AP Statistics – 9.2a	Name: 2020 KEY
Goal: 1-Sided Significance Tests for a Population Proportion (p)	Date:

I. Right-Tail Test of Significance for Proportions

Example #1 "*Can you be confident of victory?*" Jack is a candidate for mayor running against only one other candidate, so he must gain at least 50% of the votes to be elected. Heading into the mayoral election, Jack is feeling fairly confident that he will be elected by obtaining more than 50% of the vote. Suppose that a random sample of 100 voters shows that 56 will vote for Jack. Based on a poll of voters just before the election, can Jack be confident of victory?

Step I: Set up your Test of Hypothesis (TOH).

- Define parameter: P = TRUE PROPORTION WHO VOTE FOR JACK
- Define hypothesis: $H_0: p = 0.5$
  $H_A: p > 0.5$ (JACK WINS)
- Define your Level of Significance: $\alpha = 0.5$ ← USE WHEN α NOT GIVEN

Step II: Check the conditions for carrying out a significance test to determine if Jack should feel confident of victory in the mayoral election.

Random: Random sample (stated) ✓

INDEPENDENT: $n = 100$ VOTERS $\leq \frac{1}{10}$ (all voters) ✓

NORMAL: $np_0 = 100(.50) = 50 \geq 10$ ✓

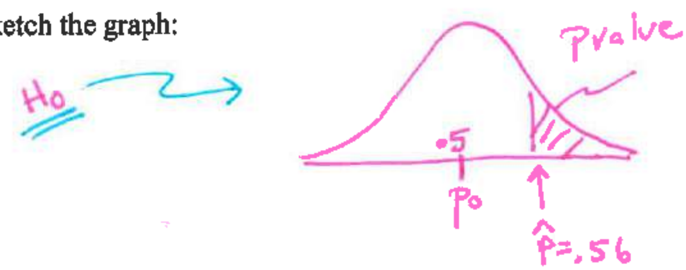
$n(1-p_0) = 100(.50) = 50 \geq 10$ ✓

Step III: Define your choice of significance test: 1 SAMPLE Z TEST FOR P

Step IV: Define Sampling Distribution. Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean.

- n= 100
- $\hat{p}$ = $56/100 = .56$
- p_0= 0.5

• Sketch the graph:



Step V: Calculate the test statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.56 - .50}{\sqrt{\frac{(.5)(.5)}{100}}} = 1.20$$

CALC: [STAT][TESTS][5: 1 PROP Z TEST] $p_0 = .5$ $x = 56$ $n = 100$ $> p_0$

$\Longrightarrow$ $Z = 1.2$ $\hat{p} = .56$ $n = 100$ $p = 0.1151$ ← pvalue

Step VI: Calculate the p-value (write as a probability statement):

p-value = $P(z > 1.20) = 0.1151$

→ use NormalCdf(1.20, E99, 0, 1)

Step VII: Interpret the P-value. ~~Decision: Reject or Fail to Reject the null hypothesis.~~

p_0

ASSUMING THE PROPORTION OF VOTERS WHO VOTE YES IS .50, THERE IS A 0.1151 probability OF GETTING A SAMPLE PROPORTION AS FAR AS 0.56 OR FURTHER PURELY BY CHANCE.

$\hat{p}$

Step VIII: Interpret your significance test decision in context. (4 PARTS MUST BE INCLUDED!)

→ SINCE THE PVALUE (0.1151) IS GREATER THAN H_0 WE FAIL TO REJECT H_0.

→ WE DO NOT HAVE CONVINCING EVIDENCE JACK WILL WIN THE MAYOR ELECTION.

II. Left-Tail Test of Significance for Proportions

Example #2 Eleven percent of the products produced by an industrial process over the past several months have failed to conform to specifications. The company modifies the process in an attempt to reduce the rate of nonconformities. In a random sample of 300 items from a trial run, the modified process producess16 nonconforming item. Do these results provide convincing evidence that the modification is effective? Support your conclusion with a test of significance.

- Use the "Test of Significance Template" to work through these steps:

1. **Parameter of Interest**
2. **Level of Significance**
3. **Choice of Test**
4. **Null Hypothesis** (symbols and words)
5. **Alternative Hypothesis** (symbols and words)
6. **Conditions of Test**
7. **Sampling Distribution** (Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)
8. **Test Statistic** (clearly show calculation)
9. **P-value** (Use correct probability notation.)
10. **Meaning of the P-value** (Reject or Fail to reject null hypothesis)
11. **Conclusions** (in context)

Test of Significance Template

Parameter of Interest	p = TRUE PROPORTION OF NON CONFORMING PRODUCTS
Choice of Test	1 SAMPLE Z TEST FOR p (Proportions)
Level of Significance	$\alpha = 0.05$ ← USE WHEN NOT GIVEN α
Null Hypothesis	English: $H_0: p = .11$ ← Company Claims 11% Products FAIL TO CONFORM TO STANDARDS Symbols:
Alternative Hypothesis	English: $H_A: p < .11$ ← HAVE THE MODIFICATIONS REDUCED THE PROPORTIONS Symbols:
Conditions of Test	Random: Random sample (stated) Independent: We must assume each trial run must have more than 10(300) = 3,000 ITEMS. Normal: $p_0 = .11$ < $(.11)(300) = 33 \geq 10$ ✓, $(.89)(300) = 267 \geq 10$
Sampling Distribution	Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean: $n = 300$ $\hat{p} = \frac{16}{300} = 0.053$ 
Test Statistic	Formula: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ Plug-ins & Value: $z = \dfrac{.053 - 0.11}{\sqrt{\dfrac{(0.11)(0.89)}{300}}} = -3.16$ Hand calc CALC: $z = -3.14$, pvalue ≈ 0 OR 0.00085
P-value	Use correct probability notation. Pvalue $= P(Z \leq -3.16) = 0.00079$
Meaning of the P-value	ASSUMING THE TRUE PROPORTION OF NONCONFORMING ITEMS IS 11%, THERE IS ONLY A 0.00079 Probability of getting a sample proportion AS FAR FROM 0.056 OR FURTHER PURELY BY CHANCE.
Conclusions	☒ Reject null hypothesis ☐ Fail to reject null hypothesis English: Since the pvalue (0.00079) is less than $\alpha = .05$, We reject Ho. We have convincing evidence that the true proportion of nonconforming items is less than 0.11.