Section 8.3
Sample Means

Learning Objectives

After this section, you should be able to…

- ✓ FIND the mean and standard deviation of the sampling distribution of a sample mean
- ✓ CALCULATE probabilities involving a sample mean when the population distribution is Normal
- ✓ EXPLAIN how the shape of the sampling distribution of sample means is related to the shape of the population distribution
- ✓ APPLY the central limit theorem to help find probabilities involving a sample mean

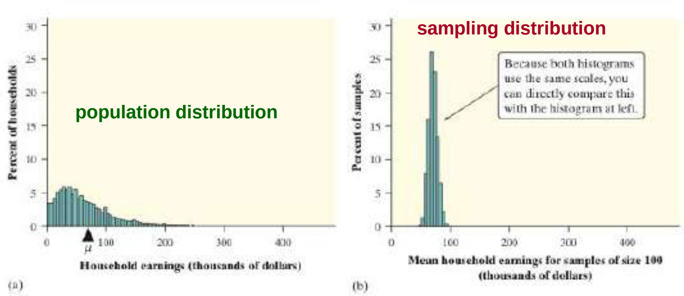
Sample Means – Comparing Population and Sampling Distributions

1. **Sample proportions** arise most often when we are interested in **categorical variables.**
2. **Sample means** are used use **quantitative variables** we are interested in other statistics such as the median or mean or standard deviation of the variable.

Example: **Consider the mean household earnings for samples of size 100.**

Compare the **population distribution on the left** with the **sampling distribution on the right.**

What do you notice about the shape, center, and spread of each?



The Sampling Distribution of $\bar{x}$

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- ***A Mind Twister*; Make sure you understand what this is saying!!!!!!**
- **When we choose many SRSs from a population,**
- **the sampling distribution of the sample mean**
- **is centered at the population mean μ and**
- **is less spread out than the population distribution. Here are the facts:**

Mean and Standard Deviation of the Sampling Distribution of Sample Means

Suppose that $\bar{x}$ is the mean of an SRS of size n drawn from a large population with mean μ and standard deviation σ. Then:

The **mean** of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu$

The standard deviation of the sampling distribution of $\bar{x}$ is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

as long as the *10% condition* is satisfied: *n* ≤ (1/10)*N*.

Note: These facts about the mean and standard deviation of $\bar{x}$ are true *no matter what shape the population distribution has*.

Sampling from a Normal Population

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1) We have described the mean and standard deviation of the sampling distribution of the sample mean $\bar{x}$ but not its shape.
That's because the shape of the distribution of $\bar{x}$ depends on the shape of the population distribution.

2) AN IMPORTANT CASE:
If the population distribution is Normal, then so is the sampling distribution of $\bar{x}$.
This is true no matter what the sample size is.

Sampling Distribution of a Sample Mean from a Normal Population

Suppose that a population is Normally distributed with mean μ and standard deviation σ. Then the sampling distribution of $\bar{x}$ has the Normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, provided that the 10% condition is met.

- **Let's look at the simulation we did on sampling distributions of sample means:**
- **Has the 10% condition been met?**
- **Find the mean and standard deviations for n=3, 5, and 8.**
- **What happens to means and standard deviations when sample size increases.**

Example: Young Women's Heights

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The height of young women follows a Normal distribution with mean $\mu = 64.5$ inches and standard deviation $\sigma = 2.5$ inches.

1. Find the probability that a randomly selected young woman is taller than 66.5 inches.
   - Define the Random Variable.
   - Calculate Probability (draw a picture, use zscores, answer in context)

2. Find the probability that the mean height of an SRS of 10 young women exceeds 66.5 inches.
   - Check conditions; Calculate mean and standard deviation for the sampling distribution
   - Calculate Probability (draw a picture, use zscores, answer in context)

