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NEW JERSEY CENTER
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AP Physics 1

2 Dimensional Work and Energy

2016-05-12

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Review of One Dimensional Forces, Work and Energy

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Topics to Review

This chapter assumes that you have already studied Work and Energy in One Dimension - which introduced the concepts of a system, the environment and the forces acting between them - and the definitions of Energy and Work.

But, only the forces acting in one dimension were considered - and life is both more complicated and interesting than that.

We will start with a review of the basic concepts. If you require more than just a review, please refer to the Algebra Based Physics unit on Work and Energy.

Topics to Review

- System and Environment
- Energy
- Work
- Conservation Principles

System and Environment

The concept of a system was introduced in the Algebra Based Physics module, but it needs to be specified in more detail for this course.

A <u>system</u> is a small segment of the universe that will be considered in solving a specific problem, and we will erect a boundary around it. Any force or object outside this boundary will not be considered.

The <u>environment</u> is everything outside the system boundary.

The system can be a particle, a group of particles, an object, an area of space, and its size and shape is totally determined by how you want to solve the problem.

Why are we defining a system and its environment?

System and Environment

So we can make the problem solvable.

By defining an appropriate system, we can isolate the forces that are within the system from the forces that act on the system from the environment.

If the forces are internal to the system, then there is no change in the energy of the system (as long as we don't consider thermal energy - which we won't for now).

If the forces are external, then there will be a change in the energy of the system.

But.....what is energy?

Energy

It turns out that energy is so fundamental, like space and time, that there is no good answer to this question. However, just like space and time, that doesn't stop us from doing very useful calculations with energy.

There are some things we can say about it:

- It is the ability to do work.
- It can be stored.
- It can be changed from one form to another (light to thermal energy, mechanical to thermal energy, gravitational potential energy to kinetic energy).

Did you notice a term in the bullet list above that hasn't been defined yet?

1 Which of the following are characteristics of energy?
Select two answers.

A Thermal energy can never be changed into another form.

B Energy can be changed from one form to another.

C It has the ability to do work.

D Energy cannot be stored; it must always be used.

Answer

2 A system is defined as:

A All of the forces that are external to the boundary between it and the rest of the universe.

B A small segment of the universe that has no internal forces acting on it.

C A small segment of the universe that is chosen to facilitate the solution to a problem. Forces internal to the system can change its total energy.

D A small segment of the universe that is chosen to facilitate the solution to a problem. Forces internal to the system can not change its total energy.

Answer

Work

The previous slide on Energy mentioned Work, which has the ability to change Energy.

What is Work?

It is not what is talked about in common language. It is unfortunate that sometimes Physics uses words that are used everyday - in a quite different fashion. For example - if you're holding up a heavy box, do you think you're doing work?

Work

You're not!

Work, in physics terms, is defined as the exertion of a force over a displacement. And, to further complicate it, only the component of the force in the direction of the motion is counted.

If you're just holding a box, you are certainly exerting an upward force on the box (to keep gravity from pulling it to the ground), but it's not moving, so there is no displacement. Therefore, there is no work.

The equation for this is Work = Force x $\text{Displacement}_{\text{parallel}}$ or

$$W = Fd_{\text{parallel}}$$

Work

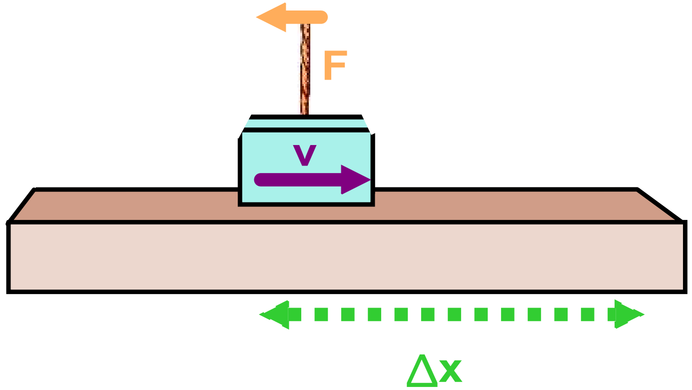
$$W = Fd_{parallel}$$

Three comments about this definition:

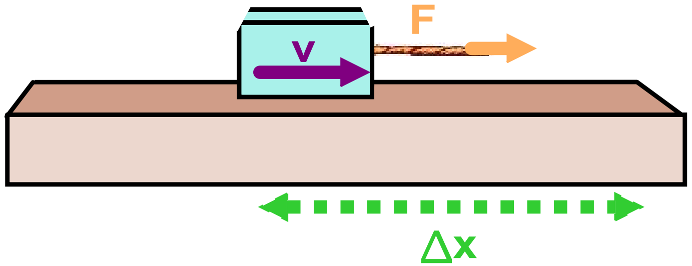
- If the force acts in the same direction as the object's motion, then the work done is positive.
- If the force acts in the opposite direction as the object's motion, then the work done is negative.
- If the object does not move, then zero work is done.

The next slide illustrates these points.

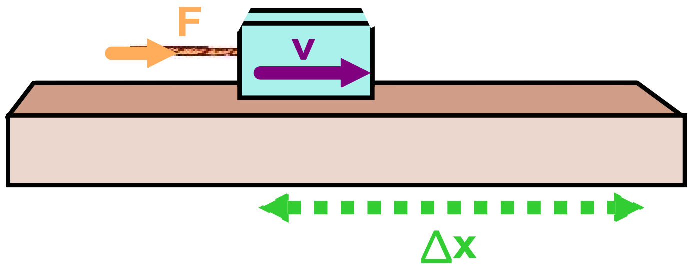
Force and Work



$W = Fd_{parallel} = 0$



$W = Fd_{parallel} = F\Delta x$



$W = Fd_{parallel} = -F\Delta x$

3 In which of the following cases is positive work done by the applied force?

A A softball player catches a ball in her glove.

B A home owner is applying his force to move his lawnmower from rest.

