AP Statistics – 7.2 (2020 version)	Name:
Goal: Understanding Sample Proportion Sampling Distributions	Date:

I. Compare the means and standard deviations for our "Bean Activity"

Green Sheet: $\mu_{\hat{p}} = p$ $\qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

1) SRS n=3. Find the mean and standard deviation.

$p = 1/2 \rightarrow \mu_{\hat{p}} = 1/2$ $\qquad \sigma_{\hat{p}} = \sqrt{\frac{.5(.5)}{3}} = .289$

2) SRS n=5. Find the mean and standard deviation.

$p = 1/2 \rightarrow \mu_{\hat{p}} = 1/2$ $\qquad \sigma_{\hat{p}} = \sqrt{\frac{(.5)(.5)}{5}} = .224$

3) SRS n=20. Find the mean and standard deviation.

$p = 1/2 \rightarrow \mu_{\hat{p}} = 1/2$ $\qquad \sigma_{\hat{p}} = \sqrt{\frac{.5(.5)}{5}} = .112$

4) How do the means compare?

The means are the same.

$\mu_{\hat{p}}$ is an unbiased estimator of the true population p because by definition an unbiased estimator must EQUAL THE population parameter

5) How do the standard deviations compare?

AS THE SAMPLE SIZE OF THE SRS, THE VARIABILITY DECREASES AND $\sigma_{\hat{p}}$ DECREASES.

II. Important Ideas - Sampling Distributions of $\hat{p}$

#1 P and $\hat{p}$ are %'s μ and $\bar{x}$ are averages. #2 MEAN + S.D $\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	#3 NORMAL Condition Large Counts $np \geq 10$ and $n(1-p) \geq 10$	#4 PROBABILITY IF THE SAMPLING DISTRIBUTION OF $\hat{p}$ is approx. NORMAL: USE $Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

IF THE 10% CONDITION MET.

III. **Sampling Distributions o f $\hat{p}$** **(CYU on page 437):**

ALWAYS DEFINE THE POPULATION PARAMETER
(TIP: WRITE DIRECTLY FROM QUESTION)

p = TRUE PROPORTION OF YOUNG ADULT INTERNET USERS (18-29) WHO WATCH ONLINE VIDEOS

① $p = .75 \longrightarrow \mu_{\hat{p}} = .75$

② To calculate S.D, the independent condition MUST BE MET!

10% CONDITION - SRS $\boxed{n=1,000}$. IT IS FAIR TO ASSUME THE population is over 10,000 young adults.

$$\sigma_{\hat{p}} = \sqrt{\frac{(.75)(.25)}{1,000}} = \underline{\underline{.0137}}$$

③ NORMAL Condition - LARGE COUNTS FOR PROPORTIONS

$np = 1000(.75) = 750 \geq 10$ ✓

$nq = 1000(.25) = 250 \geq 10$ ✓

MUST Check BOTH!

④ SRS $n = 9,000$

$$\sigma_{\hat{p}} = \sqrt{\frac{(.75)(.25)}{9000}} = \underline{\underline{.0046}}$$ (The SD decreased when n increased)

IV. Using the Normal Approximation for $\hat{p}$ (page 395):

TIP: Do NOT skip any of these steps !!!!!!!!!

Example #1: A polling organization asks an SRS of 1500 first-year college students how far away their home is. Suppose that 35% of all first-year students actually attend college within 50 miles of home. What is the probability that the random sample of 1500 students will give a result within 2 percentage points of this true value?

1) What is the Population Parameter(s)?

P = TRUE % 1ST YEAR COLLEGE STUDENTS WHO LIVE WITHIN 50 miles of home.

$\boxed{p = .35}$

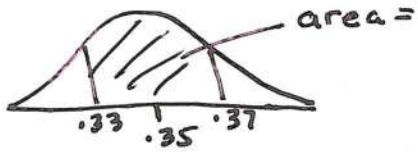
2) What Sample information is given?

SRS $n = 1{,}500$

$\hat{p} = ?$

3) State the probability of interest (in a probability statement). $P(.33 \le \hat{p} \le .37)$

Sketch Graph



4) What conditions must you check? Have they been met?

