AP Statistics – 9.2a	Name: KEY
Goal: 1-Sided Significance Tests for a Population Proportion (p)	Date:

I. Right-Tail Test of Significance for Proportions

Example #1 "*Can you be confident of victory?*" Jack is a candidate for mayor running against only one other candidate, so he must gain at least 50% of the votes to be elected. Heading into the mayoral election, Jack is feeling fairly confident that he will be elected by obtaining more than 50% of the vote. Suppose that a random sample of 100 voters shows that 56 will vote for Jack. Based on a poll of voters just before the election, can Jack be confident of victory?

Step I: Set up your Test of Hypothesis (TOH).

- Define parameter: p = true proportion who vote for Jack
- Define hypothesis: $H_0: p = .5$
  $H_A: p > .5$
- Define your Level of Significance: $\alpha = .05$ ← when not given use $\alpha = .05$.

Step II: Check the conditions for carrying out a significance test to determine if Jack should feel confident of victory in the mayoral election.

Solution: The 3 required conditions are

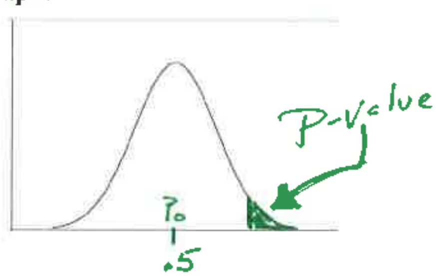
- Random: A random sample of voters was selected for the poll.
- Independent: Assuming the poll was done confidentially, one response should not affect other responses. We must assume there are more than 10(100) = 1000 voters since we are sampling without replacement.
- Normal: $np_0 = 100(0.5) = 50 \geq 10$ and $n(1 - p_0) = 100(1 - 0.5) = 50 \geq 10$

Step III: Define your choice of significance test: 1 Sample Z test for a proportion

Step IV: Define Sampling Distribution. Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean.

- n= 100
- $\hat{p} = \frac{56}{100} = .56$
- $p_o = .50$

- Sketch the graph:



Step V: Calculate the test statistic:

$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$	Because the hypothesized value is $p_0 = 0.5$, the standardized test statistic is: $z = \dfrac{0.56 - 0.5}{\sqrt{\dfrac{0.5(1 - 0.5)}{100}}} = 1.20$

CALC COMMAND

[STAT] [TESTS] [5: 1-Prop Z Test] $p_0 = .5$ x = 56 n = 100 $> p_0$

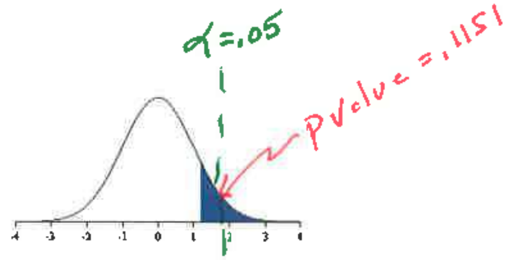
[Calculate] z = 1.2 p = .1151 [DRAW]

$\hat{p} = .56$ n = 100

Step VI: Calculate the p-value (write as a probability statement):

P-value = P(Z>1.20) =0.1151

- Using the TI-84→ normalcdf(1.20,E99,0,1) = 0.1151
- The *P*-value is the shaded area under the curve



Step VII: Interpret the P-value. Decision - Reject or Fail to Reject the null hypothesis.

- P-Value: If exactly 50% of all voters support Jack, there is about an 11.5% chance that 56% or more voters would support Jack in a random sample of size 100.
- Decision: Since the p-value (.1151) is very large, and greater than our predetermined significance level(α=.05), we "fail to reject Ho."

Step VIII: Interpret your significance test decision in context.

- There is not sufficient evidence to conclude that Jack will get more than 50% of the vote.

II. Left-Tail Test of Significance for Proportions

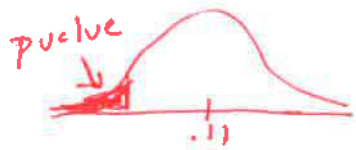
Example #2 Eleven percent of the products produced by an industrial process over the past several months have failed to conform to specifications. The company modifies the process in an attempt to reduce the rate of nonconformities. In a random sample of 300 items from a trial run, the modified process producess16 nonconforming item. Do these results provide convincing evidence that the modification is effective? Support your conclusion with a test of significance.