C A driver applies the brakes to slow his car.

D A student holds his textbook in front of him and does not move.

Answer

Work

$$W = Fd_{parallel}$$

You have to very specific about using Work.

The system or environment that the work is acting on needs to be specified. For example:

"An applied force does 12 J of Work on a box."

"Gravity does -5 J of work on a box that is being raised up."

This is not a complete statement. *What's missing?*

"An external force does 6 J of work."

The system or the environment that the work is acting on must be described.

Work

Net non-zero work can only be done to a system by an external force; a force from the environment outside the system.

So if our system is a box sitting on a table and I come along and push the box, I can increase the energy of the box - I am doing net non-zero work on the box.

Why are none of the internal forces (forces within the box, such as the box molecules moving about and colliding with each other) involved in increasing the energy of the system? The molecules are certainly exerting forces on each other, and they are causing each other to move.

Work

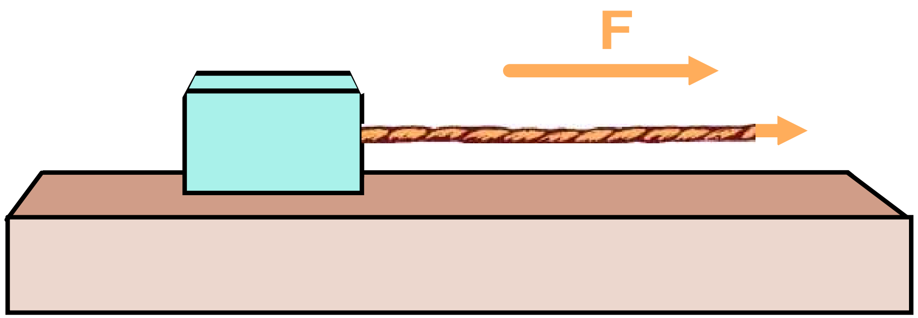
Newton's Third Law!

Every time a molecule in the box strikes another molecule, it exerts a force on it, and moves it. However, the second molecule exerts an equal and opposite force on the first one.

Thus, the work done internal to the system equals zero - it all cancels out.

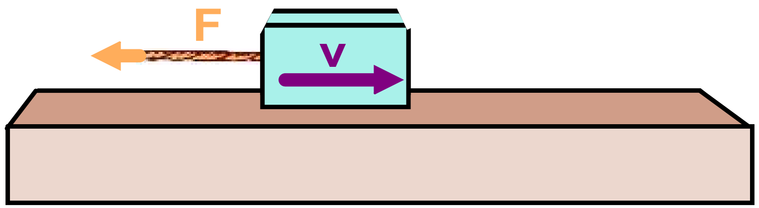
Thermal Energy complicates this picture, but as stated earlier, it will not be considered in this unit.

4 A 36.0 N force is applied to an object that moves 11.0 m in the same direction as the applied force on a frictionless surface. How much work is done on the object by the force?



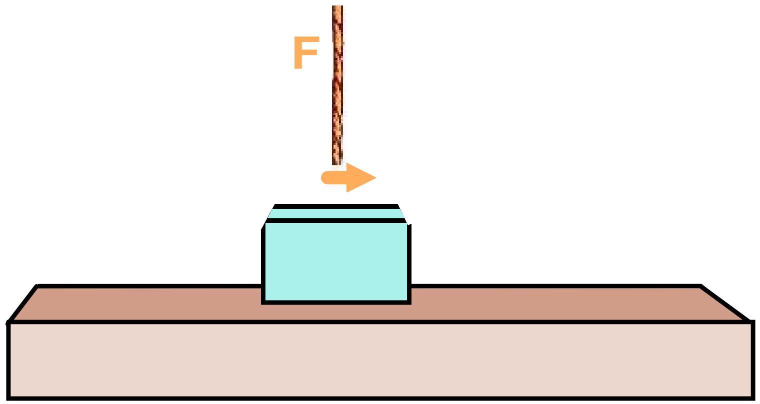
Answer

5 A 36.0 N force is applied to an object that moves 11.0 m in the opposite direction of the applied force on a frictionless surface. How much work is done on the object by the force?



Answer

6 A 36 N force is applied to an object that remains stationary. How much work is done on the object by the applied force?



Answer

7 A 2.0 kg block slides 4.5 m to the right on a frictionless table with a constant velocity of 5.0 m/s. What is the net work on the block?

Answer

8 Which law explains why internal forces to a system do not change its total mechanical energy?

A Newton's First Law

B Newton's Second Law

C Newton's Third Law

D Newton's Law of Universal Gravitation

Answer

9 A book is held at a height of 2.0 m for 20 s by a librarian. How much work is done on the book by the librarian?

A 400 J

B 200 J

C 20 J

D 0 J

Answer

10 A book of mass, m, is lifted upwards at a constant velocity, a displacement, h, by a librarian. How much work is done by the librarian on the book?

A -mg

B -mgh

C mgh

D 0

Answer

11 A book of mass, m, is lifted upwards at a constant velocity, a displacement, h, by a librarian. How much work is done by the gravitational force on the book?

A -mg

B -mgh

C mgh

D 0

Answer

12 A book of mass, m, is lifted upwards at a constant velocity, a displacement, h, by a librarian. How much net work is done by the gravitational force and the librarian on the book?

A -mg

B -mgh

C mgh

D 0

Answer

Putting it all together

Now, let's relate the four concepts of system, environment, work and energy, in terms of Conservation Principles.

The most powerful concepts in physics are called Conservation Principles. These principles allow us to solve problems without worrying too much about the details of a process.

