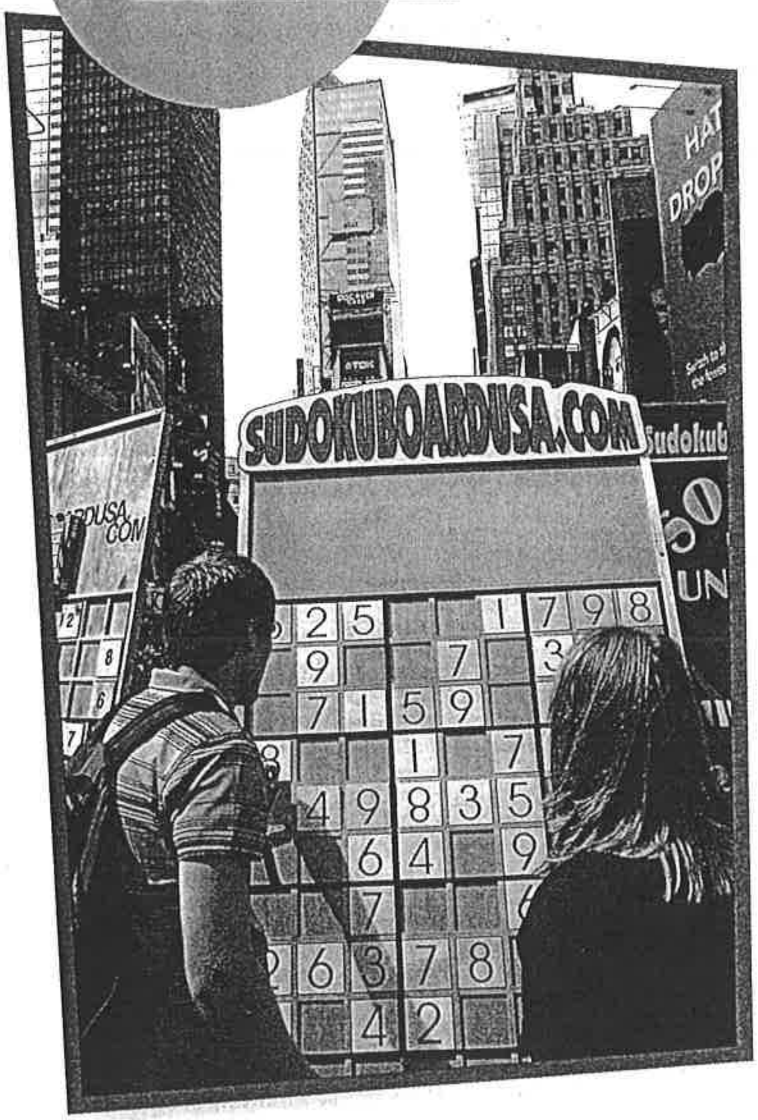
TPS TEXT BOOK SOLUTIONS

Chapter 2



Answers to Odd-Numbered Section 2.1 Exercises

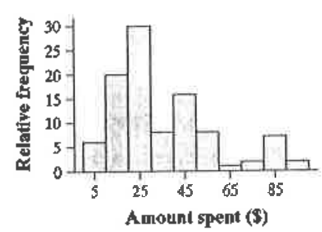
1. (a) The girl with 22 pairs of shoes is the 6th smallest. Her percentile is 0.25. 25% of girls have fewer pairs of shoes. (b) The boy with 22 pairs has more shoes than 17 people. His percentile is 0.85. 85% of boys have fewer pairs of shoes. (c) The boy is more unusual because only 15% of the boys have as many or more than he has, while the girl has a value that is more centered in the distribution. 25% have fewer and 75% have as many or more.

3. According to the *Los Angeles Times*, the speed limits on California highways are such that 85% of the vehicle speeds on those stretches of road are less than the speed limit.

5. The girl in question weighs more than 48% of girls her age, but is taller than 78% of the girls her age. Since she is taller than 78% of girls, but only weighs more than 48% of girls, she is probably fairly skinny.

7. (a) The highlighted student sent about 212 text messages in the two-day period which placed her at about the 80th percentile. (b) The median number of texts is the same as the 50th percentile. Locate 50% on the *y* axis, read over to the points and then find the relevant place on the *x* axis. The median is approximately 115 text messages.

