AP STATISTICS CHAPTER 5 – PROBABILITY REVIEW #1

1

A group of 50 college students were classified according to hair and eye color. The results are summarized in the joint frequency table below-

Cells are joints

Marginal Totals

	Red hair	Blond hair	Brown hair	
Blue eyes	6	12	4	22
Green eyes	10	8	10	28
	16	20	14	50 = TT

Marginal totals

(a) What is the probability that a randomly selected student has brown hair?

(b) What is the probability that a randomly selected has red hair or brown hair?

(c) What is the probability that a randomly selected student has red hair or green eyes?

(d) What is the probability that a randomly selected student has red hair and blue eyes?

(e) What is the probability that a randomly selected student with green eyes has blond hair? Restricted sample – Conditional Prob

(f) Are the two events "red hair" and "green eyes" independent? Show work to support your conclusion.

To see if independent create Segmented bar graph:

	Red	BLOND	BROWN
BLUE	6/22=.27	12/22=.55	4/22=.18
GREEN	10/28=.36	8/28=.29	10/28=.36

* THE GRAPHS WILL BE SIMILAR IF THEY ARE INDEPENDENT.

a) $P(BH) = \frac{MT}{TT} = \boxed{\frac{14}{50}}$

EVENTS ARE MUTUALLY EXCLUSIVE (disjoint)

b) $P(RH \text{ OR } BH) = \frac{16}{50} + \frac{14}{50} = \boxed{\frac{30}{50}}$ OR means "add" $\cup$ Union

c) $P(RH \text{ OR } GE) = \frac{16}{50} + \frac{28}{50} - \frac{10}{50} = \boxed{\frac{34}{50}}$

d) $P(RH \text{ and } BE) = \boxed{\frac{6}{50}}$ And means "mult" $\cap$ intersection

EVENTS Intersect

e) $P(BH \mid GE) = \boxed{\frac{8}{28}}$ ← Conditional Probability CELL/marginal

f) Independent $P(A \mid B) = P(A)$

$P(RH \mid GE) = P(RH)$

$\frac{10}{28} = \frac{16}{50}$

$.36 \neq .32$

$P(GE \mid RH) = P(GE)$

$\frac{10}{16} \overset{?}{=} \frac{28}{50}$

$.625 \neq .56$

NOT INDEPENDENT

2

An exploratory oil well is being drilled. The probability of drilling through shale is 0.4. If the well goes through shale the probability of striking oil is 0.3. If the well does not go through shale, the probability of striking oil is only 0.1.

table tree

(a) What is the probability of striking oil?

$P(\text{OIL}) = .18 = .12 + .06 = .18$

(b) What is the probability of drilling through shale and striking oil?

$P(\text{SHALE AND OIL}) = .12$

(c) What is the probability of going through shale given that we strike oil? $P(\text{SHALE} \mid \text{OIL}) = \frac{.12}{.18} = .67$ = Tree use formula

see Green sheet:

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$P(\text{shale} \mid \text{oil}) = \frac{P(\text{shale} \cap \text{OIL})}{P(\text{OIL})} = \frac{.12}{.12 + .06} = \frac{.12}{.18}$

$.67$

STEP I write what you know

$P(\text{SHALE}) = .4$ marginal prob

$P(\text{OIL} \mid \text{SHALE}) = .3$ conditional prob

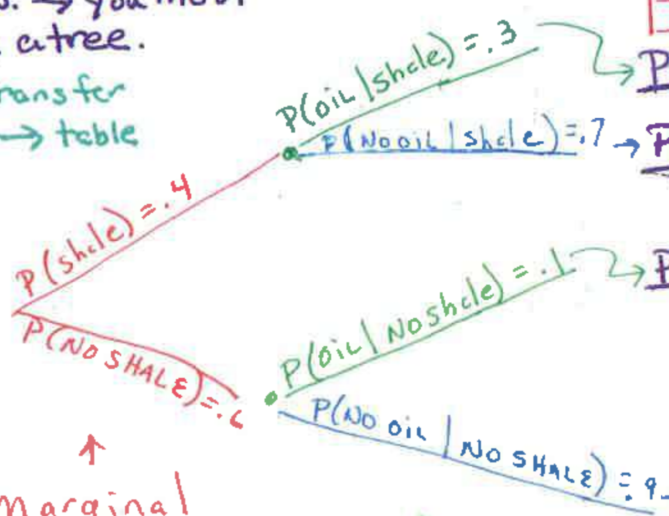
$P(\text{OIL} \mid \text{NO SHALE}) = .1$ conditional prob