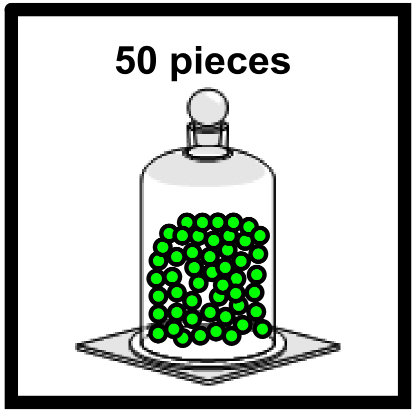
We just have to take a snapshot of a system initially, and then after various forces have acted upon the system, we take another snapshot.

By comparing those two snapshots, we can learn a lot.

Conservation Principles

A good example is a jar of candy.

First, define the system, the system boundary and the environment.

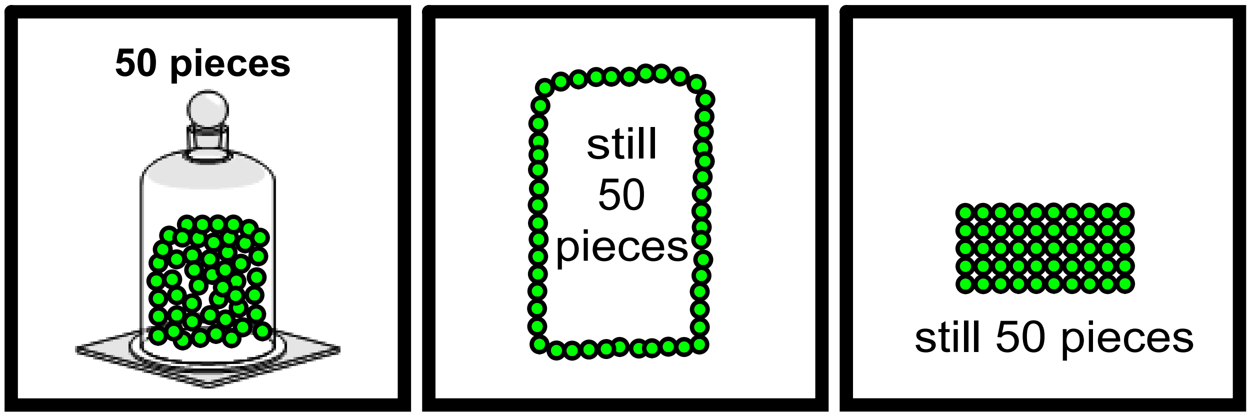


The system includes the candy pieces inside the jar. The jar is the system boundary and everything outside the jar is the environment.

Conservation Principles

If you know that there are fifty pieces of candy at the beginning and that that no pieces have been taken out or added (crossed the system boundary), then you know that there must be 50 pieces at the end.

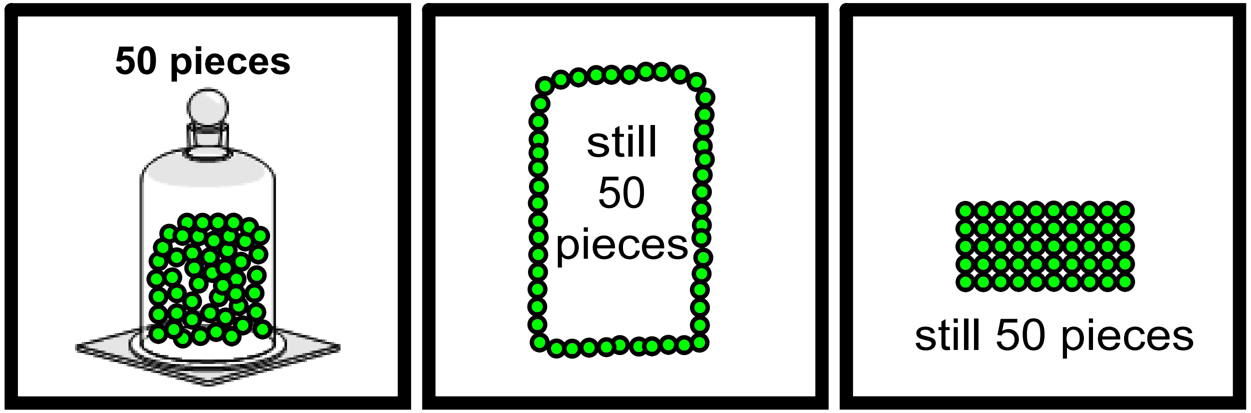
Now, you could change the way you arrange them or you could move them around, but you will still have 50 pieces.



Conservation Principles

In that case we would say that the number of pieces of candy is conserved.

That is, we should always get the same amount, regardless of how they are arranged as long as we take into account whether any have crossed the system boundary.



Conservation of Energy

Energy is a conserved property of nature. It is not created or destroyed, so in a system where nothing crosses the system boundary, we will always have the same amount of energy. This is called a closed system.

The only way the energy of a system can change is if it is open to the environment - if energy has been added or taken away.

Because of this conservation principle, we can do very useful calculations with energy.

And, unlike force, energy is a scalar. It can be positive or negative, but we don't have to worry about vectors.

Conservation of Energy

Define the amount of energy that we start with as E_o and the amount we end up with as E_f; then if no energy is added to or taken away from a system, we have the Law of the Conservation of Energy.

$$E_o = E_f$$

It turns out that there are only *three* ways to change the energy of a system. One is with heat, which won't be dealt with here; the other is with work.

What is the third one?

Conservation of Energy

As shown by Einstein's mass energy equivalence formula, Energy can be changed into mass and vice versa; $E = mc^2$.

This is evidenced in nuclear reactions (both controlled nuclear reactors and nuclear weapons), and at the quantum level, where particles may "pop" in and out of existence as long as they don't violate the Heisenberg Uncertainty Principle.

But, this is Classical Physics - and we won't consider those cases now.

Conservation of Energy

If energy is added to or taken away from a system by work, then we have the "Work-Energy" equation.

$$E_o + W = E_f$$

If positive work is done on a system, then the energy of the system increases. If negative work is done on a system, then the energy of the system decreases.

This equation also shows that the units of work and energy are the same.

