7.5G HW

NAME

BINOMIAL DISTRIBUTIONS:

85. Aircraft engines Engineers define reliability as the probability that an item will perform its function under specific conditions for a specific period of time. A certain model of aircraft engine is designed so that each engine has probability 0.999 of performing properly for an hour of flight. Company engineers test an SRS of 350 engines of this model. Let $X =$ the number that operate for an hour without failure.

(a) Explain why X is a binomial random variable.

(b) Find the mean and standard deviation of X. Interpret each value in context.

(c) Two engines failed the test. Are you convinced that this model of engine is less reliable than it's supposed to be? Compute $P(X \leq 348)$ and use the result to justify your answer.

91.*On the Web What kinds of Web sites do males aged 18 to 34 visit most often? Half of male Internet users in this age group visit an auction site such as eBay at least once a month.[11] A study of Internet use interviews a random sample of 500 men aged 18 to 34. Let $X =$ the number in the sample who visit an auction site at least once a month.

(a) Show that X is approximately a binomial random variable.

(b) Check the conditions for using a Normal approximation in this setting.

(c) Use the Normal approximation to find the probability that at least 235 of the men in the sample visit an online auction site at least once a month.

GEOMETRIC DISTRIBUTIONS:

95. Geometric or not? Determine whether each of the following scenarios describes a geometric setting. If so, define an appropriate geometric random variable.

(a) A popular brand of cereal puts a card with one of five famous NASCAR drivers in each box. There is a 1/5 chance that any particular driver's card ends up in any box of cereal. Buy boxes of the cereal until you have all 5 drivers' cards.

(b) In a game of 4-Spot Keno, Lola picks 4 numbers from 1 to 80. The casino randomly selects 20 winning numbers from 1 to 80. Lola wins money if she picks 2 or more of the winning numbers. The probability that this happens is 0.259. Lola decides to keep playing games of 4-Spot Keno until she wins some money.

96. Geometric or not? Determine whether each of the following scenarios describes a geometric setting. If so, define an appropriate geometric random variable.

(a) Shuffle a standard deck of playing cards well. Then turn over one card at a time from the top of the deck until you get an ace.

(b) Lawrence is learning to shoot a bow and arrow. On any shot, he has about a 10% chance of hitting the bull's-eye. Lawrence's instructor makes him keep shooting until he gets a bull's-eye.

97. Cranky mower To start her old mower, Rita has to pull a cord and hope for some luck. On any particular pull, the mower has a 20% chance of starting.

(a) Find the probability that it takes her exactly 3 pulls to start the mower. Show your work.

(b) Find the probability that it takes her more than 10 pulls to start the mower. Show your work.

99. Roulette Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot.

(a) How many spins do you expect it to take until Marti wins? Justify your answer.

(b) Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

STATISTICAL SAMPLING

87. Airport security The Transportation Security Administration (TSA) is responsible for airport safety. On some flights, TSA officers randomly select passengers for an extra security check before boarding. One such flight had 76 passengers—12 in first class and 64 in coach class. Some passengers were surprised when none of the 10 passengers chosen for screening were seated in first class. Can we use a binomial distribution to approximate this probability? Justify your answer.

88. Scrabble In the game of Scrabble, each player begins by drawing 7 tiles from a bag containing 100 tiles. There are 42 vowels, 56 consonants, and 2 blank tiles in the bag. Cait chooses her 7 tiles and is surprised to discover that all of them are vowels. Can we use a binomial distribution to approximate this probability? Justify your answer.

7.5 G HW

(85) X = # that operate for an hour WITHOUT FAILURE

(a) ✓ BINARY – Success – operates for an hour successfull
FAILURE – does NOT

✓ INDEPENDENT – THE OPERATION OF ONE ENGINE DOES NOT AFFECT ANOTHER ENGINE

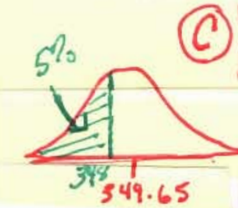
✓ Number – fixed number of trials $n = 350$

✓ Success – fixed probability of succes $p = .999$

(b) $B(350, .999)$ $\mu_x = np = (350)(.999) = \boxed{349.65}$

$\sigma_x = \sqrt{npq} = \sqrt{350(.999)(.001)} = .5913$

IF WE WERE TO TEST 350 ENGINES OVER AND OVER AGAIN, WE WOULD EXPECT THAT, ON AVERAGE, 349.65 of them would OPERATE FOR 1 HOUR WITHOUT FAILURE. IN INDIVIDUAL TESTS, WE WOULD EXPECT TO FIND THE NUMBER OF ENGINES THAT OPERATE FOR AN HOUR WITHOUT FAILURE TO VARY FROM 349.65 by an average of .591.



