CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_{\frac{\pi}{4}}^{x} \cos(2t)\,dt =$

(A) $\cos(2x)$ (B) $\frac{\sin(2x)-1}{2}$ (C) $\cos(2x)-1$ (D) $\sin(2x)$ (E) $\frac{\sin(2x)}{2}$

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2. What are the coordinates of the point of inflection on the graph of $y = x^3 - 15x^2 + 33x + 100$?

(A) (9, 0) (B) (5, –48) (C) (1, 119) (D) (9, –89) (E) (5, 15)

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3. If $3x^2 - 2xy + 3y = 1$, then when $x = 2$, $\frac{dy}{dx} =$

(A) -12 (B) -10 (C) $-\frac{10}{7}$ (D) 12 (E) 32

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4. $\int_1^3 \frac{8}{x^3}\,dx =$

(A) $\frac{32}{9}$ (B) $\frac{40}{9}$ (C) 0 (D) $-\frac{40}{9}$ (E) $-\frac{32}{9}$

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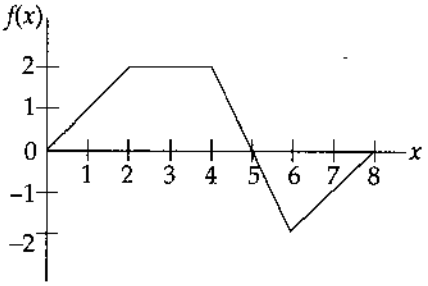
5. 

The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

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6. If f is continuous for $a \leq x \leq b$, then at any point $x = c$, where $a < c < b$, which of the following must be true?

(A) $f(c) = \dfrac{f(b) - f(a)}{b - a}$

(B) $f(a) = f(b)$

(C) $f(c) = 0$

(D) $\displaystyle\int_a^b f(x)\,dx = f(c)$

(E) $\lim_{x \to c} f(x) = f(c)$

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7. If $f(x) = x^2\sqrt{3x+1}$, then $f'(x) =$

(A) $\dfrac{-3x^2 - 2x}{\sqrt{3x+1}}$

(B) $\dfrac{9x^2 + 2x}{\sqrt{3x+1}}$

(C) $\dfrac{-9x^2 + 4x}{2\sqrt{3x+1}}$

(D) $\dfrac{15x^2 + 4x}{2\sqrt{3x+1}}$

(E) $\dfrac{-9x^2 - 4x}{2\sqrt{3x+1}}$

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8. What is the instantaneous rate of change at $t = -1$ of the function f, if $f(t) = \dfrac{t^3 + t}{4t + 1}$?

(A) $\dfrac{12}{9}$ (B) $\dfrac{4}{9}$ (C) $-\dfrac{20}{9}$ (D) $-\dfrac{4}{9}$ (E) $-\dfrac{12}{9}$

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9. $\displaystyle\int_2^{e+1}\left(\frac{4}{x-1}\right)dx =$

(A) 4 (B) $4e$ (C) 0 (D) $-4e$ (E) -4

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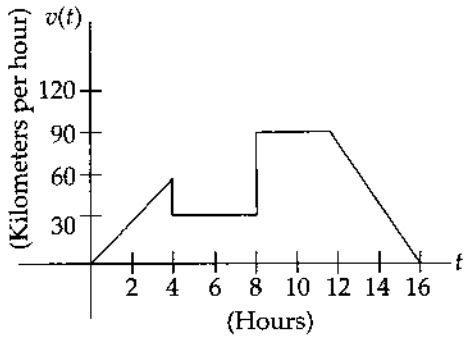
10. 

A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360 (B) 390 (C) 780 (D) 1000 (E) 1360

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11. $\frac{d}{dx}\tan^2(4x) =$

(A) $8\tan(4x)$
(B) $4\sec^4(4x)$
(C) $8\tan(4x)\sec^2(4x)$
(D) $4\tan(4x)\sec^2(4x)$
(E) $8\sec^4(4x)$

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12. What is the equation of the line tangent to the graph of $y = \sin^2 x$ at $x = \frac{\pi}{4}$?

