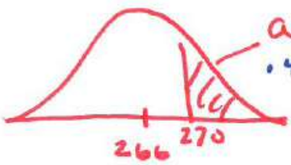
AP Statistics – 7.3	Name:
Goal: Understanding Sample Means	Date:

• **Sampling Distributions of Sample Means** **(CYU on page 448):**

① X = LENGTH OF PREGNANCY IN DAYS

$X \sim N(266, 16)$



$P(X \geq 270)$

$z = \frac{270-226}{16} = 0.25$

$P(Z \geq .25) = .4013$

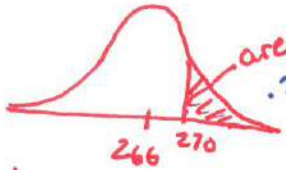
∴ THE PROBABILITY OF RANDOMLY CHOOSING A PREGNANT WOMEN WHO'S PREGNANCY IS FOR THAN 270 DAYS (9 mos) IS ABOUT 40%

② The mean of the sampling distr. for $\bar{X} \rightarrow \boxed{\mu_{\bar{X}} = \mu = 266 \text{ days}}$

③ THE 10% CONDITION IS REQUIRED TO CALC. S.D. $\boxed{n=6}$

- It is reasonable there are more than 6(10) = 60 preg women
  OR $6 \leq \frac{1}{10}$(preg. women)
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{6}} \simeq 6.53$ days

④ $\bar{X}$ = Random sample of 6 preg. women $\longrightarrow N(266, 6.53)$



$P(\bar{X} \geq 270)$

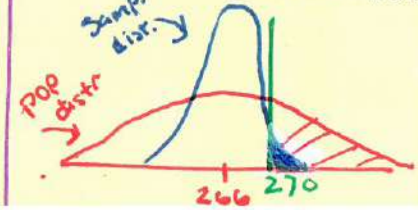
$Z = \frac{270-266}{6.53} = .6126$

$P(Z \geq .6126) = .2701$

THE PROBABILITY OF A sample of 6 pregnant women's mean days pregnant is more than 270 dayas is about 27%

IMPORTANT CONCEPT

The avg of several observations has less Variability than 1 INDIVIDUAL observation



- **The Central Limit Theorem** *Describe CLT in your words:*

CLT SAYS THAT WHEN "n" IS LARGE, THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN $\bar{x}$ IS APPROXIMATELY NORMAL

- *How does CLT apply to sample proportions and sample means?*

NO YES

CLT APPLIES TO $\bar{X}$.

CLT DOES NOT APPLY TO $\hat{p}$

- *How does CLT apply to normal distributions and non-normal distributions?*

(1) NORMAL DISTR. OF POPULATION DOES NOT NEED CLT

(2) CLT IS FOR NON-NORMAL OR UNKNOWN POP DISTR. WHEN $n \geq 30$.

- *How does CLT apply to sample size?*

RULE OF THUMB $\longrightarrow n \geq 30$

III. Example: Making Auto Parts *A grinding machine in an auto parts plant prepares axles with a target diameter μ = 40.125mm. The machine has some variability, so the standard deviation is σ = 0.002mm. The machine operator inspects random samples of 4 axles each hour for quality control purposes. Assume the process is working properly. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution for the sample means to be σ = 0.0005 mm. Justify your answer.*

$\mu = 40.125$ mm $\sigma = .002$ mm SRS $n=4$

Want $\sigma_{\bar{x}} = .0005$

10% condition met – reasonable the shop has more than $4(10) = 40$ axles

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$ ← Green Sheet

⇓

$$.0005 = \frac{.002}{\sqrt{n}} \xrightarrow[\text{algebra}]{\text{use}} \frac{.0005\sqrt{n}}{.0005} = \frac{.002}{.0005}$$

$$(\sqrt{n})^2 = 4^2 \quad n = 16$$

So you need to take samples of size 16, if we want a standard deviation of .0005 m

"FRAPPY"
{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Consider the sampling distribution of a sample mean obtained by random sampling from an infinite population. This population has a distribution that is highly skewed toward the larger values.

(a) How is the mean of the sampling distribution related to the mean of the population?