Example: Young Women's Heights

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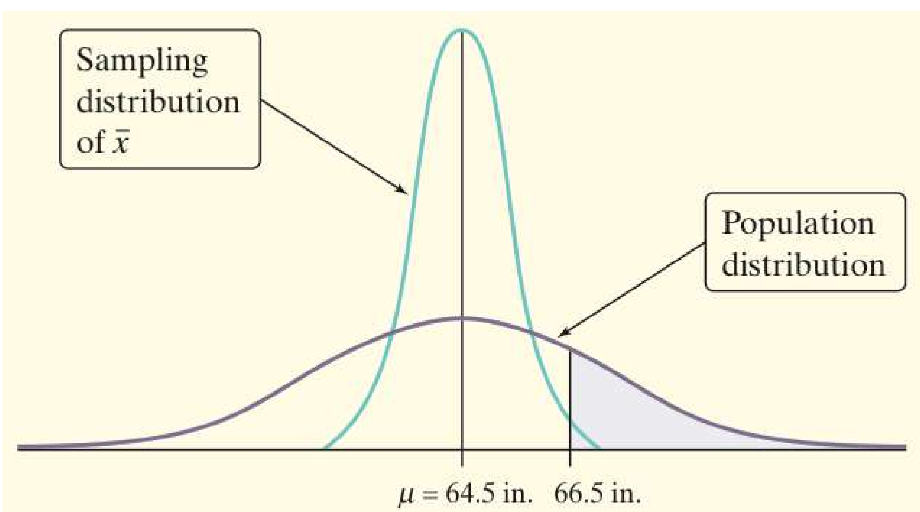
The height of young women follows a Normal distribution with mean $\mu = 64.5$ inches and standard deviation $\sigma = 2.5$ inches.



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The Central Limit Theorem

Most population distributions are not Normal. What is the shape of **the sampling distribution of sample means** when the population distribution isn't Normal?

- It is a remarkable fact that **as the sample size increases, THEN…**
- the **distribution of sample means changes its shape**: it looks <u>less like that of the population</u> and **more like a Normal distribution**!
- **When the sample is large enough, the distribution of sample means is very close to Normal**, ***no matter what shape the population distribution has*****, as long as the population has a finite standard deviation.**

> **Definition:**
> Draw an SRS of size n from any population with mean μ and finite standard deviation σ. The **central limit theorem (CLT)** says that when n is large, the sampling distribution of the sample mean $\bar{x}$ is approximately Normal.

Note: How large a sample size *n* is needed for the sampling distribution to be close to Normal depends on the shape of the population distribution. More observations are required if the population distribution is far from Normal.

$\boxed{\text{Normal in most cases if } n \geq 30.}$

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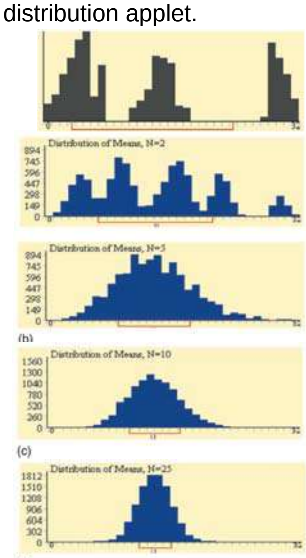
The Central Limit Theorem

Consider the strange population distribution from the Rice University sampling distribution applet.



Describe the shape of the sampling distributions as *n* increases. What do you notice?

Normal Condition for Sample Means

If the population distribution is Normal, then so is the sampling distribution of $\bar{x}$. This is true no matter what the sample size n is.

If the population distribution is not Normal, the central limit theorem tells us that the sampling distribution of $\bar{x}$ will be approximately Normal in most cases if $n \geq 30$.

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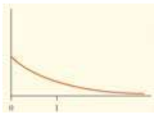
Example: Servicing Air Conditioners



Based on service records from the past year, the time (in hours) that a technician requires to complete preventative maintenance on an air conditioner follows the distribution that is strongly right-skewed, and whose most likely outcomes are close to 0. The mean time is $\mu = 1$ hour and the standard deviation is $\sigma = 1$

Your company will service an SRS of 70 air conditioners. You have budgeted 1.1 hours per unit. Will this be enough?

- •Check conditions
- •Calculate sampling distribution mean and standard deviation
- •Calculate Probability (draw a picture, use zscores, answer in context)

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Example: Making Auto Parts

A grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$mm. The machine has some variability, so the standard deviation is $\sigma = 0.002$mm. The machine operator inspects random samples of 4 axles each hour for quality control purposes. Assume the process is working properly.

How many axles would you need to sample if you wanted the standard deviation of the sampling distribution for the sample means to be $\sigma = 0.0005$ mm. Justify your answer.

APPENDIX

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Solutions to Problems in the notes

Example: Young Women's Heights

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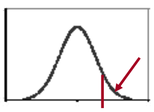
The height of young women follows a Normal distribution with mean µ = 64.5 inches and standard deviation σ = 2.5 inches.

1. **Find the probability that a randomly selected young woman is taller than 66.5 inches.**



Let *X* = the height of a randomly selected young woman. *X* is *N*(64.5, 2.5)

$$z = \frac{66.5 - 64.5}{2.5} = 0.80 \qquad P(X > 66.5) = P(Z > 0.80) = 1 - 0.7881 = 0.2119$$

The probability of choosing a young woman at random whose height exceeds 66.5 inches is about 0.21.