(A) $y - \frac{1}{2} = -\left(x - \frac{\pi}{4}\right)$

(B) $y - \frac{1}{2} = \left(x - \frac{\pi}{4}\right)$

(C) $y - \frac{1}{\sqrt{2}} = \left(x - \frac{\pi}{4}\right)$

(D) $y - \frac{1}{\sqrt{2}} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y - \frac{1}{2} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

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13. If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1; x \le 1 \\ ax^4 - 4bx^2 - 3x; x > 1 \end{cases}$ is differentiable for all real values of x, then $b =$

(A) $-\frac{11}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{7}{16}$ (D) 0 (E) $-\frac{1}{4}$

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14. The graph of $y = x^4 + 8x^3 - 72x^2 + 4$ is concave down for

(A) $-6 < x < 2$

(B) $x > 2$

(C) $x < -6$

(D) $x < -3 - 3\sqrt{5}$ or $x > -3 + 3\sqrt{5}$

(E) $-3 - 3\sqrt{5} < x < -3 + 3\sqrt{5}$

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15. If $f(x) = \dfrac{x^2 + 5x - 24}{x^2 + 10x + 16}$, then $\lim_{x \to -8} f(x)$ is

(A) 0 (B) 1 (C) $-\dfrac{3}{2}$ (D) $\dfrac{11}{6}$ (E) Nonexistent

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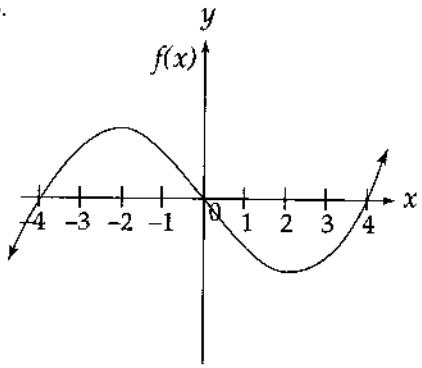
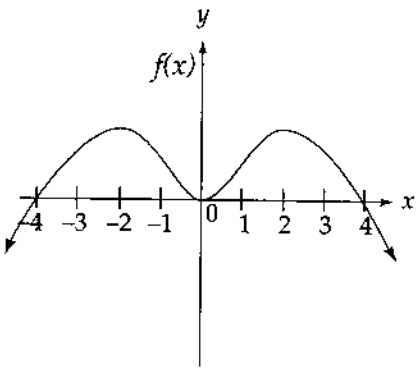
16.

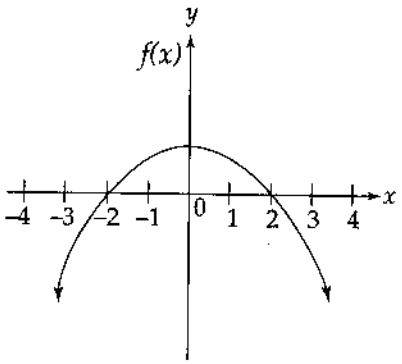


The graph of $f(x)$ is shown in the figure above. Which of the following could be the graph of $f'(x)$?

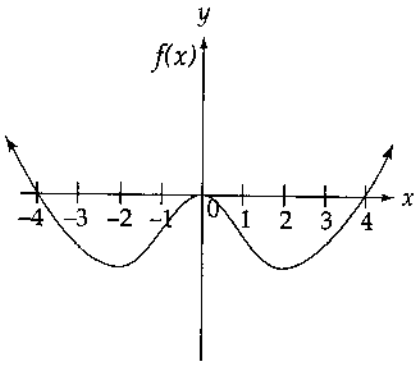
(A)



(B)



(C)



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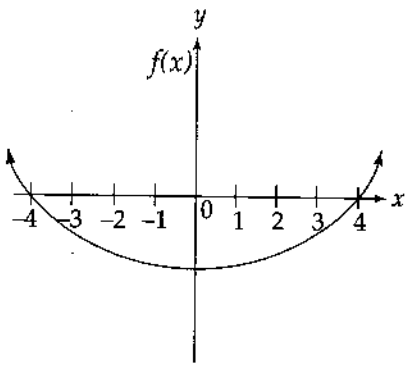
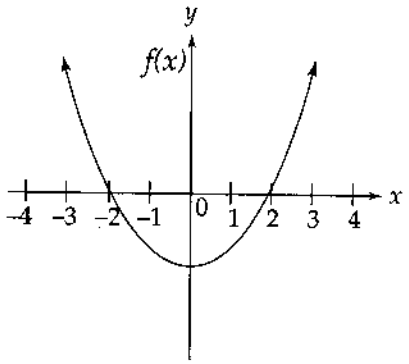


(E)



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17. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

(A) $-3\csc(3x)$
(B) $3\sec(3x)$
(C) $3\tan(3x)$
(D) $-3\tan(3x)$
(E) $-3\cot(3x)$

18. If $f(x) = \int_0^{x+1} \sqrt[3]{t^2 - 1}\, dt$, then $f'(-4) =$

(A) $\sqrt[3]{-9}$ (B) -2 (C) 2 (D) $\sqrt[3]{15}$ (E) 0

19. A particle moves along the x-axis so that its position at time t, in seconds, is given by $x(t) = t^2 - 7t + 6$. For what value(s) of t is the velocity of the particle zero?