9. (a) First find the quartiles. The first quartile is the 25th percentile. Find 25 on the *y* axis, read over to the line and then down to the *x* axis to get about \$19. The 3rd quartile is the 75th percentile. Find 75 on the *y* axis, read over to the line and then down to the *x* axis to get about \$50. So the interquartile range is \$50 − \$19 = \$31. (b) Approximately the 26th percentile. (c) Here is a histogram.



11. Eleanor's standardized score, $z = 1.8$, is higher than Gerald's standardized score, $z = 1.5$.
13. (a) Judy's bone density score is about one and a half standard deviations below the average score for all women her age. The fact that your standardized score is negative indicates that your bone density is below the average for your peer group. The magnitude of the standardized score tells us how many standard deviations you are below the average (about 1.5). (b) $\sigma = 5.52$ grams/cm^2.
15. (a) Since 22 salaries were less than Lidge's salary, his salary is at the 75.86 percentile. (b) $z = 0.79$. Lidge's salary was 0.79 standard deviations above the mean salary of \$3,388,617.
17. (a) In the national group, about 94.8% of the test takers scored below 65. Scott's percentiles, 94.8th among the national group and 68th within the school, indicate that he did better among all test takers than he did among the 50 boys at his school. (b) Scott's z-scores are $z = 1.57$ among the national group and $z = 0.62$ among the 50 boys at his school.
19. (a) The mean and the median both increase by 18 so the mean is 87.188 and the median is 87.5. The distribution of heights just shifts by 18 inches. (b) The standard deviation and IQR do not change. For the standard deviation, note that although the mean increased by 18, the observations each increased by 18 as well so that the deviations did not change. For the IQR, Q_1 and Q_3 both increase by 18 so that their difference remains the same as in the original data set.
21. (a) To give the heights in feet, not inches, we would divide each observation by 12 (12 inches = 1 foot). Thus the mean and median are divided by 12. The new mean is 5.77 feet and the new median is 5.79 feet. (b) To find the standard deviation in feet, note that each deviation in terms of feet is found by dividing the original deviation by 12.

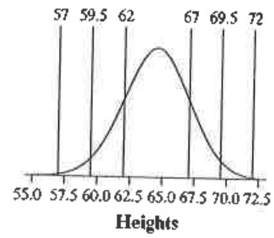
$$\text{standard deviation}_{\text{New}} = \sqrt{\frac{(\text{first deviation (ft)})^2 + \cdots + ((\text{last deviation (ft)})^2)}{n-1}}$$
$$= \sqrt{\frac{\left(\frac{\text{first deviation (in)}}{12}\right)^2 + \cdots + \left(\frac{\text{last deviation (in)}}{12}\right)^2}{n-1}}$$
$$= \frac{1}{12} \cdot \text{standard deviation}_{\text{Old}} = \frac{3.2}{12} = 0.27 \text{ feet}$$

The first and third quartiles are still the medians of the first and second halves of the data, these values must simply be converted to feet. To do this, divide the first and third quartiles of the original data set by 12: $Q_1 = \frac{67.75}{12} = 5.65$ feet and $Q_3 = \frac{71}{12} = 5.92$ feet. So the interquartile range is $IQR = 5.92 - 5.65 = 0.27$ feet.
23. Mean in degrees Fahrenheit is 77. Standard deviation in degrees Fahrenheit is 3.6.
25. Sketches will vary.
27. (a) It is on or above the horizontal axis everywhere and the area beneath the curve is 1. (b) $\frac{1}{3}$ (c) Since $(1.1 - 0.8) \cdot \frac{1}{3} = 0.1$, one-tenth of accidents occur next to Sue's property.
29. Both are 1.5.
31. (a) Mean is C, median is B. (b) Mean is B, median is B.
33. c
35. c
37. d
39. The distribution is skewed to the right since most of the values are 25 minutes or less, but the values stretch out up to about 90 minutes. The data are centered roughly around 20 minutes and the range of the distribution is close to 90 minutes. The two largest values appear to be outliers.

Section 2.2

Answers to Check Your Understanding

Page 114 1. Here is the graph.



