

Chapter 5 AP Statistics Practice Test

Section I: Multiple Choice Select the best answer for each question.

- T5.1. Dr. Stats plans to toss a fair coin 10,000 times in the hope that it will lead him to a deeper understanding of the laws of probability. Which of the following statements is true?
- (a) It is unlikely that Dr. Stats will get more than 5000 heads.
 - (b) Whenever Dr. Stats gets a string of 15 tails in a row, it becomes more likely that the next toss will be a head.
 - (c) The fraction of tosses resulting in heads should be close to $1/2$.
 - (d) The chance that the 100th toss will be a head depends somewhat on the results of the first 99 tosses.
 - (e) All of the above statements are true.

PROBABILITY ONLY TELLS US WHAT HAPPENS APPROXIMATELY IN THE LONG RUN, NOT WHAT WILL HAPPEN IN THE SHORT RUN.

T5.2. China has 1.2 billion people. Marketers want to know which international brands they have heard of. A large study showed that 62% of all Chinese adults have heard of Coca-Cola. You want to simulate choosing a Chinese at random and asking if he or she has heard of Coca-Cola. One correct way to assign random digits to simulate the answer is:

- (a) One digit simulates one person's answer; odd means "Yes" and even means "No."
- (b) One digit simulates one person's answer; 0 to 6 mean "Yes" and 7 to 9 mean "No."
- (c) One digit simulates the result; 0 to 9 tells how many in the sample said "Yes."
- (d) Two digits simulate one person's answer; 00 to 61 mean "Yes" and 62 to 99 mean "No."
- (e) Two digits simulate one person's answer; 00 to 62 mean "Yes" and 63 to 99 mean "No."

YOU NEED EXACTLY 62 OF THE 100 2-DIGIT NUMBERS TO REPRESENT THE EVENT "HAVING HEARD OF COKE"

T5.3. Choose an American household at random and record the number of vehicles they own. Here is the probability model if we ignore the few households that own more than 5 cars:

Number of cars:	0	1	2	3	4	5
Probability:	0.09	0.36	0.35	0.13	0.05	0.02

$$P(\text{MORE THAN 2}) = .13 + .05 + .02 = .20$$

20%

A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?

- (a) 7%
- (b) 13%
- (c) 20%
- (d) 45%
- (e) 55%

T5.4. Computer voice recognition software is getting better. Some companies claim that their software correctly recognizes 98% of all words spoken by a trained user. To simulate recognizing a single word when the probability of being correct is 0.98, let two digits simulate one word; 00 to 97 mean "correct." The program recognizes words (or not) independently. To simulate the program's performance on 10 words, use these random digits:

60970 70024 17868 29843 61790 90656 87964 18883

The number of words recognized correctly out of the 10 is

- (a) 10
- (b) 9
- (c) 8
- (d) 7
- (e) 6

9 out of 10 Correct

Questions T5.5 to T5.7 refer to the following setting. One thousand students at a city high school were classified according to both GPA and whether or not they consistently skipped classes. The two-way table below summarizes the data.

Skipped Classes	GPA		
	<2.0	2.0-3.0	>3.0
Many	80	25	5
Few	175	450	265
	255		1000

T5.5. What is the probability that a student has a GPA under 2.0?

- (a) 0.227 (b) 0.255 (c) 0.450 (d) 0.475 (e) 0.506

T5.6. What is the probability that a student has a GPA under 2.0 or has skipped many classes?

- (a) 0.080 (b) 0.281 (c) 0.285 (d) 0.365 (e) 0.727

T5.7. What is the probability that a student has a GPA under 2.0 given that he or she has skipped many classes?

- (a) 0.080 (b) 0.281 (c) 0.285 (d) 0.314 (e) 0.727

T5.8. For events A and B related to the same chance process, which of the following statements is true?

- (a) If A and B are mutually exclusive, then they must be independent.
 (b) If A and B are independent, then they must be mutually exclusive.
 (c) If A and B are not mutually exclusive, then they must be independent.
 (d) If A and B are not independent, then they must be mutually exclusive.
 (e) If A and B are independent, then they cannot be mutually exclusive.

