AP Statistics – 7.2	Name:
Goal: Understanding Sample Proportions	Date:

I. How is **THE STATISTIC** $\hat{p}$ related to the binomial random variable X (page 436)?

- **Binomial random variable X →** $\mu_x = np$ $\sigma_x = \sqrt{np(1-p)}$
- $\hat{p}$ **=X/n= (1/n)X** **(X is our random variable)**
  - $\mu_{\hat{p}} = (1/n) \bullet \mu_x$
  - $\sigma^2_{\hat{p}} = \left(\frac{1}{n} \bullet \sigma_x\right)^2$
- **Use your algebra skills to simplify to find the mean and standard deviation of $\hat{p}$:**
  - $\mu_{\hat{p}} = (1/n) \bullet np = p$
  - $\sigma^2_{\hat{p}} = \left(\frac{1}{n} \bullet \sigma_x\right)^2 = \frac{1}{n^2} \bullet np(1-p) = \frac{np(1-p)}{n}$

II. **Compare the means and standard deviations for our Bean Activity**

Green Sheet: $\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

1) SRS n=3. Find the mean and standard deviation.
   - Since the population parameter… p=½ → $\mu_{\hat{p}} = p = ½$
   - $\sigma_{\hat{p}} = \sqrt{\frac{.5(.5)}{3}} = .289$
2) SRS n=5. Find the mean and standard deviation.
   - p=½ → $\mu_{\hat{p}} = ½$
   - $\sigma_{\hat{p}} = \sqrt{\frac{.5(.5)}{5}} = .224$
3) SRS n=20. Find the mean and standard deviation.
   - p=½ → $\mu_{\hat{p}} = ½$
   - $\sigma_{\hat{p}} = \sqrt{\frac{.5(.5)}{20}} = .112$
4) How do the means and standard deviations compare?
   - $\mu_{\hat{p}} = ½$ is the same in all SRS's.
   - **As the size of the SRS increases the variability decreases and $\sigma_{\hat{p}}$ decreases.**

III. **Sampling Distributions of $\hat{p}$** (CYU on page 437):

ⓐ <u>Given information</u>: $p = .75$ (the population parameter for a proportion)

① The mean of the sampling distribution ($\mu_{\hat{p}}$) is the same as the population proportion $\longrightarrow$ $\underline{\underline{\mu_{\hat{p}} = .15}}$

② 10% Condition: SRS = 1,000 AND IT IS FAIR TO ASSUME THE POPULATION IS OVER 10,000 young adults

$$\underline{\underline{\sigma_{\hat{p}}}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.75)(.25)}{1000}} = \underline{\underline{.0137}}$$

③ The sampling distribution is approximately normal because Normal conditions met:

$np = 1000(.75) = 750 \geq 10$ ✓

$nq = 1000(.25) = 250 \geq 10$ ✓

④ SRS $n = 9,000$

$$\underline{\underline{\sigma_{\hat{p}}}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.75)(.25)}{9000}} = \underline{\underline{.0046}}$$ (NOTICE IT DECREASES)

IV. Using the Normal Approximation for $\hat{p}$ (page 395):

TIP: Do the work on another sheet of paper and do NOT skip any of these steps !!!!!!!!!

Example #1: A polling organization asks an SRS of 1500 first-year college students how far away their home is. Suppose that 35% of all first-year students actually attend college within 50 miles of home. What is the probability that the random sample of 1500 students will give a result within 2 percentage points of this true value?

1) What are the Population Parameter(s)?

$p = .35$ = % of college students within 50 miles from home

2) What Sample information is given?

SRS $n = 1{,}500$
What's the probability results ± 2% of the true value.

3) State the probability of interest (in a probability statement).

$P(.33 \le \hat{p} \le .37)$

4) What conditions must you check? Have they been met?

① 10% Condition met $n = 1{,}500$ to ensure independence

N = population must be at least 15,000 students which seems reasonable.

② Can the Normal Model be used?

$np = 1500(.35) = 525 \geq 10$ ✓
$nq = 1500(.65) = 975 \geq 10$ ✓

Yes, we can use the Normal Model

5) Find the Mean and Standard Deviation. Clearly show your work.

$\mu_{\hat{p}} = p = .35$

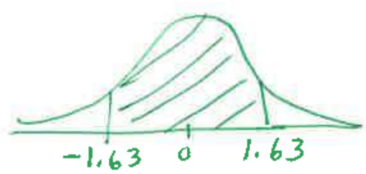
$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.35)(.65)}{1500}} \approx .0123$

model and parameters shortcut: $N(.35, .0123)$

6) Calculate the Z-Scores. Remember to label Z=

$Z_L = \frac{.33 - .35}{.0123} = -1.63$ $Z_u = \frac{.37 - .35}{.0123} = 1.63$

7) Draw an appropriate diagram.



8) Estimate the probability with the 68-95-99.7 rule.

$+/- 1\sigma \approx 68\%$
$+/- 2\sigma \approx 95\%$

probability probably around 85-90%

9) Restate the probability statement using the z-scores→
& find the probability →

$P(-1.63 \le z \le 1.63) = \boxed{.8969}$

normalcdf(-1.63, 1.63, 0, 1)

10) Write conclusion (in context)

About 90% of all SRS's of size 1,500 will give a result within 2% of the true population parameter (35%) of first year college students who live within 50 miles of home.

Example #2: The Superintendent of a large school wants to know the proportion of high school students in her district are planning to attend a four-year college or university. Suppose that 80% of all high school students in her district are planning to attend a four-year college or university. What is the probability that an SRS of size 125 will give a result within 7 percentage points of the true value?

1) What are the Population Parameter(s)? $p = .80$

2) What Sample information is given? SRS $n = 125$
$+/- 7\%$ of $\hat{p}$

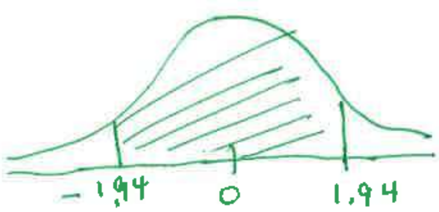
3) State the probability of interest (in a probability statement).
$P(.73 \leq \hat{p} \leq .87)$

4) What conditions must you check? Have they been met?
① 10% Condition for independence: $n = 125$ N = It is reasonable that a large school population is greater than 1,250 students
② We can use the normal model since np and nq are greater than 10.
$np = 125(.80) = 100 \geq 10$ ✓ $nq = 125(.20) = 25 \geq 10$ ✓

5) Find the Mean and Standard Deviation. Clearly show your work.
$\mu_{\hat{p}} = .8$
$\sigma_{\hat{p}} = \sqrt{\frac{(.8)(.2)}{125}} = .036$
$\boxed{N(.8, .036)}$

6) Calculate the Z-Scores. Remember to label Z=
$Z = \frac{.73 - .80}{.036} = -1.94$ $Z = \frac{.87 - .80}{.036} = 1.94$

7) Draw an appropriate diagram.



8) Estimate the probability with the 68-95-99.7 rule.
About 95%

9) Restate the probability statement using the z-scores→
& find the probability →
$P(-1.94 \leq Z \leq 1.94) = \boxed{.9476}$
normalcdf(-1.94, 1.94, 0, 1)

10) Write conclusion (in context)
About 95% of all SRS's of size 125 will give a sample proportion within 7 points of the true population parameter (the true value) of high school students who are planning to attend a 4-year College or University.