Conservation of Energy

Motion problems can certainly be solved through the application of Newton's Laws to free body diagrams.

However - for non constant forces or objects moving in two dimensions, the calculations can be complex.

By using the Conservation of Energy, wherever possible, it doesn't matter what the forces are doing, or how the objects are moving - you just need to consider the original and final states.

This, along with the fact that Energy is a scalar, makes the solutions much easier.

13 How much force must be applied to an object such that it gains 100.0 J over a displacement of 20.0 m?

Answer

14 Over what displacement must a 400.0 N force be applied to an object such that it gains 1600 J of energy?

Answer

15 A vacuum cleaner is moved from the ground floor to the second floor of an apartment building. In which of the following cases is the most work done by the person moving the vacuum?

A The vacuum cleaner is pushed up an incline set over the stairs.

B The person carries the cleaner up the stairs in his arms

C The cleaner brings the cleaner to the third floor, by mistake, then back to the second floor.

D The work is the same in each case.

Answer

16 When using the Conservation of Energy to solve a system problem, what needs to be considered? **Select two answers.**

A The initial energy of the system.

B The final energy of the system.

C The direction of the internal applied forces.

D The magnitude of the internal applied forces.

Answer

Work Related to Energy Change

$$W = Fd_{parallel}$$

If the object that is experiencing the force does not move (if $d_{parallel} = 0$) then no work is done: $W = 0$. The energy of the system is unchanged.

If the object moves in the direction opposite the direction of the force (for instance if F is positive and $d_{parallel}$ is negative) then the work is negative: $W < 0$. The energy of the system is reduced.

If the object moves in the same direction as the direction of the force (for instance if F is positive and $d_{parallel}$ is also positive) then the work is positive: $W > 0$. The energy of the system is increased.

Units of Work and Energy

$$W = Fd_{parallel}$$

This equation gives us the units of work. Since force is measured in Newtons (N) and distance is measured in meters (m) the unit of work is the Newton-meter (N-m).

And since N = kg-m/s^2; a N-m also equals a kg-m^2/s^2.

In honor of James Joule, who made critical contributions in developing the idea of energy, the unit of work and energy is also known as the Joule (J).

1 Joule = 1 Newton-meter = 1 kilogram-meter^2/second^2

$$1 \text{ J} = 1 \text{ N-m} = 1 \text{ kg-m}^2/\text{s}^2$$

James Prescott Joule

Joule was instrumental in showing that different forms of energy can be converted into other forms - most notably mechanical to thermal energy.

Before Joule, it was commonly accepted that thermal energy is conserved. This was disproved by Joule's extremely accurate and precise measurements showing how thermal energy is just another form of energy.

This was made possible by his experience as a brewer which relied on very accurate measurements of temperature, time and volume!

17 Which are valid units for Work? **Select two answers.**

A Joule

B Watt

C Newton

D Newton-meter

Answer

18 An athlete is holding a football. He throws it to a teammate who catches it. Describe the work done on the football by both players, starting from when the football is at rest before it is thrown, and when it is at rest after it is caught.

Answer

Two
Dimensional Forces
and Work

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Two Dimensional Forces and Work

In the previous section, we learned that the amount of work done to a system, and therefore the amount of energy increase that the system experiences, is given by:

$$\text{Work} = \text{Force} \times \text{Displacement}_{parallel}$$

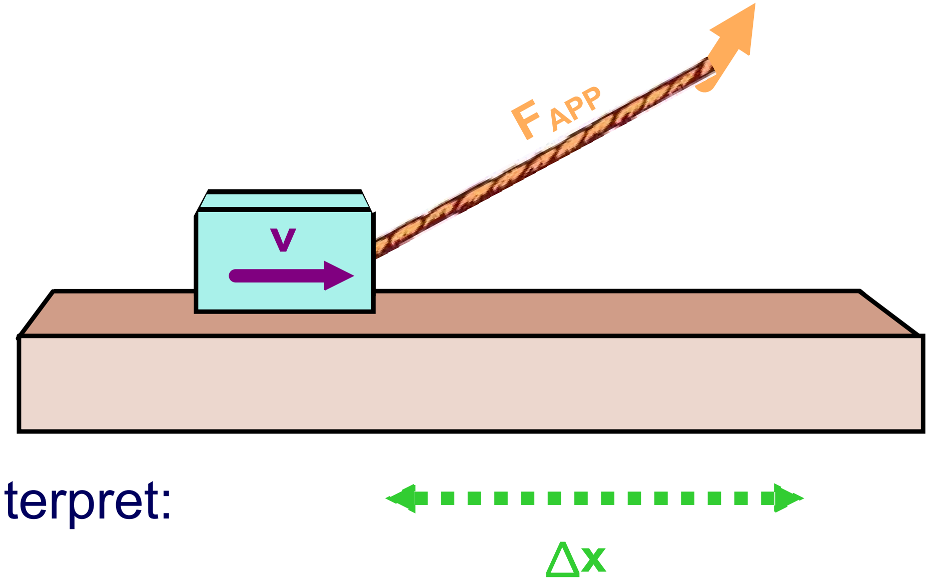
or

$$W = Fd_{parallel}$$

This is still valid, but we have to bring a new interpretation to that equation based on what we know about vector components.

Two Dimensional Forces and Work

Instead of pulling the object horizontally, what if it is pulled at an angle to the horizontal?