T5.9. Choose an American adult at random. The probability that you choose a woman is 0.52. The probability that the person you choose has never married is 0.25. The probability that you choose a woman who has never married is 0.11. The probability that the person you choose is either a woman or has never been married (or both) is therefore about

- (a) 0.77. (b) 0.66. (c) 0.44. (d) 0.38. (e) 0.13.

T5.10. A deck of playing cards has 52 cards, of which 12 are face cards. If you shuffle the deck well and turn over the top 3 cards, one after the other, what's the probability that all 3 are face cards?

- (a) 0.001 (b) 0.005 (c) 0.010 (d) 0.012 (e) 0.02

$$P(<2.0) = \frac{255}{1000} = 0.255$$

$$P(<2.0 \text{ or Skipped Many Classes}) = \frac{80 + 25 + 5 + 175}{1000} = \frac{285}{1000} = 0.285$$

$$P(\text{GPA} < 2.0 | \text{Skipped Many Classes}) = \frac{80}{110} = 0.727$$

IF A and B are independent, then we don't know whether B has occurred if A occurred. But if A and B are mutually exclusive, then if B has occurred then we know that A couldn't have occurred.

$$\begin{aligned} P(\text{Women}) &= 0.52 \\ P(\text{Never married}) &= 0.25 \\ P(\text{Women and never married}) &= 0.11 \\ P(\text{Women or never married}) &= 0.52 + 0.25 - 0.11 = 0.66 \end{aligned}$$

$$P(1^{\text{st}} \text{ FACE and } 2^{\text{nd}} \text{ FACE and } 3^{\text{rd}} \text{ FACE}) =$$

$$\frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} \approx \frac{1320}{132600} = 0.00995$$

1. The probability of flipping four coins and getting four "heads" is $\frac{1}{16}$.

(a) Interpret this probability.

IF 4 COINS WERE FLIPPED MANY, MANY TIMES, THE PROPORTION OF TIMES ALL 4 COINS WOULD COME UP "HEADS" WOULD BE ABOUT $1/16$.

(b) You flip four coins 32 times. Are you guaranteed to get four "heads" twice? Explain.

NO WHILE WE CAN PREDICT THE PROPORTION OF TIMES WE GET 4 HEADS IN THE LONG RUN, IN THE SHORT RUN THAT PROPORTION IS UNPREDICTABLE.

2. You are playing a board game with some friends in which each turn begins with rolling two dice. In this game, rolling "doubles"—the same number on both dice—is especially beneficial. You've rolled doubles on your last three turns, and one of your friends says, "No way you'll roll doubles this time, it would be nearly impossible." Explain to your friend what he doesn't seem to understand about probability.

SINCE DICE ROLLS ARE INDEPENDENT, PREVIOUS ROLLS HAVE NO IMPACT ON THE PROBABILITY OF THE NEXT ROLL. ONLY IN THE VERY LONG RUN CAN BE CONFIDENT THAT THE PROPORTION OF DOUBLES WILL APPROACH THE EXPECTED VALUE.

3. A school's debate club has 10 members, 6 females and 4 males. If the team decides to pick two members randomly to participate in a debate, what is the probability that both of the chosen members are female? We want to use simulation to estimate this probability. Describe the simulation procedure below, then use the random number table on the next page to carry out 10 trials of your simulation and estimate the probability. Mark on or above each line of the table so that someone can clearly follow your method.

① ASSIGN THE DIGITS 0 TO 5 (OR 1-6) TO FEMALES AND 6 TO 9 (7-9, 0) TO MALES.

② CHOOSE 2 NUMBERS FROM THE RANDOM DIGIT TABLE, IGNORING REPEATS.

③ DETERMINE THE GENDER OF THE 2 CLUB MEMBERS CHOSEN

④ DO THIS 10 TIMES AND CALCULATE THE PROPORTION OF TIMES BOTH ARE FEMALE

F: 0-5 m: 6-9

Random number table for question 3.