STEP II Create a table or Tree

- TRY FILLING IN A TABLE NOTICE YOU CAN NOT FILL IN MUCH
- Since we have conditional prob. → you MUST use a tree.
- Tip: transfer tree → table

Table

	OIL	NO OIL	
Shale	.12	.28	.4
No shale	.06	.54	.6
	.18	.82	1.0



$P(\text{SHALE} + \text{OIL}) = .4(.3) = .12$

$P(\text{SHALE} + \text{NO OIL}) = .4(.7) = .28$

$P(\text{OIL} + \text{NO SHALE}) = .6(.1) = .06$

$P(\text{NOT OIL} + \text{NOT SHALE}) = (.6)(.9) = .54$

$\Sigma = 1.$

Marginal probabilities

Conditional Probabilities

Joint probabilities

A2

	BLUE	NOT BLUE	
PREGNANT	.038	.002	.04
NOT PREGNANT	.048	.912	.96
	.086	.914	1.00

3 Medi-Mart has just come out with a new pregnancy test that registers blue (indicating a pregnancy) in 95% of users who are pregnant. However, the new monitor also registers blue in 5% of the users who are not pregnant. Suppose that, in reality, only 4% of women using this test are pregnant.

(a) Construct a table or draw a tree diagram that reflects the situation, including probabilities or counts.

(b) What is the probability that a randomly selected woman who uses this test gets a "blue" result?

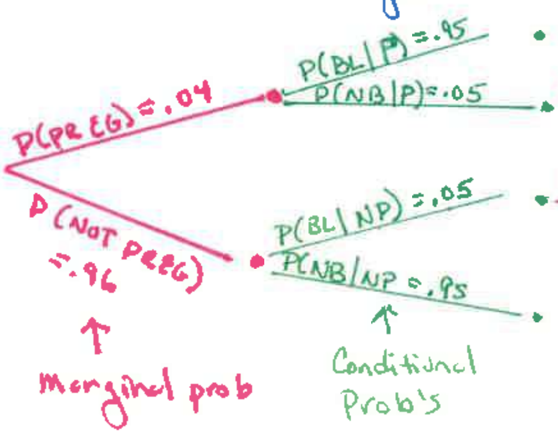
(c) What is the probability that the woman actually is pregnant given that the test registers blue?

IDENTIFY PROBABILITIES GIVEN:

P(BLUE | PREG) = .95 (CONDITIONAL)
P(BLUE | NOT PREG) = .05 (CONDITIONAL)
P(PREG) = .04 (marginal)

Since conditional probabilities are given, need to start with a tree (then can convert to a table)

A1



- P(PREG and BLUE) = .04 (.95) = .038 *
- P(PREG and Not Blue) = .04 (.05) = .002
- P(NOT PREG and Blue) = .96 (.05) = .048 *
- P(NOT PREG and Not Blue) = .96 (.95) = .912

JOINT PROBABILITIES

Σ = 1.00 ✓ to see =' s 1.0

$\boxed{\text{NOW IT IS EASY TO CREATE A TABLE TO ANSWER ?'s (see above)}}$

b) P(BLUE) = .086 FROM TREE = .038 + .048 = .086

c) P(PG | BLUE) = $\frac{.038}{.086}$ = .442 FROM TREE (USING GREEN SHEET)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(PG \mid BLUE) = \frac{P(PG \text{ and } BLUE)}{P(BLUE)} = \frac{.038}{.038+.04} = .442$$

$P(A \text{ and } B) = P(A \cap B)$

and $\xrightarrow{\text{means}}$ MULT.