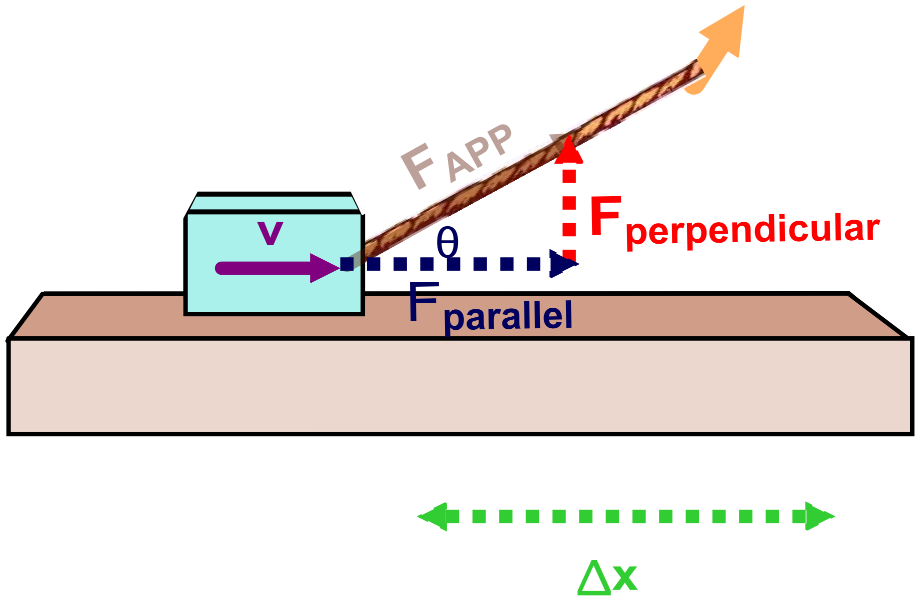
How would we interpret:

$W = Fd_{parallel}$

for this case?

Two Dimensional Forces and Work

After breaking F_{APP} into components that are parallel and perpendicular to the direction of motion, we can see that no work is done by the perpendicular component; work is only done by the parallel component.

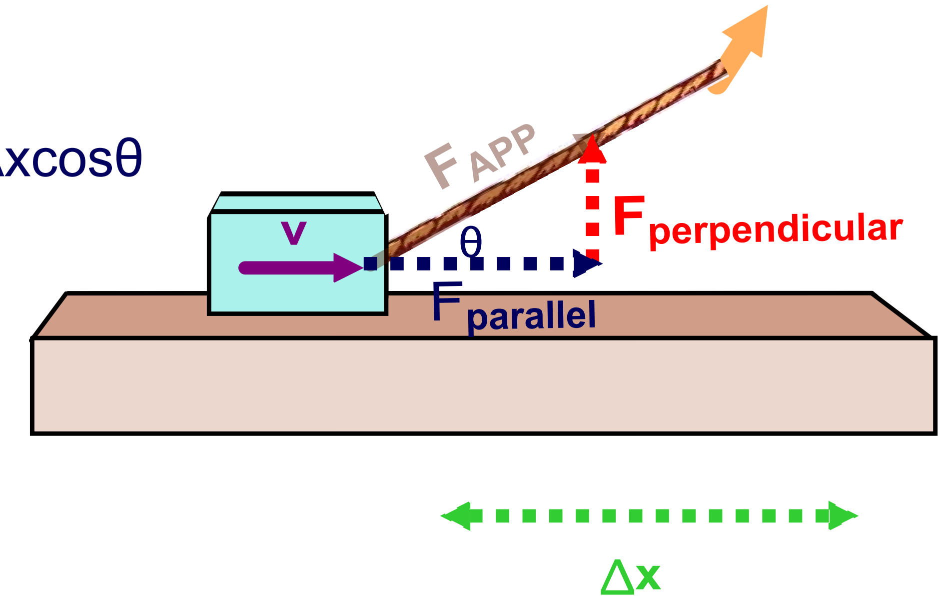


Using trigonometry, we find that
$F_{parallel} = F_{APP}\cos\theta$

Two Dimensional Forces and Work

$W = F_{parallel}d$ becomes:

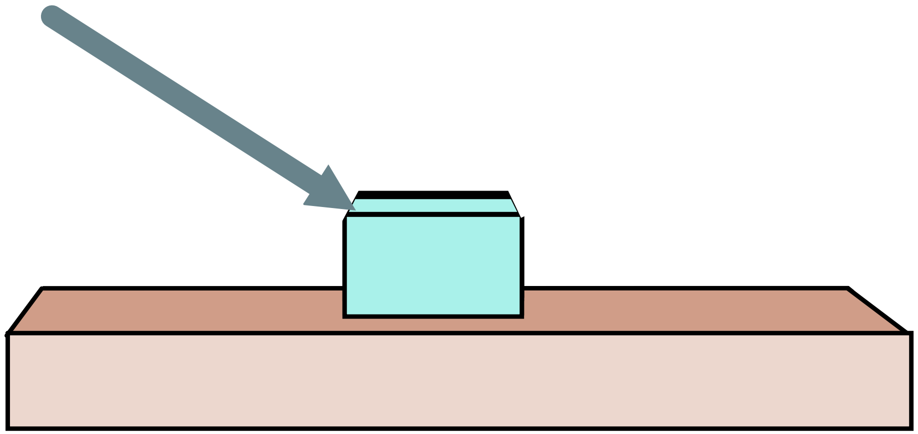
$W = (F_{APP}\cos\theta)\Delta x = F_{APP}\Delta x\cos\theta$



In words, the work done on an object by a force is the product of the magnitude of the force and the magnitude of the displacement times the cosine of the angle between them.