141	96767	35964	23822	96012	94591	65194	50842	53372
142	72829	50232	97892	63408	77919	44575	24870	04178
143	88565	42628	17797	49376	61762	16953	88604	12724
144	62964	88145	83083	69453	46109	59505	69680	00900

L → mm
 mm
 mf
 fm
 mf
 ff*
 mf
 fm
 mf
 ff*

Probability (2 Females) = $\left\lfloor \frac{2}{10} \right\rfloor$ or $\boxed{.20}$

1. The table below is a probability model for the number of cars in a randomly-selected household in the United States. (Based on U.S. Census 2000 data).

Number of cars	0	1	2	3	4	5 or more
Probability	0.07	0.19	0.47	.19 ?	0.06	0.02

- (a) What is the probability that a randomly selected household has three cars? (That is, fill in the space marked with a "?") Show your work.

ALL THE PROBABILITIES MUST ADD TO 1:
 $P(3 \text{ cars}) = 1 - .07 - .19 - .47 - .06 - .02 = .19$

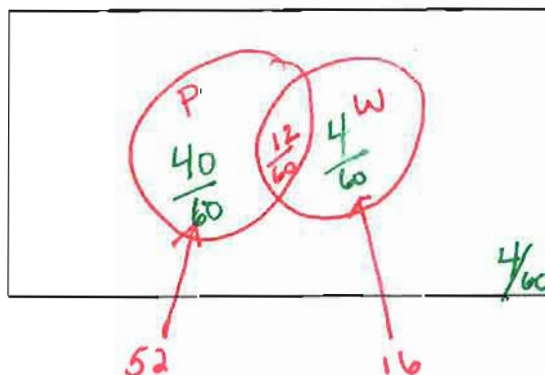
- (b) What is the probability that a randomly-selected household has at least 2 cars? Show your work.

$P(\text{at least 2 cars}) = .47 + .19 + .06 + .02 = .74$

TO GET FULL CREDIT ① write the probability statement
 ② show work.

2. Last Saturday at Pasquale's Pizzas and Wings, 60 customers were served over the course of the evening. Fifty-two customers ordered pizza and 16 ordered buffalo wings. Twelve of these customers ordered both pizza and wings. Suppose we select a customer from last Saturday at random.

- (a) Fill in the Venn diagram below so that it describes the chance process involved here. Let P = the event "ordered pizza" and W = the event "ordered wings."



- (b) What is the probability that a randomly-chosen customer did not order wings or pizza?
Justify your answer with appropriate calculations.

$$\begin{aligned}
 P(\text{Neither Wings or Pizza}) &= 1 - P(\text{Wings or Pizza}) \\
 &= 1 - \left(\frac{52}{60} + \frac{16}{60} - \frac{12}{60} \right) \\
 &= 1 - \frac{56}{60} \\
 &\approx \boxed{.067}
 \end{aligned}$$

3. The table below gives the counts (in thousands) of earned degrees in the United States in a recent year, classified by level and by the gender of the degree recipient.

	Degree (thousands)				Total
	Bachelor's	Master's	Professional	Doctoral	
Female	616	194	30	16	856
Male	529	171	44	26	770
Total	1145	365	74	42	1626

Suppose one degree recipient from this group is selected randomly.

- (a) List two mutually exclusive events for this chance process.

① Female and Male
② Any pair of degree types

- (b) What is the probability that the person selected earned a Master's degree?

$$P(\text{MASTERS DEGREE}) = \frac{365}{1626} \approx \boxed{.224}$$

- (c) What is the probability that the person selected earned a Professional or Doctoral degree?

$$P(\text{Professional or Doctoral}) = \frac{74 + 42}{1626} = \boxed{.071}$$

- (d) What is the probability that the person selected is female or earned a Master's degree?

$$\begin{aligned}
 P(\text{female or Masters}) &= \frac{856}{1626} + \frac{365}{1626} - \frac{194}{1626} \\
 &\approx \boxed{.632}
 \end{aligned}$$

1. What age groups use social networking sites? A recent study produced the following data about 768 individuals who were asked their age and which of three social networking sites they used most often. (People who did not use such sites were excluded from the study).

Web site	Age Group (Years)				Totals
	0 – 24	25 – 44	45 – 64	Over 65	
Facebook	77	105	114	12	308
Twitter	46	110	81	7	244
LinkedIn	15	97	95	9	216
Totals	138	312	290	28	768

Suppose one subject from this study was selected at random.