Tip:
$P(\text{AT LEAST}) = 1 - P(\text{NONE})$

SAMPLING WITHOUT REPLACEMENT

4. **Another hand.** You pick three cards at random from a deck. Find the probability of each event described below.

a) You get no aces.
b) You get all hearts.
c) The third card is your first red card.
d) You have at least one diamond.

P(No ace) · P(No ace) ·

ⓐ $P(\text{NO ACE'S}) = \frac{52-4}{52} = \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} = \boxed{.783}$

1ST pick, 2ND pick, 3RD pick

ⓑ $P(\text{ALL HEARTS}) = \frac{13}{52}\left(\frac{12}{51}\right)\left(\frac{11}{50}\right) = \boxed{.013}$

26 Red, 24 BLACK

ⓒ $P(\text{3RD CARD is the 1ST RED}) = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{26}{50} = \boxed{.127}$

1ST pick BLACK, 2ND pick BLACK, 1ST Red

ⓓ $P(\text{at least 1 diamond}) = 1 - (\text{no diamond})$

$1 - \frac{39}{52}\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) = \boxed{.586}$

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5. **Shirts.** The soccer team's shirts have arrived in a big box, and people just start grabbing them, looking for the right size. The box contains 4 medium, 10 large, and 6 extra-large shirts. You want a medium for you and one for your sister. Find the probability of each event described.

a) The first two you grab are the wrong sizes.
b) The first medium shirt you find is the third one you check.
c) The first four shirts you pick are all extra-large.
d) At least one of the first four shirts you check is a medium.

GIVEN: 4 medium, 10 large, 6 XL — Want 2 mediums — 20 SHIRTS

ⓐ $P(\text{Not m}) \cdot P(\text{NOT m}) = \frac{16}{20} \cdot \frac{15}{19} = \boxed{.632}$

ⓑ $P(\text{NOT m}) \cdot P(\text{NOT m}) \cdot P(m) =$

$\frac{16}{20} \cdot \frac{15}{19} \cdot \frac{4}{18} = \boxed{.140}$

ⓒ $P(XL) \cdot P(XL) \cdot P(XL) \cdot P(XL) =$

$\frac{6}{20} \cdot \frac{5}{19} \cdot \frac{4}{18} \cdot \frac{3}{17} = \boxed{.003}$

ⓓ $P(\text{at least 1 of 4 shirts is medium}) = 1 - (\text{none are medium})$

$P(NM) \cdot P(NM) \cdot P(NM) \cdot P(NM)$

$1 - \left(\frac{16}{20}\right)\left(\frac{15}{19}\right)\left(\frac{14}{18}\right)\left(\frac{13}{17}\right) =$

.624

NM = NOT medium

DISJOINT AND INDEPENDENCE

6 **Eligibility.** A university requires its biology majors to take a course called BioResearch. The prerequisite for this course is that students must have taken either a Statistics course or a computer course. By the time they are juniors, 52% of the Biology majors have taken Statistics, 23% have had a computer course, and 7% have done both.

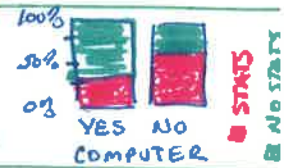
a) What percent of the junior Biology majors are ineligible for BioResearch? Asked for %
b) What's the probability that a junior Biology major who has taken Statistics has also taken a computer course? ← (restricted sample)
c) Are taking these two courses disjoint events? Explain.
d) Are taking these two courses independent events? Explain.

GIVEN
P(STATS) = .52
P(Computer) = .23
P(Stats and computer) = .07

CREATE A TABLE

STATS \ COMPUTER	Y	N	
Y	.07	.45	.52
N	.16	.32	.48
	.23	.77	1.00

Use segmented bar graph to see the relationship.
*Graphs NOT similar when not independent.



a) P(INELIGIBLE) = P(NEITHER) = 32%

b) P(Computer | Stats) = $\frac{.07}{.52}$ = .135 or 13.5%

c) No. Taking the 2 courses is NOT disjoint (or mutually exclusive) because they have outcomes in common – 7% took both courses Stats + computers.

d) The 2 courses are not independent because
P(COMPUTER) $\overset{?}{=}$ P(COMPUTER | STATS) or P(STATS) $\overset{?}{=}$ P(STAT | COMPUTER)
.23 ≠ $\frac{.07}{.52}$ = .135 .52 ≠ $\frac{.07}{.23}$ = .30

Note: only have to check 1!