2. Since 67 inches is one standard deviation above the mean, approximately $\frac{1-0.68}{2} = 16\%$ of young women have heights greater than 67 inches. 3. Since 62 is one standard deviation below the mean and 72 is three standard deviations above the mean, approximately $\frac{0.68}{2} + \frac{0.997}{2} = 84\%$ of young women have heights between 62 and 72 inches. ***Page 119*** 1. The proportion is 0.9177. 2. The proportion is 0.9842. 3. The proportion is $0.9649 - 0.2877 = 0.6772$. 4. The z-score for the 20th percentile is -0.84. 5. The 55th percentile is the z-value where 45% are greater than that value. $z = 0.13$.

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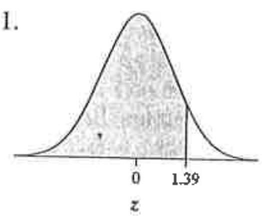

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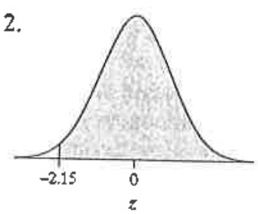

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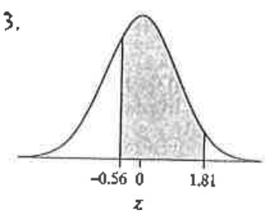

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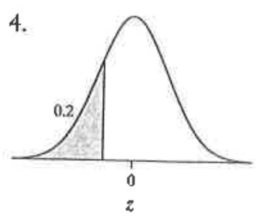

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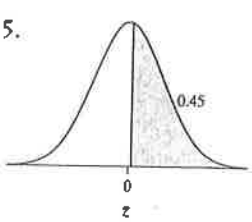
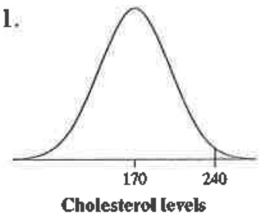


Page 124 1. For 14-year old boys with cholesterol 240, the z-score is $z = \frac{240-170}{30} = 2.33$. The proportion of z-scores above 2.33 is

$1 - 0.9901 = 0.0099$. 2. The z-score for a cholesterol level of 200 is $z = \frac{200 - 170}{30} = 1$. The proportion of z-scores between 1 and 2.33 is $0.9901 - 0.8413 = 0.1488$. 3. The 80th percentile of the standard Normal distribution is 0.84. This means that the distance, x, of Tiger Woods's drive lengths that satisfies the 80th percentile is the solution to $0.84 = \frac{x - 304}{8}$. Solving for x, we get 310.72 yards.

1.



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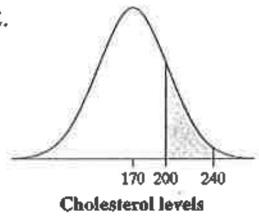

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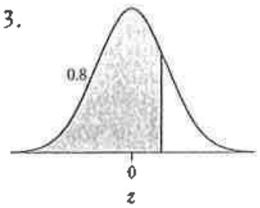
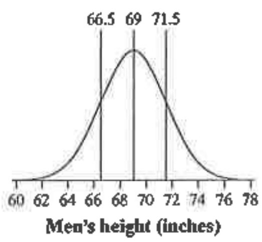


Answers to Odd-Numbered Section 2.2 Exercises

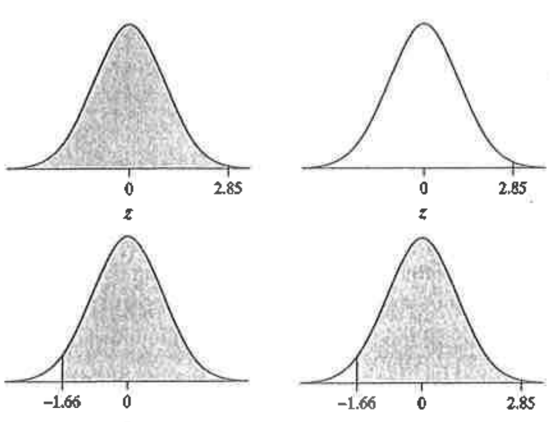
41. Here is the graph.