(A) 1 (B) 6 (C) 1 or 6 (D) 3.5 (E) 1 or 3.5 or 6

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20. $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin^2 x}dx =$

(A) e (B) $e-1$ (C) $1-e$ (D) $e+1$ (E) 1

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21. The average value of $\sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ is

(A) $\frac{8}{\pi}$

(B) $\frac{12\sqrt{3}-12}{\pi}$

(C) $\frac{12-4\sqrt{3}}{\pi}$

(D) $\frac{6\sqrt{2}-6}{\pi}$

(E) $\frac{6-6\sqrt{2}}{\pi}$

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22. Find the area of the region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$.

(A) 9 (B) 27 (C) 6 (D) –9 (E) –18

23. The function f is given by $f(x) = x^4 + 4x^3$. On which of the following intervals is f decreasing?

(A) (-3, 0) (B) $(0, \infty)$ (C) $(-3, \infty)$ (D) $(-\infty, -3)$ (E) $(-\infty, 0)$

24. $\lim_{x \to 0} \frac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0 (B) $\frac{3}{5}$ (C) 1 (D) $\frac{6}{5}$ (E) Nonexistent

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25. If the region enclosed by the y-axis, the curve $y = 4\sqrt{x}$, and the line $y = 8$ is revolved about the x-axis, the volume of the solid generated is

(A) $\frac{32\pi}{3}$ (B) 128π (C) $\frac{128}{3}$ (D) 128 (E) $\frac{128\pi}{3}$

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26. The maximum velocity attained on the interval $0 \le t \le 5$ by the particle whose displacement is given by $s(t) = 2t^3 - 12t^2 + 16t + 2$ is

(A) 286 (B) 46 (C) 16 (D) 0 (E) –8

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27. The value of c that satisfies the Mean Value Theorem for derivatives on the interval $[0,5]$ for the function $f(x) = x^3 - 6x$ is

(A) $-\frac{5}{\sqrt{3}}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) $\frac{5}{\sqrt{3}}$

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28. If $f(x) = \sec(4x)$, then $f'\left(\frac{\pi}{16}\right)$ is

(A) $4\sqrt{2}$ (B) $\sqrt{2}$ (C) 0 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{4}{\sqrt{2}}$

STOP

END OF PART A SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. If $f(x)$ is the function given by $f(x) = e^{3x} + 1$, at what value of x is the slope of the tangent line to $f(x)$ equal to 2?

(A) −.135 (B) 0 (C) .231 (D) −.366 (E) .693

30. The graph of the function $y = x^3 + 12x^2 + 15x + 3$ has a relative maximum at $x =$

(A) −10.613 (B) −.248 (C) −7.317 (D) −1.138 (E) −.683

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31. The side of a square is increasing at a constant rate of 0.4 cm/sec. In terms of the perimeter, P, what is the rate of change of the area of the square, in cm^2 / sec?

(A) $0.05P$ (B) $0.2P$ (C) $0.4P$ (D) $6.4P$ (E) $51.2P$

32. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1?

(A) 1.099 (B) .086 (C) 0 (D) −.086 (E) −1.099

33. Given f and g are differentiable functions and

$$f(a) = -4,\ g(a) = c,\ g(c) = 10,\ f(c) = 15$$

$$f'(a) = 8,\ g'(a) = b,\ g'(c) = 5,\ f'(c) = 6$$

If $h(x) = f(g(x))$, find $h'(a)$

(A) $6b$ (B) $8b$ (C) $-4b$ (D) 80 (E) $15b$

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34. What is the area of the region in the first quadrant enclosed by the graph of $y = e^{\frac{-x^2}{4}}$ and the line $y = 0.5$?

(A) 0.240 (B) 0.516 (C) 0.480 (D) 1.032 (E) 1.349

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

36. The second derivative of a function f is given by $f''(x) = x \sin x - 2$. How many points of inflection does f have on the interval $(-10,10)$?