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7 **Phone service.** According to estimates from the federal government's 2003 National Health Interview Survey, based on face-to-face interviews in 16,677 households, approximately 58.2% of U.S. adults have both a landline in their residence and a cell phone, 2.8% have only cell phone service but no landline, and 1.6% have no telephone service at all.

* ARE HAVING A CELL PHONE AND A LANDLINE INDEPENDENT? EXPLAIN.

GIVEN:
P(Cell and Land) = .582
P(Cell and No LL) = .028
P(NO PHONE) = .016

CREATE A TABLE:

Cell \ Land Line	YES	No	
Yes	.582	.028	.61
NO	.374	.016	.39
	.956	.044	1.000

To check for independence: P(A) = P(A | B)

P(Cell) = P(Cell | Land)
.61 $\overset{?}{=}$ $\frac{.582}{.956}$
.610 $\overset{?}{=}$ .609 ✓

OR

P(LANDLINE) = P(LL | CELL)
.956 = $\frac{.582}{.61}$
.956 ≅ .954 ✓

Conclusion: Since the probabilities are about the same, it appears cell and landlines are independent. That is knowing someone owns a cell phone has no effect on the chance they have a land line and vice versa.

Validate with a segmented bar graph

*Graphs will be similar when independent

8

At a certain college 80% of the freshman are enrolled in English, 70% are enrolled in Mathematics, and 65% are enrolled in both courses. A freshman is to be randomly selected.

(a) Construct a two-way table showing this information.

(b) What is the probability that a freshman is enrolled in English or Mathematics?

(c) What is the probability that the freshman is enrolled in English, given that the student is enrolled in Mathematics?

GIVEN

P(ENG) = .8

P (MATH) = .7

P (BOTH) = .65

(A)

		MATH YES	MATH NO	
ENGLISH	YES	.65	.15	.80
	NO	.05	.15	.20
		.70	.30	1.00

(B) P(ENGLISH OR MATH) = P(ENG) + P(MATH) − P(ENG and math)

= .8 + .7 − .65 = .85

(C) P(ENGLISH | MATH) = $\frac{.65}{.70}$ = .93

9

Optic City sells six brands of digital cameras. Each brand is available in 2, 3, or 4 megapixels.

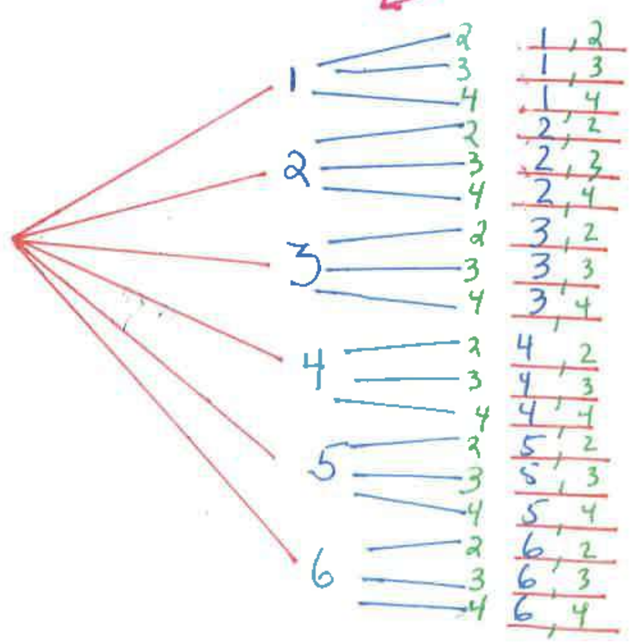
(a) How many different digital cameras are sold by Optic City?

\# Camera's = (# brands)(# models) = 6(3) = $\boxed{\text{18 TYPES OF CAMERAS}}$

(b) Make a tree diagram or a two-way table showing all of the different cameras.

(c) What is the probability of randomly selecting any one of the digital cameras?

Model \ BRAND	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)



(C) P(1 CAMERA) = $\boxed{\frac{1}{18}}$