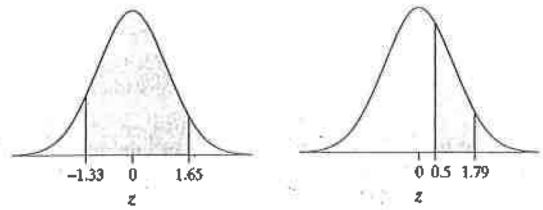
43. (a) Approximately 2.5% (b) 64 and 74 inches (c) Approximately 13.5% (d) 84th percentile.

45. The standard deviation is approximately 0.2 for the tall, more concentrated one and 0.5 for the short, less concentrated one.

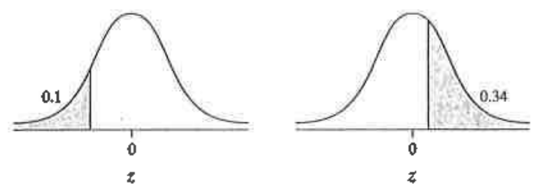
47. (a) 0.9978 (b) 0.0022 (c) 0.9515 (d) 0.9493



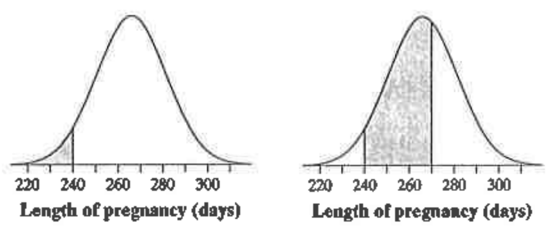
49. (a) 0.8587 (b) 0.2718



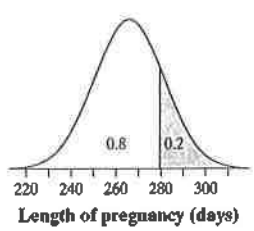
51. (a) $z = -1.28$ (b) $z = 0.41$.



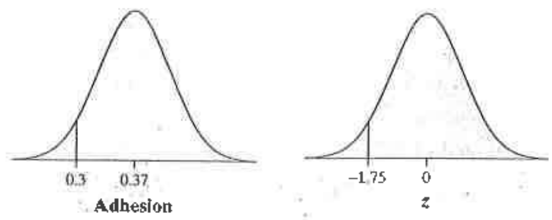
53. (a) **State:** Let x = the length of pregnancies. The variable x has a Normal distribution with $\mu = 266$ days and $\sigma = 16$ days. We want the proportion of pregnancies that last less than 240 days. **Plan:** The proportion of pregnancies lasting less than 240 days is shown in the graph. **Do:** For $x = 240$ we have $z = -1.63$, so $x < 240$ corresponds to $z < -1.63$. Using Table A, we see that the proportion of observations less than -1.63 is 0.0516 or about 5.2%. **Conclude:** About 5.2% of pregnancies last less than 240 days which means that 240 is approximately the 5th percentile. (b) **Do:** From part (a) we have that for $x = 240$, $z = -1.63$. For $x = 270$, we have $z = 0.25$. Using Table A we see that the proportion of observations less than 0.25 is 0.5987. So the proportion of observations between -1.63 and 0.25 is about 55%. **Conclude:** Approximately 55% of pregnancies last between 240 and 270 days.



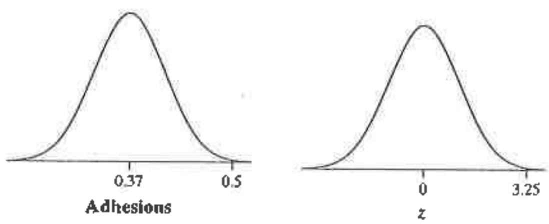
(c) **Do:** Using Table A, the 80th percentile for the standard Normal distribution is 0.84. Therefore, the 80th percentile for the length of human pregnancies can be found by solving the equation $0.84 = \frac{x - 266}{16}$ for x. Thus, $x = (0.84)16 + 266 = 279.44$. **Conclude:** The longest 20% of pregnancies last approximately 279 or more days.



55. (a)



We would expect trains to arrive on time about 96% of the time. (b) $z = 3.25$. We would expect trains to arrive early 0.06% of the time.