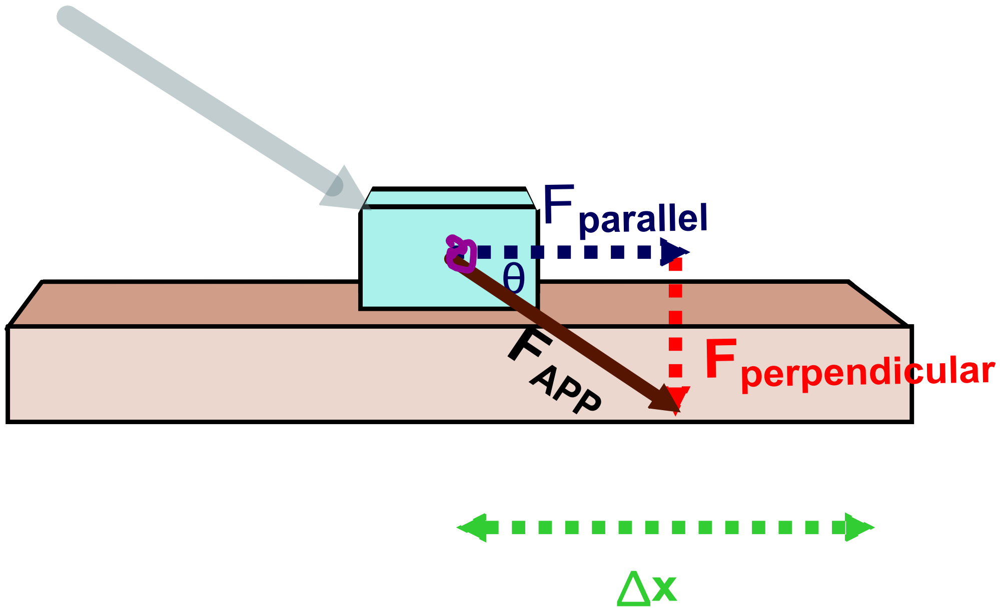
Two Dimensional Forces and Work

Instead of pulling the object at an angle to the horizontal, what if it is pushed?



This is really no more difficult a case. We just have to find the component of force that is parallel to the object's displacement.

Two Dimensional Forces and Work

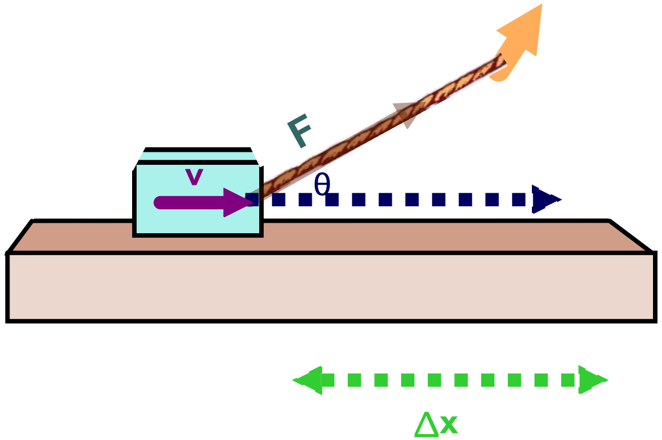


The interpretation is the same, just determine the angle between the force and displacement and use:

$W = F_{APP}\Delta x\cos\theta$

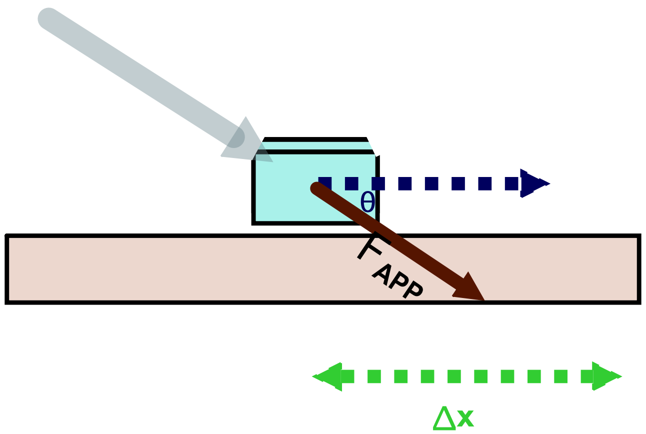
Even though $F_{perpendicular}$ is in the negative direction (it was positive when the object was pulled), it does not affect the work - as only the parallel component contributes to the work.

19 A 40.0 N force pulls an object at an angle of $\theta = 37.0^0$ to its direction of motion. Its displacement is $\Delta x = 8.00$ m. How much work is done by the force on the object?



Answer

20 An object is pushed with an applied force of 36.0 N at an angle of $\theta = 60.0^0$ to the horizontal and it moves $\Delta x = 3.40$ m. What work does the force do on the object?



Answer

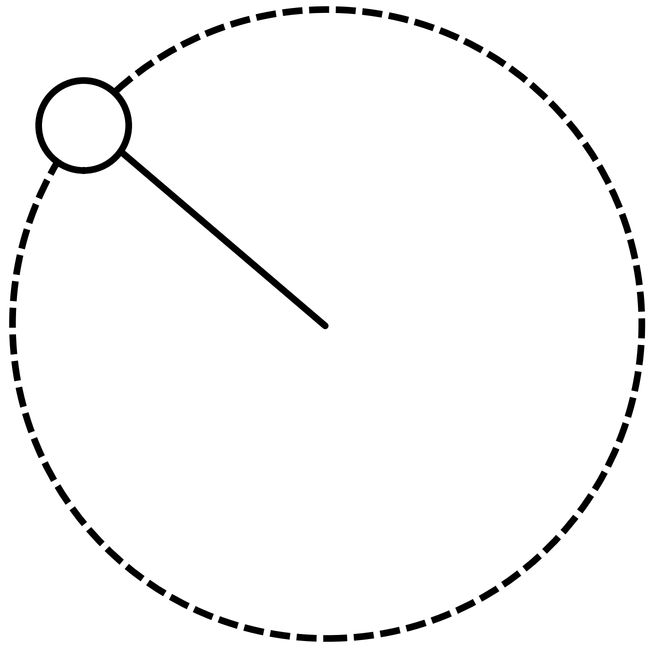
21 A ball is swung around on a string, covering a displacement of 2.0 m in 5.0 s. What is the work done on the ball by the string?

A 0 J

B 2.5 J

C 5.0 J

D 10 J



22 Assume the earth moves around the sun in a perfect circular orbit (a good approximation). Use the direction of the gravitational force between the two celestial objects and describe the work done by the sun on the earth and how that impacts the earth's orbital speed. Remember to consider the direction of the gravitational force and the direction that the earth is moving.

