USING SAMPLE SPACE

$E(T) = \mu_T = 3(.14) + 4(.35) + 5(.06) + 6(.15) + 7(.21) + 9(.09) = 5.3$

Now use the relationship between T, X, Y

$\mu_T = \mu_x + \mu_y = 2.7 + 2.6 = 5.3$ Notice these are EQUAL ✓

(b) $VAR(T) = (3-5.3)^2(.14) + \ldots (9-5.3)^2(.09) = 3.25$

Tip: USE LISTS $L1 = T_i$ $\quad$ $L4 = (L3)^2$

$L2 = p_i$ $\quad$ $L5 = L4 \cdot L2$

$L3 = L_1 - 5.4$ $\quad$ 1-VARSTAT $\boxed{L5}$

$\Sigma x = 3.25$ $\quad \sigma^2$

Now use RV's: T, X, Y

$SD(T) = \sigma_x + \sigma_y = 1.55 + .917 = 2.467$ X Can't Do!

$VAR(T) = \sigma_x^2 + \sigma_y^2 = (1.55)^2 + (.917)^2 = 3.24$ OFF DUE TO ROUNDING ERROR

NOT EQUAL

$SD(T) = \sigma_T = \sqrt{3.24} = 1.8$

6.2

FOR #48 ONLY DO A

48A

A

x	1	2	5
$P(x)$	.2	.5	.3

y	2	4
$P(y)$	.7	.3

$D = x - y$

Sample space

$1-2 = -1$
$1-4 = -3$
$2-2 = 0$
$2-4 = -2$
$5-2 = 3$
$5-4 = 1$

T	PROBABILITY
-3	.06
-2	.15
-1	.14
0	.35
1	.09
3	.21
	1.00

$P(D=-3) = P(x=1) \cdot P(y=4) = .2(.3)$
$P(D=-2) = P(x=2) \cdot P(y=4) = .5(.3)$
$P(D=-1) = P(x=1) \cdot P(y=2) = .2(.7)$
$P(D=0) = P(x=2) \cdot P(y=2) = .5(.7)$
$P(D=1) = P(x=5) \cdot P(y=4) = .3(.3)$
$P(D=3) = P(x=5) \cdot P(y=2) = .3(.7)$

B COMPARE FIND $E(T)$ and $VAR(T)$ ① Using RV'S ② Using CALC

$E(T) = \mu_T = \mu_x - \mu_y = 2.7 - 2.6 = 0.1$

$VAR(T) = \sigma_T^2 = \sigma_x^2 + \sigma_y^2 = (1.55)^2 + (0.917)^2 = 3.24$

$\sigma_T = 1.80$

NOW USE CALC TO Check: ① $L1 = T_i$ $L2 = p_i$

② 1-VAR STAT LIST: L1 FREQ LIST: L2

$\bar{x} = .1$* $\Sigma x = .1$
$\sigma x = 1.80$*
*Checks w/ RV calc's

6.2

49

49. **Checking independence** In which of the following games of chance would you be willing to assume independence of X and Y in making a probability model? Explain your answer in each case.

(a) In blackjack, you are dealt two cards and examine the total points X on the cards (face cards count 10 points). You can choose to be dealt another card and compete based on the total points Y on all three cards.

(b) In craps, the betting is based on successive rolls of two dice. X is the sum of the faces on the first roll, and Y is the sum of the faces on the next roll.

(a) BLACK JACK IS DEPENDENT.
SINCE THE CARDS ARE BEING DRAWN FROM A DECK WITHOUT REPLACEMENT, THE NATURE OF THE 3RD CARD WILL DEPEND UPON THE NATURE OF THE 1ST 2 CARDS THAT WERE DRAWN

(b) CRAPS IS INDEPENDENT. X RELATES TO THE OUTCOME OF THE 1ST ROLL, Y TO THE OUTCOME OF THE SECOND ROLL. THE INDIVIDUAL DICE ROLLS ARE INDEPENDENT. (THE DICE HAVE NO MEMORY).

50. **Checking independence** For each of the following situations, would you expect the random variables X and Y to be independent? Explain your answers.

(a) X is the rainfall (in inches) on November 6 of this year, and Y is the rainfall at the same location on November 6 of next year.

(b) X is the amount of rainfall today, and Y is the rainfall at the same location tomorrow.

(c) X is today's rainfall at the airport in Orlando, Florida, and Y is today's rainfall at Disney World just outside Orlando.

50 (A) INDEPENDENT. WEATHER CONDITIONS A YEAR APART SHOULD BE INDEPENDENT.

B NOT INDEPENDENT. WEATHER PATTERNS TEND TO PERSIST FOR SEVERAL DAYS

C NOT INDEPENDENT. ORLANDO AND DISNEY ARE VERY CLOSE TOGETHER AND WOULD LIKELY HAVE SIMILAR WEATHER CONDITIONS

51. **His and her earnings** A study of working couples measures the income X of the husband and the income Y of the wife in a large number of couples in which both partners are employed. Suppose that you knew the means μ_X and μ_Y and the variances σ_X^2 and σ_Y^2 of both variables in the population.

(a) Is it reasonable to take the mean of the total income $X + Y$ to be $\mu_X + \mu_Y$? Explain your answer.