- Use the "Test of Significance Template" to work through these steps:

1. **Parameter of Interest**
2. **Level of Significance**
3. **Choice of Test**
4. **Null Hypothesis** (symbols and words)
5. **Alternative Hypothesis** (symbols and words)
6. **Conditions of Test**
7. **Sampling Distribution** (Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)
8. **Test Statistic** (clearly show calculation)
9. **P-value** (Use correct probability notation.)
10. **Meaning of the P-value** (Reject or Fail to reject null hypothesis)
11. **Conclusions** (in context)

9.2a EXAMPLE #2

Test of Significance Template

Parameter of Interest	p = true proportion of nonconforming items
Choice of Test	1 Sample Z Test for a proportion
Level of Significance	$\alpha = .05$
Null Hypothesis	English: THE TRUE PROPORTION OF NONCONFORMING ITEMS IS .11. Symbols: $H_0: p = .11$
Alternative Hypothesis	English: THE PROPORTION IS < .11 Symbols: $H_a: p < .11$
Conditions of Test	Random: SRS $n = 300$ Independent: We assume 1 items specification has no impact on another item. A trial run is at least $10(300) = 3{,}000$ items Normal: $np = 300(.11) = 33 \geq 10$ $n(1-p) = 300(.89) = 267 \geq 10$
Sampling Distribution	Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean: $n = 300$ $\hat{p} = \frac{16}{300} = .053$ 
Test Statistic	Formula: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ Plug-ins & Value: $z = \frac{.053 - .11}{\sqrt{\frac{(.11)(.89)}{300}}} = -3.16$
P-value	Use correct probability notation. $P(Z > -3.16) = .00079$
Meaning of the P-value	A pvalue (.0008) so small and less than $\alpha = .05$, so we reject Ho.
Conclusions	☒ Reject null hypothesis ☐ Fail to reject null hypothesis ☒ Significant result ☐ Not Significant result $.0008 < .05$ English: We reject H_0 and conclude that there is convincing evidence that the true proportion of nonconforming items is less than .11.

normalcdf$(-e99, -3.16, 0, 1) = .00079$

SOURCE: QUIZ 9.2A #1

See 3 student examples of a 4, 3 and 2

2005 Problem 4

"FRAPPY"

{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free video rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief that the proportion of boxes with vouchers is less than 0.2? Provide statistical evidence to support your answer.

$\boxed{\text{4 PARTS TO ANSWER}}$

Scoring:

a p= proportion of boxes of this brand of breakfast cereal that include a voucher for a free video rental

$H_0: p = .2$

$H_A: p < .2$

a **E I**

b Identify correct Test (by Name or by formula)

* one sample Z-test for p OR $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

b **E I**

*Conditions (as stated in rubric)

1 $np_0 = 65(.2) = 13 > 10$ and $n(1-p_0) = 65(.8) = 52 > 10$

2 It is reasonable to assume that the company produces more than $65 \times 10 = 650$ boxes of this cereal

3 It is reasonable to assume that the 65 boxes are random samples of all boxes of this cereal

c **E I**

d **E I**

Total:__/4

(c) Correct mechanics and calculations and provide p-value.

$$\hat{p} = 11/65 = .169$$

$$z = \frac{.169 - .2}{\sqrt{\frac{.2(1-.2)}{65}}} = -.62$$

$$\text{Pvalue} = P(Z < -.62) = .2676$$

normalcdf(-∞, -.62, 0, 1)

Tips:

(1) Check w/ calc

STAT TESTS

5:1 prop Z test

$p_0 = .2$ $< p_0$

$x = 11$

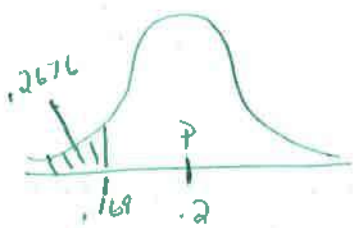
$n = 65$

⇓

$z = -.62$ ✓

$p = .2676$ ✓

(2) Sketch graph



(d) State a correct conclusion, with results of test, in context.

Since the p-value = .2676 is larger than any reasonable significance level (e.g. $\alpha = .05$), we cannot reject the companies claim. That is, we do not have statistically significant evidence to support the student's belief that the proportion of cereal boxes is less than 20 percent.

"FRAPPY"

{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free video rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief that the proportion of boxes with vouchers is less than 0.2? Provide statistical evidence to support your answer.

Scoring:

E I

E I

E I

E I

Total:__/4