- (a) Find the probability that the selected subject preferred Twitter.

$$P(\text{Twitter}) = \frac{244}{768} \approx .318$$

- (b) Find the probability that the selected subject preferred Twitter, given that he or she was in the 45 – 64 age group.

$$P(\text{Twitter} | 45-64) = \frac{81}{290} \approx .279$$

- (c) Are the events “preferred Twitter” and “age group 45 – 64” independent? Explain.

NO (from probabilities found in a+b)

$$P(\text{Twitter}) \neq P(\text{Twitter} | 45-64)$$

$$.318 \neq .279$$

- (d) Are the events “preferred Twitter” and “age group 45 – 64” mutually exclusive? Explain.

NO There are 81 individuals that prefer Twitter and are between 45-64.

That is, the occurrence of one event does not

preclude the occurrence of the other. That is $P(\text{Twitter} \cap 45-64) \neq 0$

$$P(\text{Twitter}) = \frac{244}{768}$$

- (e) If a random sample of two subjects were selected, what is the probability that neither preferred Twitter?

$$P(\text{NOT TWITTER} \cap \text{NOT TWITTER}) = \frac{524}{768} \cdot \frac{523}{767} = .465$$

2. Some days, Ramon drives to work. The rest of the time he rides his bike. Suppose we choose a random work day. The following table gives the probabilities of several events.

Event	Probability
Drives to work	0.20
Drives and is late for work	0.05
Late for work, given he bikes	0.30

- (a) Find the probability that Ramon is late for work, given that he drives.

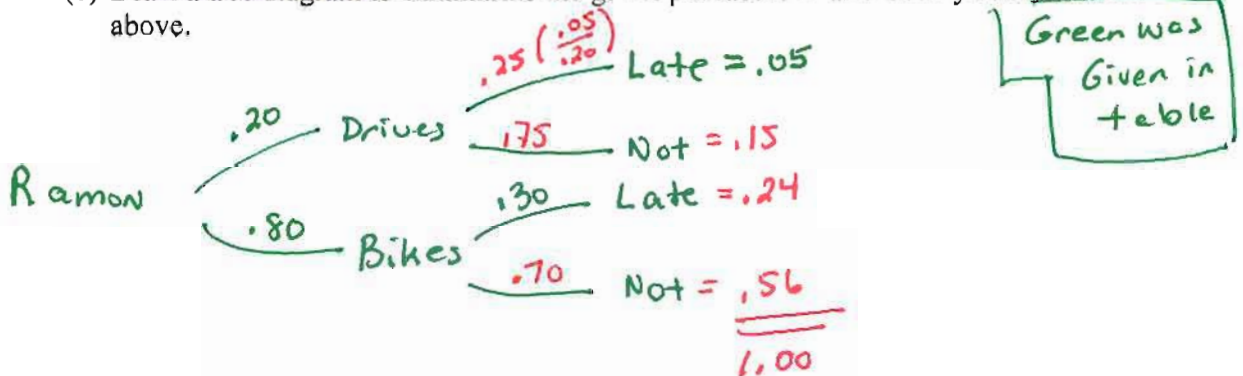
$$P(\text{LATE} | \text{DRIVES}) = \frac{P(\text{LATE and Drives})}{P(\text{Drives})} = \frac{.05}{.20} = .25$$

- (b) Find the probability that Ramon is not late for work, given that he drives.

$$P(\text{NOT LATE} | \text{DRIVES}) = \frac{P(\text{NOT LATE and Drives})}{P(\text{Drives})} = \frac{.15}{.20} = .75 \quad \leftarrow \text{FROM TREE}$$

$$\text{ALSO } P(\text{NL} | \text{D}) = 1 - P(\text{L} | \text{D}) = 1 - .25 = .75$$

- (c) Draw a tree diagram to summarize the given probabilities and those you determined above.



- (d) Find the probability that Ramon drove to work, given that he is late.

$$P(\text{DROVE} | \text{LATE}) = \frac{P(\text{DROVE and Late})}{P(\text{LATE})} = \frac{.05}{.05 + .24} \approx .172$$