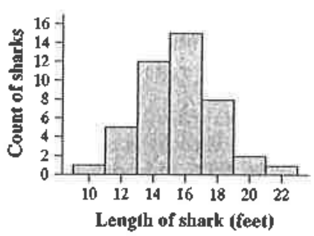
(c) It makes sense to try to have the value found in part (a) larger. We want the train to arrive at its destination on time, but not to arrive at the switch point early.

57. (a) Solve $z = \frac{0.30 - \mu}{0.04} = -2.05$ for μ. The mean adhesion should be 0.382. (b) Solve $z = \frac{0.30 - 0.37}{\sigma} = -2.05$ for σ. The standard deviation of the adhesions values should be 0.034. (c) To compare the options, we want to find the area under the Normal distribution to the right of 0.50. Under option (a), $z = 2.95$ and the area is $1 - 0.9984 = 0.0016$. Under option (b), $z = 3.82$ and the area is $1 - 0.9999 = 0.0001$. Therefore, we prefer option (b).

59. (a) ± 1.28 (b) $-1.28(2.5) + 64.5 = 61.3$ inches and $1.28(2.5) + 64.5 = 67.7$ inches

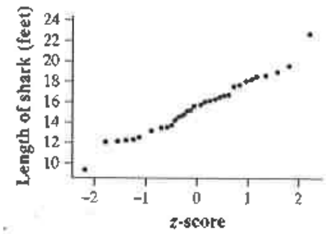
61. $\mu = 41.43$ minutes; $\sigma = 17.86$ minutes

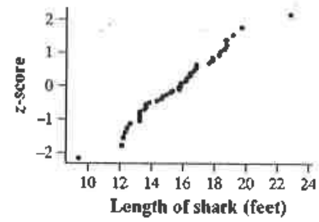
63. (a) Below are descriptive statistics and a histogram.



Variable	N	Mean	StDev	Minimum	Q_1	Median	Q_3	Maximum
shlength	44	15.586	2.550	9.400	13.55	15.750	17.200	22.800

The distribution of shark lengths is roughly symmetric with a peak at 16 and varies from 9.4 feet to 22.8 feet. (b) 68.2% of the lengths fall within one standard deviation of the mean, 95.5% of the lengths fall within two standard deviations of the mean, and 100% of the lengths fall within 3 standard deviations of the mean. These are very close to the 68–95–99.7 rule. (c) A Normal probability plot is shown.





Except for one small shark and one large shark, the plot is fairly linear, indicating that the Normal distribution is appropriate. (d) The graphical display in (a), check of the 68–95–99.7 rule in (b), and Normal probability plot in (c) indicate that shark lengths are approximately Normal.

65. The plot is nearly linear. The smallest value is smaller than we would expect and the largest two values are larger than we would expect. There is also a cluster of points around 125 bpm that are a little larger than expected.

67. (a) 11.47% (b) 15.34% (c) 12.30%

69. d

71. b

73. c

75. For both kinds of cars, the highway miles per gallon is higher than the city miles per gallon. The two-seater cars have a wider spread of miles per gallon values than the minicompact cars do, both on the highway and in the city. Also, the miles per gallon values are slightly lower for the two-seater cars than for the minicompact cars. This difference is greater on the highway than it is in the city. All four distributions are roughly symmetric.

Answers to Chapter 2 Review Exercises

R2.1 (a) The only way to obtain a z-score of 0 is if the x-value equals the mean. Thus, the mean is 170 cm. To find the standard deviation, use the fact that a z-score of 1 corresponds to a height of 177.5. Then $1 = \frac{177.5 - 170}{\sigma}$ so $\sigma = 7.5$. Thus, the standard deviation of the height distribution for 15-year-old males is 7.5 cm. (b) Since $2.5 = \frac{x - 170}{7.5}$, $x = 2.5(7.5) + 170 = 188.75$ cm. A height of 188.75 cm has a z-score of 2.5.

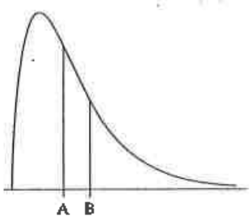
R2.2 (a) $z = 1.20$. Paul is somewhat taller than average for his age. His height is 1.20 standard deviations above the average male height for his age. (b) 85% of boys Paul's age are shorter than Paul's height.