The mean of the sampling distribution is equal to the mean of the population.

$$\boxed{\mu_{\bar{x}} = \mu}$$

(b) How is the standard deviation of the sampling distribution related to the standard deviation of the population?

The standard deviation of the sampling distribution EQUAL TO THE POPULATION STANDARD DEVIATION DIVIDED BY THE SQUARE ROOT OF THE SAMPLE SIZE.

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

OR CLEARLY STATES THAT THE STANDARD DEVIATION OF THE SAMPLING DISTRIBUTION DECREASES AS THE SAMPLE SIZE INCREASES.

(c) How is the shape of the sampling distribution affected by the sample size?

① FOR SMALL SAMPLE SIZES, THE SAMPLING DISTRIBUTION IS SKEWED.

② AS THE SAMPLE SIZE GETS LARGER FOR THE SAMPLING DISTRIBUTION THE SHAPE GETS MORE AND MORE NORMAL-LIKE (BELL-SHAPE)

(AKA CLT)

Total: __/4

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A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.

$\mu = 3.9$ $\quad n = 40$
$\sigma = 1.1$

Scoring:

(a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

Center: $\mu_{\bar{x}} = \mu = 3.9$ minutes

Spread: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.1}{\sqrt{40}} \approx 0.174$ minutes

E P I

Shape: The CLT applies since the sample size is 40 which is relatively large. Therefore the shape is approximately normal.

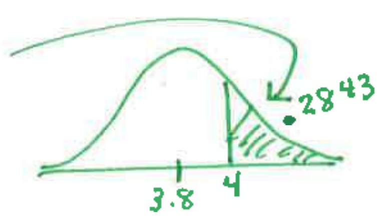
(b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

$N(3.9, 0.174)$

music time = 160 min. $\Rightarrow \bar{X} = \frac{160}{40} = 4$ min/song
\# of songs = 40

$P(\bar{X} > 4.0) = P\left(z > \frac{4-3.9}{.174} = .57\right)$

normalcdf(.57, E99, 0, 1)



E P I

The probability of 40 randomly selected songs exceed the available air time is about 28%.

Total: __/4

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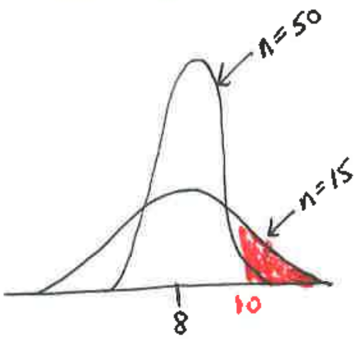
Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

Scoring:



(a) Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely?

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

• A random sample of 15 fish having a mean length that is greater than 10 inches

$N(8, \frac{\sigma}{\sqrt{15}})$

or

• A random sample of 50 fish having a mean length that is greater than 10 inches

$N(8, \frac{\sigma}{\sqrt{50}})$ will have a small S.D.

Justify your answer.

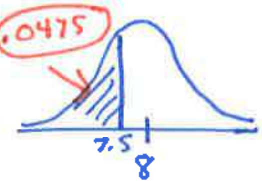
A random same of size 15 will more likely have fish greater than 10 in because the smaller sample size will have more variability and the tail area will be larger compared to the n=50. SEE THE GRAPH.

E P I

(b) Suppose the standard deviation of the sampling distribution of the sample mean for random samples of size 50 is 0.3 inch. If the mean length of the fish is 8 inches, use the normal distribution to compute the probability that a random sample of 50 fish will have a mean length less than 7.5 inches.

$N(8, .3)$ $\quad P(\bar{x} < 7.5) = P(z < \frac{7.5-8}{.3} = -1.67) = .0475$

normalcdf(-E99, -1.67, 0, 1)



There is about a 5% chance that a random sample of size 50 will result in a mean of less than 7.5 in.

E P I

(c) Suppose the distribution of fish lengths in this lake was nonnormal but had the same mean and standard deviation. Would it still be appropriate to use the normal distribution to compute the probability in part (b)? Justify your answer.

YES because the CLT states that if the sampling distribution of a sample mean is large enough the distribution will be approximately normal. IN Part (b), the sample size of 50 fish is reasonably large.

E P I

Total:__/4