

1. The diastolic blood pressure for American women aged 18 to 44 has an approximately normal distribution with mean 75 mm Hg and st.dev. 10 mm Hg. We suspect that regular exercise will lower blood pressure. A random sample of 25 women who jog at least five miles a week gives a mean of 71 mm Hg.

- (a) Is this good evidence that the mean diastolic blood pressure for the population of regular exercisers is lower than 75 mm Hg?

Population is all women in America aged 18 to 44 who exercise regularly.  
Parameter is the mean ~~diastolic blood pressure~~ <sup>diastolic blood pressure</sup>.

$$H_0: \mu = 75 \text{ mm Hg (same)}$$

$$H_a: \mu < 75 \text{ mm Hg (lower)}$$

Since  $\sigma$  is given choose z-test. This is not a SRS and we don't know their ages, so we may not be able to generalize about the population. The population is normal so the  $\bar{x}$  distribution is normal.

$$\bar{x} = 71 \quad z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{71 - 75}{10/\sqrt{25}}$$

$$z = -2$$

$$P = .0228$$

There is sufficient evidence to reject  $H_0$  at the 5% significance level (.0228 < .05) and conclude that the mean diastolic blood pressure for American women age 18 to 44 is lower when you exercise regularly.

- (b) Describe a Type I error and a Type II error in this situation.

Type I - finding the mean diastolic blood pressure to be lower when it's really the same.

Type II - Concluding that the mean diastolic blood pressure is the same when it's really lower.

- (c) Give two ways to increase the power of the test you performed in (a).

Increase  $n$

Decrease  $\sigma$

2. You measure the weights of 24 male runners. You do not actually choose an SRS, but you are willing to assume that these runners are a random sample from the population of male runners in your town or city. Here are their weights in kilograms:

67.8	61.9	63.0	53.1	62.3	59.7	55.4	58.9
60.9	69.2	63.7	68.3	64.7	65.6	56.0	57.8
66.0	62.9	53.6	65.0	55.8	60.4	69.3	61.7

Suppose that the standard deviation of the population is known to be  $\sigma = 4.5$  kg.

Construct a 95% confidence interval for  $\mu$ , the mean of the population from which the sample is drawn. Follow the Inference Toolbox.

Population is all male runners in this city or town.  
Parameter is the mean weight.

Since  $\sigma$  is given choose a z-Interval. We're assuming this random sample is good and since  $n$  is large ( $n=24$ ) the CLT states the  $\bar{x}$  distribution is approximately normal.

$$\bar{x} = 61.79 \text{ kg} \quad \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \quad 61.79 \pm 1.96 \left( \frac{4.5}{\sqrt{24}} \right)$$
$$(59.991, 63.592)$$

We are 95% confident that the mean weight for male runners in this city is between 59.991 kg and 63.592 kg.

- (b) Explain the meaning of 95% confidence in part (a).

In repeated samples the interval will contain the true mean weight 95% of the time.

- (c) Based on this confidence interval, does a test of

$$H_0 : \mu = 61.3 \text{ kg}$$

$$H_a : \mu \neq 61.3 \text{ kg}$$

reject  $H_0$  at the 5% significance level? Justify your answer.

No, since 61.3 kg is in the interval.

3. Suppose that the population of the scores of all high school seniors who took the SAT Math test this year follows a normal distribution with mean  $\mu$  and standard deviation  $\sigma = 100$ . You read a report that says, "on the basis of a simple random sample of 100 high school seniors that took the SAT-M test this year, a confidence interval for  $\mu$  is  $512.00 \pm 25.76$ ." The confidence level for this interval is

(a) 90%.  
 (a) 95%.  
 (b) 99%.  
 (c) 99.5%.  
 (e) over 99.9%.

$$25.76 = z^* \frac{100}{\sqrt{100}} \quad z^* = 2.576$$

$$\frac{25.76}{10} = \frac{z^* 10}{10}$$

4. The value of  $z^*$  required for a 70% confidence interval is

(a) 0.5244  
 (b) 1.036  
 (c) 0.5244  
 (d) 0.6179  
 (e) The answer can't be determined from the information given.  
 (f) None of the above. The answer is \_\_\_\_\_.

5. I collect a random sample of size  $n$  from a population and from the data collected compute a 95% confidence interval for the mean of the population. Which of the following would produce a new confidence interval with larger width (larger margin of error) based on these same data?

(a) Use a larger confidence level.  
 (b) Use a smaller confidence level.  
 (c) Use the same confidence level, but compute the interval  $n$  times. Approximately 5% of these intervals will be larger.  
 (d) Increase the sample size.  
 (e) Nothing can guarantee absolutely that you will get a larger interval. One can only say the chance of obtaining a larger interval is 0.05.

6. In a test of  $H_0: \mu = 100$  against  $H_a: \mu \neq 100$ , a sample of size 80 produces  $z = 0.8$  for the value of the test statistic. The  $P$ -value of the test is thus equal to:

(a) 0.20  
 (b) 0.40  
 (c) 0.29  
 (d) 0.42  
 (e) 0.21



$$2(P(z > .8)) = 2(\text{normalcdf}(.8, 1E99)) = .42$$

7. A significance test allows you to reject a hypothesis  $H_0$  in favor of an alternative  $H_a$  at the 5% level of significance. What can you say about significance at the 1% level?

(a)  $H_0$  can be rejected at the 1% level of significance.  
 (b) There is insufficient evidence to reject  $H_0$  at the 1% level of significance.  
 (c) There is sufficient evidence to accept  $H_0$  at the 1% level of significance.  
 (d)  $H_a$  can be rejected at the 1% level of significance.  
 (e) The answer can't be determined from the information given.

In the past, the mean score of the seniors at South High on the American College Testing (ACT) college entrance examination has been 20. This year a special preparation course is offered, and all 53 seniors planning to take the ACT test enroll in the course. The mean of their 53 ACT scores is 22.1. The principal believes that the new course has improved the students' ACT scores. Assume that ACT scores vary normally with standard deviation 6.

- ① Population is all seniors taking ACT that also take prep course  
Parameter of interest is the actual mean score

- ② Since  $\sigma$  is known, use  $z$  Test. We do not know if these 53 seniors are a representative SRS. So we may not be able to generalize to all seniors who take prep courses.

Since the population is normally distributed, so is our sample distribution.

- ③  $H_0: \mu = 20$  (no change in scores)

$$H_a: \mu > 20 \text{ (scores increased)}$$

$$\bar{x} = 22.1 \quad z = \frac{22.1 - 20}{6/\sqrt{53}} = 2.548$$
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad p = .0054$$

- ④ If we assume  $H_0$  is true, then the probability of obtaining a sample of 53 students whose mean is 22.1 or higher is only .0054. This gives us reason to reject  $H_0$ . It appears that seniors taking the prep course earn higher mean ACT scores.