How does your answer change if you don't make the circular orbit assumption?

Answer

Gravitational Potential Energy

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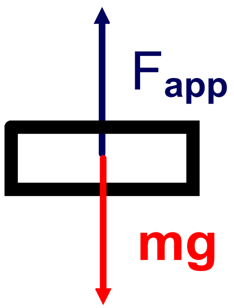
Gravitational Potential Energy

We'll start with a quick review of Gravitational Potential Energy as presented in the Algebra Based Physics course. Then, the impact of motion in two dimensions will be analyzed.



A book of mass, m, is lifted vertically upwards a displacement, h, by an external force (a person) at a constant velocity. How much work does the external force do on the book?

Gravitational Potential Energy

Apply Newtons' Second Law to the FBD:

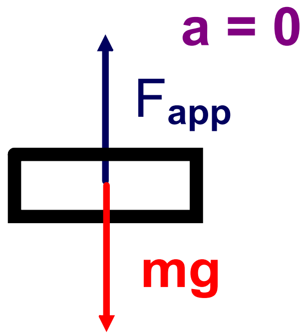
$\Sigma F_y = F_{app} - mg = ma_y = 0$ (since v = const)

$F_{app} = mg$

Use the definition of work: $W = F\Delta x_{parallel}$

$W = (mg)\Delta x_{parallel} = mg\Delta x$ (since F_{app} is parallel to Δx)

$W = mgh$ (since $\Delta x = h$)



Gravitational Potential Energy

Apply the Work-Energy equation, $E_o + W = E_f$. If the book started and finished at rest ($KE_0 = KE_f = 0$), then $E_o = 0$. The equation becomes:

$W = E_f$

But we just showed that the external force did W = mgh to lift the book... so

$mgh = E_f$ or $E_f = mgh$

We conclude that the energy of a mass is increased by an amount mgh when it is raised by a displacement, h.

Gravitational Potential Energy

The name for this form of energy is Gravitational Potential Energy (GPE).

$$GPE = mgh$$

One important thing to note is that changes in gravitational potential energy are important, but *their absolute value is not*. This is because h represents Δx, or the change in height.

You can define any point to be zero meters in height when it represents the original position of an object. This would give you zero for GPE at that point. But no matter what height you choose to call zero, changes in heights will still result in changes of GPE.

Gravitational Potential Energy

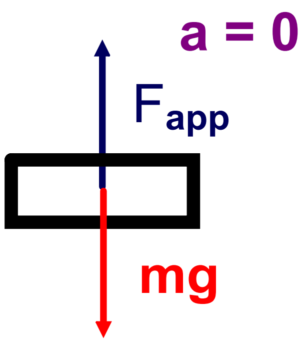
One more thing.

The potential energy of a system is conventionally defined as the negative of the work done by the field of the system (gravitational or electric).

How is this related to the derivation just performed to show that GPE = mgh?

Our calculation used the work done by an external force (a person). The conventional definition uses the work done by the force exerted by the gravitational field, F_{grav}. F_{grav} does negative work on a rising book as its force is opposite the motion. By taking the negative of this work, you get the same result - a positive GPE.

Gravitational Potential Energy



The book is moving up at a constant velocity and travels a displacement h. Note that the force exerted by gravity is in the opposite direction of its motion (anti-parallel).

So, $W_{grav} = F_{grav}\Delta x_{parallel}$

$W_{grav} = -(mg)(\Delta x) = -mgh$

$GPE = -W_{grav} = mgh$

Which, of course, gives us the same result for GPE.

23 What is the change of GPE for a 5.0 kg object which is raised from an initial height of 1.0 m above the floor to a final height of 8.0 m above the floor?

Answer

24 What is the change of GPE for an 8.0 kg object which is lowered from an initial height of 2.1 m above the floor to a final height of 1.5 m above the floor?

Answer

25 What is the change in height of a 2.0 kg object which gained 16 J of GPE?

Answer

26 A librarian takes a book off of a high shelf and refiles it on a lower shelf. Which of the following explain the work done on the book by the librarian and the earth's gravitational field as its GPE is lowered? **Select two answers.**

A The librarian does positive work on the book.

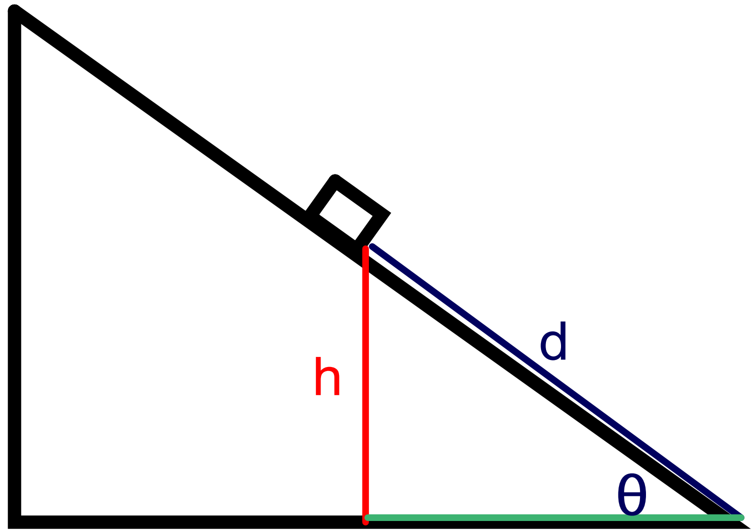
B The librarian does negative work on the book.

C The gravitational field does positive work on the book.

D The gravitational field does negative work on the book.