R2.3 (a) Approximately the 70th percentile. (b) The median (50th percentile) is about 5, Q_1 (25th percentile) is about 2.5, and Q_3 (75th percentile) is about 11. There are outliers, according to the 1.5(IQR) rule, because values exceeding $Q_3 + 1.5(IQR) = 23.75$ clearly exist.

R2.4 (a) The shape would not change. The new mean would be 13.32 meters, the median would be 12.80 meters, the standard deviation would be 3.81 meters, and the IQR would be 3.81 meters.

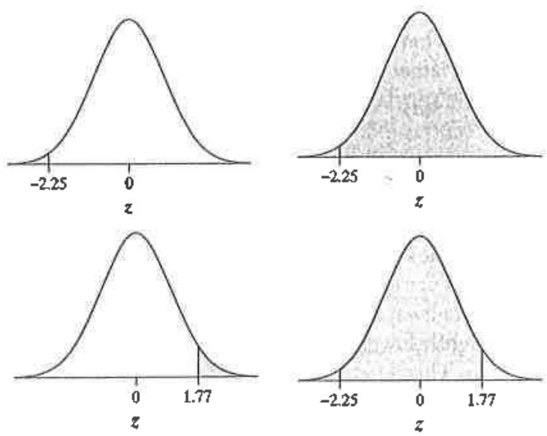
(b) The mean error would be 43.7 − 42.6 = 1.1 feet. The standard deviation of the errors would be the same as the standard deviation of the guesses, 12.5 feet, because we have just shifted the distribution, but not changed its width by subtracting 42.6 from each guess.

R2.5 (a) Answers will vary. Line A on the graph. (b) Answers will vary. Line B on the graph.

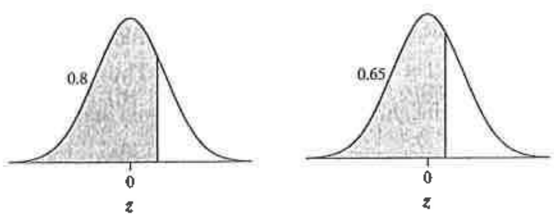


R2.6 (a) (327,345) (b) 339 is one standard deviation above the mean, so 16% of the horse pregnancies last longer than 339 days. This is because 68% are within one standard deviation of the mean, so 32% are more than one standard deviation from the mean and half of those are greater than 339.

R2.7 (a) 0.0122 (b) 0.9878 (c) 0.0384 (d) 0.9494



R2.8 (a) $z = 0.84$ (b) $z = 0.39$

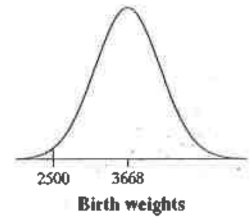


R2.9 (a) $z = -2.29$. The z-value that corresponds to a baby weight less than 2500 grams at birth is −2.29. The percent of babies weighing less than this is the area to the left. According to Table A, this is 0.0110. So, approximately 1% of babies will be identified as having low birth weight. (b) The z-values corresponding to the quartiles are −0.67 and 0.67. To find the x-values corresponding to the quartiles, we solve the following equations for x.

$$-0.67 = \frac{x-3668}{511} \Rightarrow x = -0.67(511) + 3668 = 3325.63$$

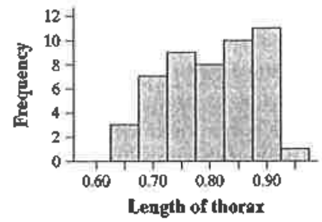
$$0.67 = \frac{x-3668}{511} \Rightarrow x = 0.67(511) + 3668 = 4010.37$$

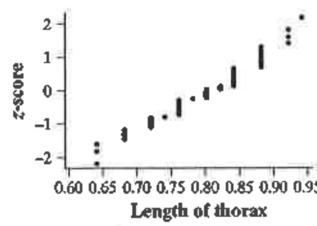
The quartiles of the birth weight distribution are 3325.6 and 4010.4.