(c) $\boxed{B(350, .999)}$ $P(x \leq 348) = \text{binomcdf}(350, .999, 348) = .0485$

THERE IS ABOUT A 5% CHANCE THAT 348 machines will OPERATE CORRECTLY. YOU WOULD EXPECT MACHINES TO BE MORE RELIABLE THAN THIS.

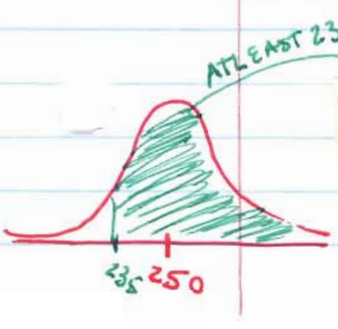
(91) X = # IN SAMPLE WHO VISIT AN AUCTION SITE AT LEAST once a MONTH

(a) X is a random binomial variable because there is a fixed number of trials (N=500); a fixed probability of success (p=.5) and participants are randomly selected (so independent)

(b) Normal conditions are met: $NP = 500(.5) = 250 > 10$ $Nq = 500(.5) = 250 > 10$

(c) $\boxed{N(250, 11.18)}$ $\mu_x = np = 500(.5) = 250$

$\sigma_x = \sqrt{npq} = \sqrt{500(\tfrac{1}{2})(\tfrac{1}{2})} = \sqrt{125} = 11.18$



$P(X \geq 235) = \text{normalcdf}(235, E99, 250, 11.18) = .9102$

THERE IS ABOUT A 91% CHANCE THAT 235 surveyed males visit an auction site once a MONTH

(95) (a) This is NOT A GEOMERTRIC SETTING (b/c NUMBER)
✓BINARY - SUCCESS: GET A CARD YOU DO NOT HAVE
FAILURE: GET A CARY YOU DO HAVE
✓INDEPENDENT - WE ASSUME CARDS ARE PUT IN BOXES RANDOMLY
NUMBER - WE ARE NOT COUNTING TO THE 1ST SUCCESS

(b) This is a Geometric Setting $\boxed{G(.259)}$
✓BINARY - SUCCESS - WINS FAILURE - DOES NOT WIN $
✓INDEPENDENT - RANDOMLY SELECTED 20 NUMBERS
✓ NUMBER - COUNT THE NUMBER OF GAMES TILL SHE WINS
✓ SUCCESS - FIXED probability = .259

(96) (a) This NO GEOMETRIC SETTING BECAUSE THE TRIALS ARE NOT INDEPENDENT. WE ARE NOT REPLACING THE PREVIOUS CARD BACK

(b) THIS A GEOMETRIC SETTING $\boxed{G(.1)}$
BINARY - SUCCESS - GET A BULLS EYE FAILURE - DOES NOT
INDEPENDENT - DIFFERENT SHOTS SHOULD BE INDEPENDENT OF EACH OTHER
NUMBER - REPEAT TRIALS UNTIL THE 1ST BULLS EYE
SUCCESS - FIXED Probability $p = .1$

(97) (a) X = # OF PULLS TO GET MOWER STARTED
$\boxed{G(.2)}$
$P(X=3) = (.8)^2(.2) = .128$ or geometpdf(.2, 3)
There is about a 13% chance the mower starts on the 3RD TRY

(b) $P(X>10) = 1 - \text{geometcdf}(.2, 10) = .1074$

(99) X = # of spins until Marti WINS $\boxed{G(1/38)}$

(a) $E(X) = \mu_x = 1/p = \frac{1}{1/38} = 38$

WE EXPECT MARTI TO SPIN 38 times, on average, to WIN.

(b) $P(X \leq 3) =$ geometcdf (1/38, 3) = .0768

ALTHOUGH THIS IS NOT AN UNUSUAL OCCURANCE, IT HAPPENS ABOUT 8% OF THE TIME, SO IT IS NOT COMPLETELY SURPRISING.

(87) Population = N = 76
Sample = n = 10

Rule: $n \leq \frac{1}{10} N$

$10 \leq \frac{1}{10}(76) = 7.6$ X

We cannot use the binomial distribution here because the sample size (10) is more than 10% of the population (76).

(88) Population = N = 100
Sample = n = 7

Rule: $n \leq 1/10\, N$

$7 \leq 1/10(100) = 10$ ✓

We can use the binomial distribution in this case because the sample size (7) is less than 10% of the population.