(A) Zero (B) Two (C) Four (D) Six (E) Eight

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37. $\lim_{h\to 0} \dfrac{\sin\left(\frac{5\pi}{6}+h\right)-\frac{1}{2}}{h}$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) $-\frac{\sqrt{3}}{2}$

38. $\dfrac{d}{dx}\int_{2x}^{5x} \cos t\, dt =$

(A) $5\cos 5x - 2\cos 2x$

(B) $5\sin 5x - 2\sin 2x$

(C) $\cos 5x - \cos 2x$

(D) $\sin 5x - \sin 2x$

(E) $\frac{1}{5}\cos 5x - \frac{1}{2}\sin 2x$

39. The base of a solid S is the region enclosed by the graph of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is

(A) $\frac{5\pi}{3}$ (B) $\frac{10\pi}{3}$ (C) $\frac{50\pi}{3}$ (D) $\frac{225\pi}{3}$ (E) $\frac{425\pi}{3}$

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40. Which of the following is an equation of the line tangent to the graph of $y = x^3 + x^2$ at $y = 3$?

(A) $y = 33x - 63$

(B) $y = 33x - 135$

(C) $y = 6.488x - 1.175$

(D) $y = 6.488x - 4.620$

(E) $y = 6.488x - 10.620$

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41. If $f'(x) = \ln x - x + 2$, at which of the following values of x does f have a relative minimum value?

(A) 5.146 (B) 3.146 (C) 1.000 (D) 0.159 (E) 0

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42. Find the total area of the region between the curve $y = \cos x$ and the x-axis from $x = 1$ to $x = 2$ radians.

(A) 0 (B) 0.068 (C) 0.249 (D) 1.751 (E) 2.592

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43. Let $f(x) = \int \cot x \, dx$; $0 < x < \pi$. If $f\left(\frac{\pi}{6}\right) = 1$, then $f(1) =$

(A) –1.861 (B) –0.480 (C) 0.134 (D) 0.524 (E) 1.521

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44. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is

(A) –41.589 (B) –0.012 (C) 0.027 (D) 0.693 (E) 98.923

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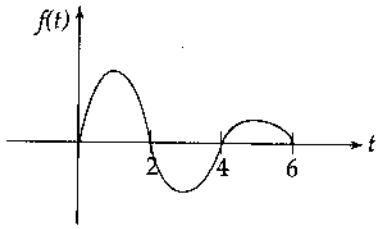
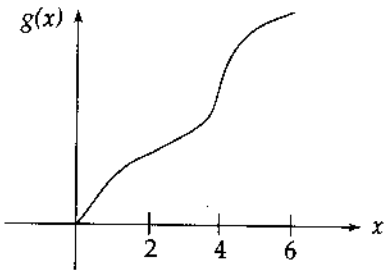
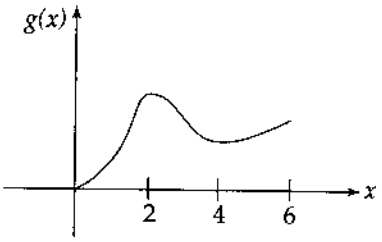
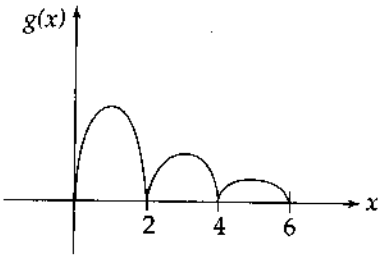
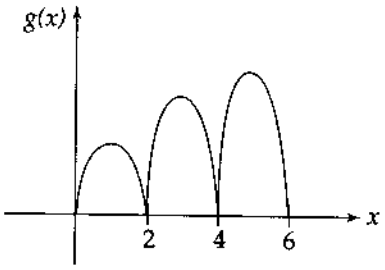
45. 

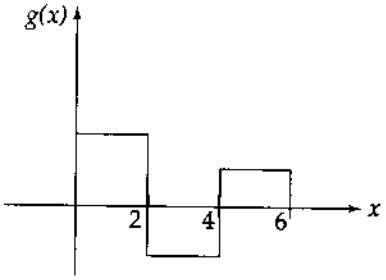
Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ has the graph shown above. Which of the following could be the graph of g ?

(A) 

(B) 

(C) 

(D) 

(E) 

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.