R2.10 If the distribution is Normal, it must be symmetric about its mean—and in particular, the 10th and 90th percentiles must be equal distances above and below the mean—so the mean is 250 points. If 225 points below (above) the mean is the 10th (90th) percentile, this is 1.28 standard deviations below (above) the mean, so the distribution's standard deviation is $\frac{225}{1.28} = 175.8$ points.

R2.11 A histogram and Normal probability plot both indicate that the data are not exactly Normally distributed. The histogram is roughly symmetric. The descriptive statistics given below indicate that the mean and median are very similar, which is consistent with rough symmetry. For most purposes, these data can be considered approximately Normal.





Variable	N	Mean	StDev	Minimum	Q_1	Median	Q_3	Maximum
length of thorax	49	0.8004	0.0782	0.6400	0.7600	0.8000	0.8600	0.9400

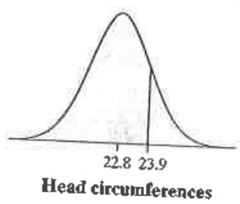
R2.12 The steep, nearly vertical portion at the bottom and the bow upward indicate that the distribution of the data is right-skewed with several outliers. In other words, these data are not Normally distributed.

Answers to Chapter 2 AP Statistics Practice Test

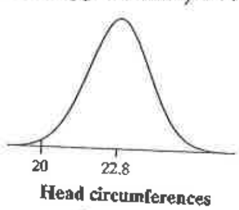
T2.1 e
T2.2 d
T2.3 b
T2.4 b
T2.5 a
T2.6 d
T2.7 c
T2.8 e
T2.9 e
T2.10 c

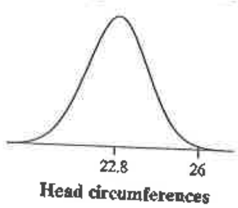
T2.11 (a) Jane's performance was better. She did more curl-ups than 85% of girls her age. This means that she qualified for both the Presidential award and for the National award. Matt did more curl-ups than 50% of boys his age. This means that less than 50% of the boys his age did better than he did, whereas less than 15% of the girls her age did better than Jane. Matt qualified for the National award, but did not qualify for the Presidential award. (b) Since Jane's position in her distribution is so much higher than Matt's position in his distribution, Jane's z-score is likely to be bigger than Matt's z-score.

T2.12 (a) The z-value that corresponds to this soldier's head circumference is $z = \frac{23.9 - 22.8}{1.1} = 1$. So the proportion of observations lower than this is 0.8413 (using Table A). This means that this soldier's head circumference is in approximately the 84th percentile.



(b) Standardizing the left endpoint we get $z = \frac{20 - 22.8}{1.1} = -2.55$. Using Table A, the area below -2.55 is 0.0054. Standardizing the right endpoint we get $z = \frac{26 - 22.8}{1.1} = 2.91$. The area below 2.91 (using Table A) is 0.9982, so the area above 2.91 is $1 - 0.9982 = 0.0018$. This means that the area in both tails is $0.0054 + 0.0018 = 0.0072$. So approximately 0.7% of soldiers require custom helmets.





(c) The quartiles of a standard Normal distribution are -0.67 and 0.67. To find the quartiles of the head circumference distribution, we solve the following equations for x.

$$-0.67 = \frac{x - 22.8}{1.1} \Rightarrow x = -0.67(1.1) + 22.8 = 22.063$$

$$0.67 = \frac{x - 22.8}{1.1} \Rightarrow x = 0.67(1.1) + 22.8 = 23.537$$

This means that $Q_1 = 22.063$ and $Q_3 = 23.537$. So $IQR = Q_3 - Q_1 = 23.537 - 22.063 = 1.474$ inches.

T2.13 No, these data do not seem to follow a Normal distribution. First, there is a large difference between the mean and the median. The mean is 48.25 and the median is 37.80. The Normal distribution is symmetric so the mean and median should be quite close in a Normally distributed data set. This data set appears to be highly skewed to the right. This can be seen by the fact that the mean is so much larger than the median. It can also be seen by the fact that the distance between the minimum and the median is $37.80 - 2 = 35.80$, but the distance between the median and the maximum is $204.90 - 37.80 = 167.10$.