AP Stats Chapter 12 Notes—More with tests

The One-Sample t Statistic and the t Distributions

Draw an SRS of size n from a population that has the Normal distribution with mean μ and standard deviation σ. The **one-sample t statistic**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has the t distribution with $n - 1$ degrees of freedom.

Suppose you carry out a significance test of versus based on a sample size of n = 20 and obtain t = 1.81.

Degrees of freedom = ________

The t test statistic falls between __________ and _________

Upper tail probability p			
df	.05	.025	.02
18	1.734	2.101	2.214
19	1.729	2.093	2.205
20	1.725	2.086	2.197

What is you were performing a test of versus based on a sample of n = 37 and obtained a t = -3.17. Keep in mind this is a two-tailed test.

Degrees of freedom = ________

The t test statistic falls between _________ and __________

Upper tail probability p			
df	.005	.0025	.001
29	2.756	3.038	3.396
30	2.750	3.030	3.385
40	2.704	2.971	3.307

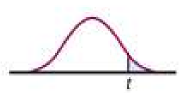
The One-Sample t Test

Draw an SRS of size n from a population having unknown mean μ. To test the hypothesis H_0: $\mu = \mu_0$ based on an SRS of size n, compute the **one-sample t statistic**

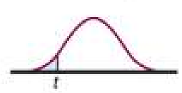
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

In terms of a random variable T having the $t(n-1)$ distribution, the P-value for a test of H_0 against

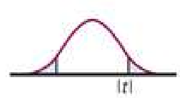
H_a: $\mu > \mu_0$ is $P(T \geq t)$



H_a: $\mu < \mu_0$ is $P(T \leq t)$



H_a: $\mu \neq \mu_0$ is $2P(T \geq |t|)$



These P-values are exact if the population distribution is Normal and are approximately correct for large n in other cases.

Diet colas are artificial sweeteners to avoid sugar. These sweeteners gradually lose their sweetness over time. Manufacturers therefore test new colas for loss of sweetness before marketing them. Trained tasters sip the cola along with drinks of standard sweetness and score the cola on a sweetness scale of 1 to 10. The cola is then stored for a month at high temperature to imitate the effect of four months' storage at room temperature. Each taster scores the cola again after storage. Our data are the differences (score before storage minus score after storage) in the tasters' scores. The bugger these differences, the bigger the loss of sweetness. Here are the sweetness losses for a new cola, as measure by 10 trained tasters:

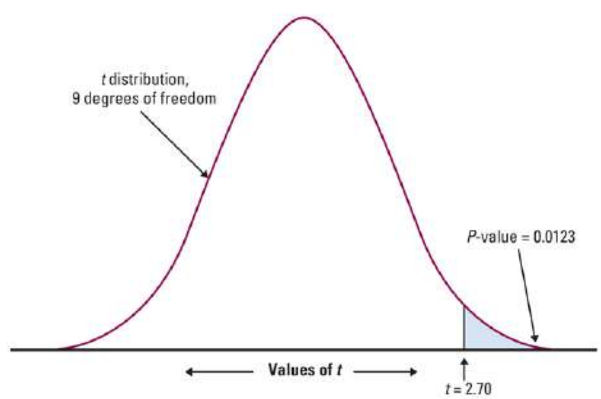
2.0	0.4	0.7	2.0	-0.4	2.2	-1.3	1.2	1.1	2.3

Most are positive. That is, most tasters found a loss of sweetness. But the losses are small, and two tasters (the negative scores) thought the cola gained sweetness. *Are these data good evidence that the cola lost sweetness in storage?*

Hypotheses:

Conditions:

Calculations:



Interpretation:

An investor with a stock portfolio worth several hundred thousand dollars sued his broker because lack of diversification in his portfolio led to poor performance. The table below gives 39 months of returns for the account that was managed by the broker. An arbitration panel compared these returns with the average of the Standard and Poor's 500 stock index for the same period. Consider the 39 monthly returns as a random sample from the monthly returns the broker would generate if he managed the account forever. Are these returns compatible with a population mean of 0.95%, the S&P average?

-8.36	1.63	-2.27	-2.93	-2.7	-2.93	-9.14	-2.64	6.82	-2.35	-3.58	6.13
7	-15.25	-8.66	-1.03	-9.16	-1.25	-1.22	-10.27	-5.11	-0.8	-1.44	1.28
-0.65	4.34	12.22	-7.21	-0.09	7.34	5.04	-7.24	-2.14	-1.01	-1.41	12.03
-2.56	4.33	2.35									

Hypotheses:

Conditions:

Calculations:

Interpretation:

Assignment: p. 745 12.1 to 12.4 and p. 754 12.5 and 12.6

Quiz 12.1A AP Statistics Name:

The developer of a new filter for filter-tipped cigarettes claims that it leaves less nicotine in the smoke than does the current filter. Because cigarette brands differ in a number of ways, he tests each filter on one cigarette of each of nine brands and records the difference in nicotine content (current filter – new filter). The mean difference is $\overline{x}$ = 1.32 milligrams (mg), and the standard deviation of the differences is s = 2.35 mg.

1. Describe the population of interest and the parameter for which inference is being performed.

2. State H_0 and H_a for this study in both symbols and words.

3. What conditions are required to carry out the significance test? Discuss the validity of each.

4. Determine the test statistic and the *P*-value. Show your work.

5. What do you conclude?

6. A 90% confidence interval for the mean amount of additional nicotine removed by the new filter is (–0.14, 2.78). Determine t^* for this interval. Then interpret the interval.

12.2 Tests about a Population Proportion

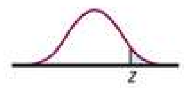
The One-Proportion z Test

Choose an SRS of size n from a large population with unknown proportion p of successes. To test the hypothesis H_0: $p = p_0$, compute the z **statistic**

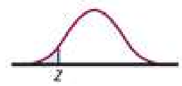
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

In terms of a random variable Z having the standard Normal distribution, the approximate P-value for a test of H_0 against

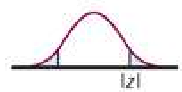
H_a: $p > p_0$ is $P(Z \geq z)$



H_a: $p < p_0$ is $P(Z \leq z)$



H_a: $p \neq p_0$ is $2P(Z \geq |z|)$



Normality condition: Use this test when the expected number of successes np_0 and failures $n(1 - p_0)$ are both at least 10.

According to the national Institute for Occupational Safety and Health, job stress poses a major threat to the health of workers. A national survey of restaurant employees found that 75% said that work stress had a negative impact on their personal lives. A random sample of 100 employees from a large restaurant chain finds that 68 answer Yes when asked, “Does work stress have a negative impact on your personal life?” Is this good reason to think that the proportion of all employees in this chain who would say Yes differs from the national proportion $p_0 = 0.75$?

Hypotheses:

Conditions:

Calculations:

Interpretation:

Assignment: p. 770 12.23, 24, 26, 29

Quiz 12.2A AP Statistics Name: ______________________

We hear that newborn babies are more likely to be boys than girls, presumably to compensate for higher mortality among boys in early life. Is this true? A random sample found 13,173 boys among 25,468 firstborn children.

1. Does this sample give sufficient evidence that boys are more common than girls in the entire population? Carry out an appropriate test to support your answer.

2. Construct and interpret a 99% confidence interval for the population proportion *p*.