2. **Find the probability that the mean height of an SRS of 10 young women exceeds 66.5 inches.**

For an SRS of 10 young women, the sampling distribution of their sample mean height will have a mean and standard deviation

$$\mu_{\bar{x}} = \mu = 64.5 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{10}} = 0.79$$

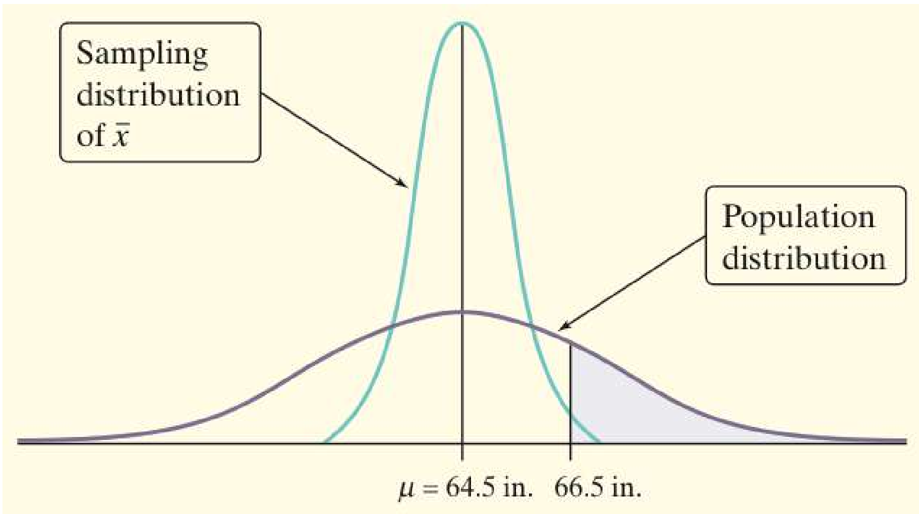
Since the population distribution is Normal, the sampling distribution will follow an *N*(64.5, 0.79) distribution.

$$z = \frac{66.5 - 64.5}{0.79} = 2.53 \qquad P(\bar{x} > 66.5) = P(Z > 2.53) = 1 - 0.9943 = 0.0057$$

It is very unlikely (less than a 1% chance) that we would choose an SRS of 10 young women whose average height exceeds 66.5 inches.

Example: Young Women's Heights

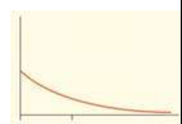
The height of young women follows a Normal distribution with mean μ = 64.5 inches and standard deviation σ = 2.5 inches.



Example: Servicing Air Conditioners

From the question:

- **Information on population: μ =1 hr and σ=1 hr**
- **Your company will service an SRS of 70 air conditioners.**
- **You have budgeted the average maintenance time as 1.1 hrs per unit.**
- **Will this be enough?**



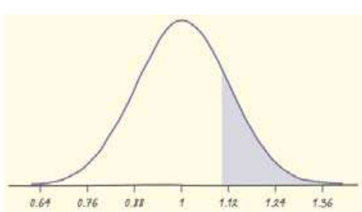
The 10% condition is met, since:
•the SRS has 70 units
•there are more than 10(70)=700 air conditioners in the population

The normal condition is met, since:
•The sampling distribution has a large enough sample with n = 70 ≥ 30.

The sampling distribution of the mean time spent working is approximately *N*(1, 0.12)

$$\mu_{\bar{x}} = \mu = 1$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{70}} = 0.12$$



We need to find P(mean time > 1.1 hours)

$$z = \frac{1.1 - 1}{0.12} = 0.83$$

$$P(\bar{x} > 1.1) = P(Z > 0.83)$$
$$= normalcdf\,(.83, e99, 0, 1) = .2033$$

If you budget 1.1 hours per unit, there is a 20% chance the technicians will NOT complete the work within the budgeted time.

Example: Making Auto Parts

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A grinding machine in an auto parts plant prepares axles with a target diameter μ = 40.125mm and σ = 0.002mm.

Need to determine how many axles needed to sample if you want a standard deviation σ = 0.0005 mm.

Justify your answer.

$$\boxed{\sigma_{\bar{x}} = .0005 = \frac{\sigma}{\sqrt{n}}} \rightarrow \boxed{.0005 = \frac{.002}{\sqrt{n}}} \rightarrow \boxed{\sqrt{n} = \frac{.002}{.0005} = 4} \rightarrow \boxed{n = 16}$$

So we need to take samples of 16, if we want a standard deviation of 0.0005 mm.