

1. A consumer watchdog organization estimates the mean weight of 1-ounce "Fun-Size" candy bars to see if customers are getting full value for their money. A random sample of 25 bars is selected and weighed, and the organization reports that a 90% confidence interval for the true mean weight of the candy bars is 0.992 to 0.998 ounces.

(a) What is the point estimate from this sample? THE POINT ESTIMATOR is the sample mean.

THE POINT ESTIMATE IS THE VALUE OF THE SAMPLE MEAN, WHICH IS THE MIDPOINT OF THE CONFIDENCE INTERVAL;

work $\frac{0.992 + 0.998}{2} = .995$

WHICH IS .995 OUNCES

(b) What is the margin of error?

THE MARGIN OF ERROR IS HALF THE WIDTH OF THE INTERVAL
 $(.998 - .992) = .006 / 2 = .003$

THE MARGIN OF ERROR IS .003 OUNCES

(c) Interpret the 90% confidence interval 0.992 to 0.998 in the context of the problem.

we are 90% confident that the interval from .992 to .998 ounces captures the true mean weight of Fun-Size candy bars

(d) Interpret the confidence level of 90% in the context of the problem.

If this method of constructing an interval were repeated many times, about 90% of the intervals constructed would contain the population mean weight of Fun-Size candy bars

2. A university health services physician is concerned about how much sleep freshman are getting in the first few months of school. She asks a simple random sample of 20 students how much sleep they got the previous night and constructs a 95% confidence interval for the mean amount of sleep in hours.

(a) Discuss whether this study meets the necessary conditions for constructing a confidence interval. If you think one of the conditions has not been met, what additional information would be required or what change in the study would you recommend?

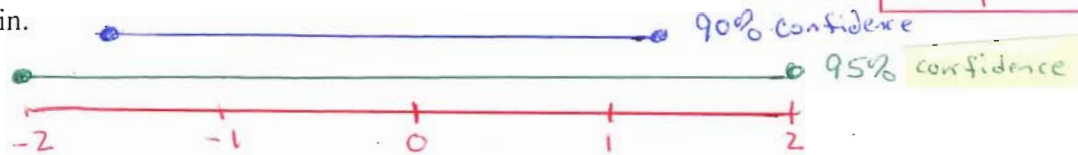
RANDOM: the problem states that an SRS was taken.

NORMAL: we do not know if the number of students are approximately normal. Since $n=20$, we can't be confident that the CLT applies. * We either need information about the population distribution's shape or we need a bigger sample.

INDEPENDENT: we are sampling students without replacement, but it seems reasonable that there are more than $10 \times 20 = 200$ students in the population, and that individual observations of amount of sleep are independent.

(b) If, instead of constructing a 95% confidence interval, the physician constructed a 90% confidence interval, would the 90% interval be wider, narrower, or the same width as the 95% interval? Explain.

USE Your Knowledge of 68-95-99.7 Rule



IF OUR INTERVAL HAS TO CAPTURE THE TRUE MEAN ONLY 90% OF THE TIME IN REPEATED SAMPLES INSTEAD OF 95% OF THE TIME, A NARROWER INTERVAL CAN BE CONSTRUCTED.

(c) How would the width of confidence interval change if the physician took a larger sample? Explain.

IF THE SAMPLE SIZE IS LARGER, * THE STANDARD DEVIATION OF THE SAMPLING DISTRIBUTION WILL BE SMALLER, SO THE CONFIDENCE INTERVAL WOULD BE SMALLER.

* Remember, larger sample sizes reduce variability in a sampling distribution.

1. Suppose you know that the distribution of finishing times for a certain crossword puzzle has a mean of 25 minutes, a standard deviation of 8 minutes, and is moderately skewed left. You take an SRS of 45 finish times from this distribution and calculate the mean finish time, \bar{x} .

- (a) Describe the shape, center, and spread of the sampling distribution of \bar{x} .

\bar{x} = mean time to finish a cross word puzzle

The sampling distribution for $n=45$:

center: $\mu_{\bar{x}} = 25 \text{ min}$

spread: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{45}} \approx 1.193 \text{ min}$

Shape: the CLT applies, since $n=45 > 30$, so the shape of the sampling distribution is approximately normal.

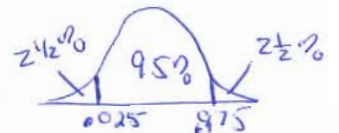
- (b) Find a number, k , such that 95% of the values in the sampling distribution will lie within k minutes of the mean of the distribution.

95% of the scores in a Normal distribution are within ± 1.96 standard deviations of the mean;

So:

$$K = 1.96 \cdot \frac{8}{\sqrt{45}} \approx \underline{\underline{2.34 \text{ min}}}$$

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$$\text{invNorm}(0.025, 0, 1) = -1.96$$

$$\text{invNorm}(0.975, 0, 1) = 1.96$$

- (c) If you take repeated samples of size 45 from this population, what proportion of the time will the interval $\bar{x} \pm k$ contain the number 25? Explain.

Since \bar{x} will be within $\pm K$ minutes of the mean of 25 minutes 95% of the time,

Then $\bar{x} \pm K$ will contain 25 minutes 95% of the time.

2. The confidence level is sometimes called the "capture rate." Explain why this is an appropriate term.

The confidence level can be interpreted as the percentage of repeated samples that capture the true mean within the interval

3. An insect ecologist reports a 95% confidence interval for the mean length of full-grown aquatic larvae of the Phantom Midge *Chaoborus albatus* to be 6.9 to 8.5 mm, based on a sample of 9 individual larvae.

- (a) What are the point estimate and margin of error associated with this confidence interval?

$$95\% \text{ CI: } 8.5 - 6.9 = 1.6 / 2 = .8 \quad 6.9 + .8 = 7.7$$

Point Estimate = mid point of interval = Sample mean = 7.7 mm

Margin of error = half the width of the interval = 0.8 mm

- (b) The ecologist stated that "all necessary conditions for constructing this confidence interval were met." What does this tell you about his methods and about the population of insect larvae?

Random: He took an SRS of larvae

Normal: The size of larvae came from a population that was approximately Normally distributed.

INDEPENDENT: the observations of larvae lengths are independent.

- (c) If the ecologist had reported a 99% confidence interval instead of a 95% interval, how would it have been different? Explain.

The 99% Confidence Interval would have to be wider in order to contain the true mean length in 99% of repeated samples, instead of only 95% of samples

- (d) The ecologist was unhappy with how wide this interval was. What should he do to produce a narrower interval with the same level of confidence? Explain.

Take a larger sample, which will reduce the standard deviation of the sampling distribution and make the interval narrower.