① INDEPENDENT – SAMPLING W/O REPLACEMENT – 1,500 < 10% (1ST COLLEGE STUDENTS)

② NORMAL – $1500(.35) = 525 \geq 10$ ✓
$1500(.65) = 975 \geq 10$ ✓

5) Find the Mean and Standard Deviation. Clearly show your work.

$\mu_{\hat{p}} = p = .35$

$\sigma_{\hat{p}} = \sqrt{\frac{(.35)(.65)}{1500}} = .0123$

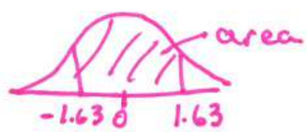
STATE MODEL $\sim N(.35, .0123)$

6) Calculate the Z-Scores. Remember to label Z=

$Z_L = \frac{.33 - .35}{.0123} = -1.63$ AND $Z_u = \frac{.37 - .35}{.0123} = 1.63$

Review Empirical Rule

7) Draw the standard normal graph N(0,1).



8) Estimate the probability with the 68-95-99.7 rule.

+/- 1 SD ≃ 68%
+/- 2 SD ≃ 95%

EST ± 1.6 ⟶ approx 80 – 90%

9) Restate the probability statement using the z-scores→ $P(-1.63 \le Z \le 1.63) = \boxed{.8969}$

& find the probability → USE NORMALCDF – NO CREDIT FOR CALC COMMAND.

10) Write conclusion (in context)

About 90% of all SRS's OF SIZE 1,500 will give a result within 2% OF THE TRUE POPULATION Parameter (35%) of 1ST YEAR COLLEGE STUDENTS WHO LIVE within 50 miles of home.

Example #2: The Superintendent of a large school wants to know the proportion of high school students in her district are planning to attend a four-year college or university. Suppose that 80% of all high school students in her district are planning to attend a four-year college or university. What is the probability that an SRS of size 125 will give a result within 7 percentage points of the true value?

1) What are the Population Parameter(s)?

p = TRUE % OF HS students in this district planning to attend a 4 year college $\boxed{p = .80}$

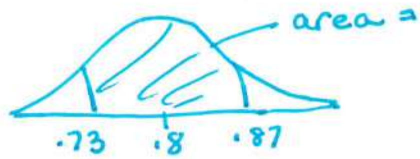
2) What Sample information is given?

SRS n = 125

$\hat{p} = ?$

3) State the probability of interest (in a probability statement). $.73 \le \hat{p} \le .87$

Sketch Graph.



4) What conditions must you check? Have they been met?

① INDEPENDENT – SAMPLING w/o REPLACEMENT

*we must assume there are more than 1,250 H.S. students in her district (10 * 125)

② Normal – $.8(125) = 100 \geq 10$ ✓

$.2(125) = 25 \geq 10$ ✓

5) Find the Mean and Standard Deviation. Clearly show your work.

$\mu_{\hat{p}} = p = .8$

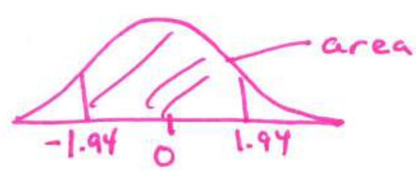
$\sigma_{\hat{p}} = \sqrt{\frac{(.8)(.2)}{125}} = .036$

$\sim N(.8, .036)$

6) Calculate the Z-Scores. Remember to label Z=

$Z_L = \frac{.73 - .80}{.036} = -1.94$ AND $Z_u = \frac{.87 - .80}{.036} = 1.94$

11) Draw the standard normal graph N (0,1).



7) Estimate the probability with the 68-95-99.7 rule.

About 95%

8) Restate the probability statement using the z-scores→ $P(-1.94 \le Z \le 1.94) = \boxed{.9476}$

& find the probability → Use normal cdf

9) Write conclusion (in context)

ABOUT 95% OF ALL SRS's OF SIZE 125 WILL GIVE A SAMPLE PROPORTION WITHIN 7 POINTS OF THE TRUE POPULATION PARAMETER (80%) OF H.S. students in this district who plan to attend a 4-yr college.