(b) Is it reasonable to take the variance of the total income to be $\sigma_X^2 + \sigma_Y^2$? Explain your answer.

51 A YES. THE MEAN OF THE SUMS IS ALWAYS EQUAL TO THE SUM OF THE MEANS.

B NO. THE VARIANCE OF THE SUM IS NOT EQUAL TO THE SUM OF THE VARIANCES, BECAUSE IT IS NOT REASONABLE TO ASSUME X AND Y ARE INDEPENDENT

$\boxed{6.2}$

Exercises 57 and 58 refer to the following setting. In Exercises 14 [redacted], we examined the probability distribution of the random variable X = the amount a life insurance company earns on a 5-year term life policy. Calculations reveal that $\mu_X = \$303.35$ and $\sigma_X = \$9707.57$.

14. Life insurance A life insurance company sells a term insurance policy to a 21-year-old male that pays \$100,000 if the insured dies within the next 5 years. The probability that a randomly chosen male will die each year can be found in mortality tables. The company collects a premium of \$250 each year as payment for the insurance. The amount Y that the company earns on this policy is \$250 per year, less the \$100,000 that it must pay if the insured dies. Here is a partially completed table that shows information about risk of mortality and the values of Y = profit earned by the company:

Age at death:	21	22	23	24	25	26 or more
Profit:	−\$99,750	−\$99,500				
Probability:	0.00183	0.00186	0.00189	0.00191	0.00193	

57. Life insurance The risk of insuring one person's life is reduced if we insure many people. Suppose that we insure two 21-year-old males, and that their ages at death are independent. If X_1 and X_2 are the insurer's income from the two insurance policies, the insurer's average income W on the two policies is

$$W = \frac{X_1 + X_2}{2} = 0.5X_1 + 0.5X_2$$

Find the mean and standard deviation of W. (You see that the mean income is the same as for a single policy but the standard deviation is less.)

$\mu_X = \$303.35$

$\sigma_X = \$9707.57$

$\boxed{57}$

$\mu_W = \frac{1}{2}(\overset{\mu_X}{\$303.35}) + \frac{1}{2}(\overset{\mu_X}{\$303.35})$

$\boxed{\mu_W = \$303.35}$

$\sigma_W = \sqrt{(\frac{1}{2} \cdot \overset{\sigma_X}{9707.57})^2 + (\frac{1}{2} \cdot \overset{\sigma_X}{9707.57})^2}$

$\boxed{\sigma_W = \$6,864.29}$

58. Life insurance If four 21-year-old men are insured, the insurer's average income is

$$V = \frac{X_1 + X_2 + X_3 + X_4}{4} = 0.25X_1 + 0.25X_2 + 0.25X_3 + 0.25X_4$$

where X_i is the income from insuring one man. Assuming that the amount of income earned on individual policies is independent, find the mean and standard deviation of V. (If you compare with the results of Exercise 57, you should see that averaging over more insured individuals reduces risk.)

$\boxed{58}$

$\mu_V = \frac{1}{4}(\mu_X) + \frac{1}{4}(\mu_X) + \frac{1}{4}(\mu_X) + \frac{1}{4}(\mu_X)$

$\boxed{\mu_V = \$303.35}$

$\sigma_V = \sqrt{(\frac{1}{4}(9707.57))^2 \times 4}$

$\boxed{\sigma_V = \$4,853.79}$

(IT IS smaller by a factor of $1/\sqrt{2}$)

6.2

63. Swim team Hanover High School has the best women's swimming team in the region. The 400-meter freestyle relay team is undefeated this year. In the 400-meter freestyle relay, each swimmer swims 100 meters. The times, in seconds, for the four swimmers this season are approximately Normally distributed with means and standard deviations as shown. Assuming that the swimmer's individual times are independent, find the probability that the total team time in the 400-meter freestyle relay is less than 220 seconds. Follow the four-step process.

Swimmer	Mean μ	Std. dev. σ
Wendy X_1	55.2	2.8
Jill X_2	58.0	3.0
Carmen X_3	56.3	2.6
Latrice X_4	54.7	2.7

4-STEP PROCESS

① STATE: What is the probability that the ~~scores~~ total team swim time is less than 220 seconds?

② PLAN:

Let T = Total Team swim time

X_1 = Wendy's time

X_2 = Jill's time

X_3 = Carmen's time

X_4 = Latrice's time

Then $T = X_1 + X_2 + X_3 + X_4$

$\mu_T = 55.2 + 58 + 56.3 + 54.7$

$\boxed{\mu_T = 224.2 \text{ seconds}}$

$\sigma_T = \sqrt{2.8^2 + 3^2 + 2.6^2 + 2.7^2}$

$\boxed{\sigma_T = 5.56 \text{ seconds}}$

3 DO:

$N(224.2, 5.56)$

Normal cdf(-1EE99, 220, 224.2, 5.56) =

.225

4 Conclude:

There is approximately a 22% chance that the swim teams time will be less than 220 seconds.

How to Organize a Statistical Problem: A Four-Step Process

STEP 4

State: What's the question that you're trying to answer?

Plan: How will you go about answering the question? What statistical techniques does this problem call for?

Do: Make graphs and carry out needed calculations.

Conclude: Give your practical conclusion in the setting of the real-world problem.

To keep the four steps straight, just remember: Statistics Problems Demand Consistency!