Answer

GPE on an Incline



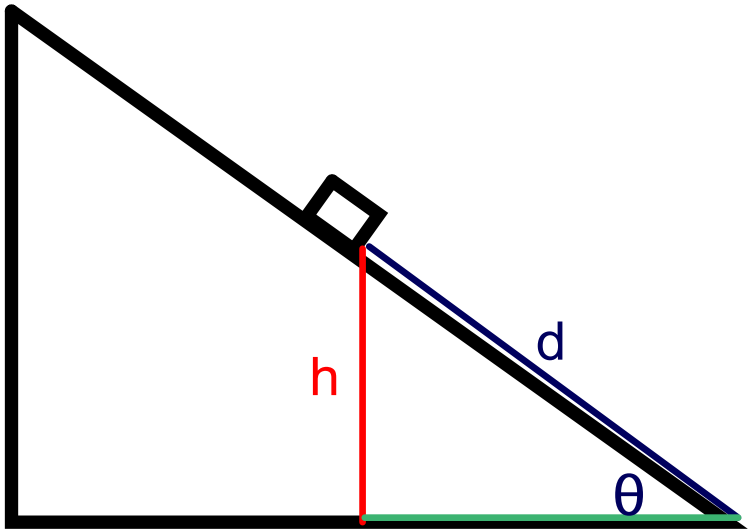
Let's put together the concepts of two dimensional motion and forces with GPE.

We'll use a box being pushed up an incline.

It all depends on what we can measure. Assume it's easier to measure the displacement (d) the box travels.

How do we find its change in GPE?

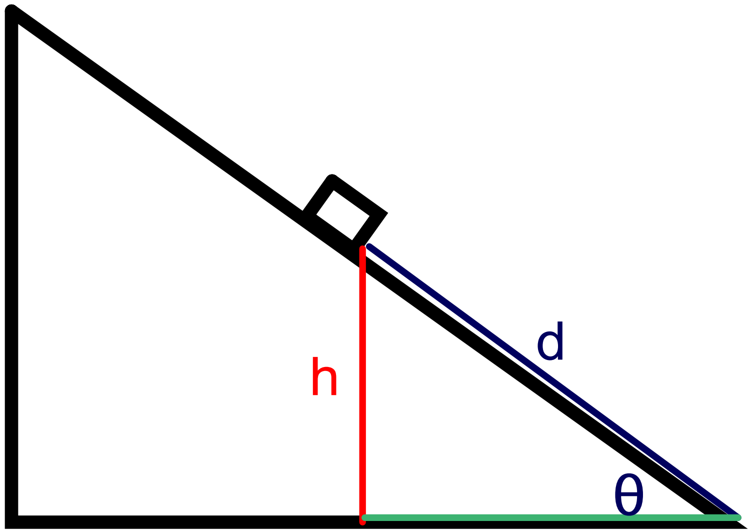
GPE on an Incline



The box starts with no velocity, and after it is pushed up a displacement d, the block slides up, and it momentarily stops before sliding back down.

Does $\Delta GPE = mgd$?

GPE on an Incline

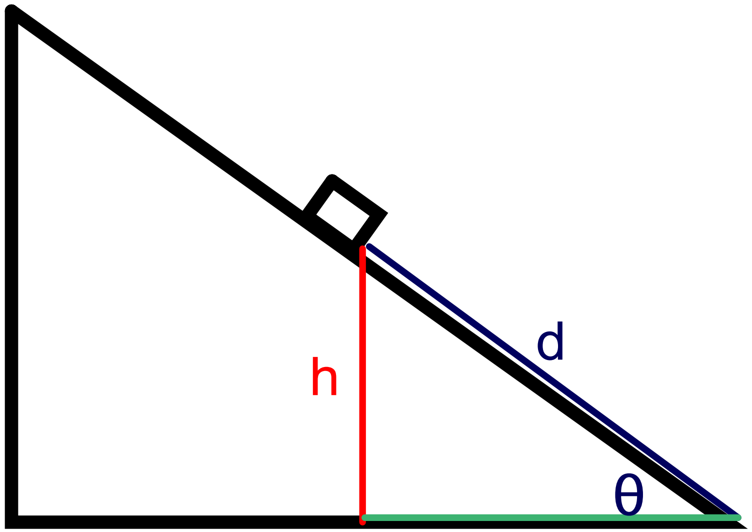


No! The formula for ΔGPE was calculated from the work formula, and it assumed the gravitational force (or the force that opposed it to lift the object) was in the same direction of the object's motion.

The gravitational force points down. Since work only includes the distance and force components that are in parallel, ΔGPE involves h and not d in the picture.

How is h calculated from d?

GPE on an Incline



$\Delta GPE = mgh$

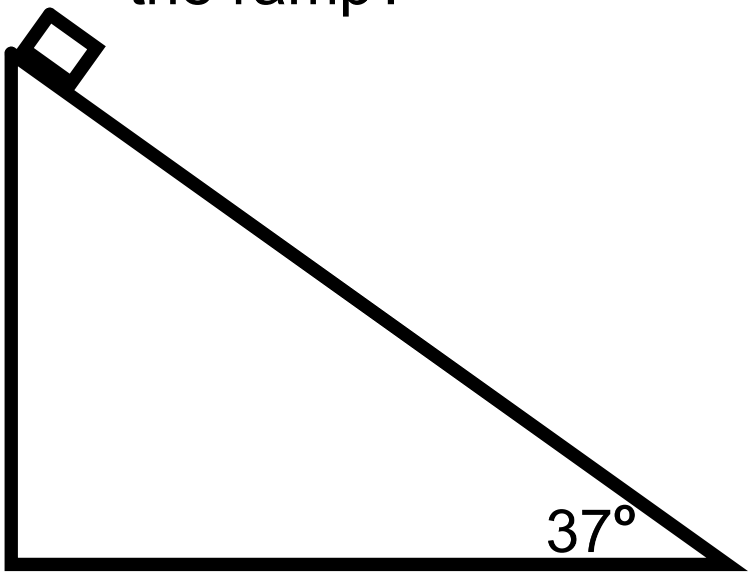
When motion is along an incline, the change in height can be related to the distance traveled using trigonometry.

$\sin\theta = h/d$

$h = d\sin\theta$

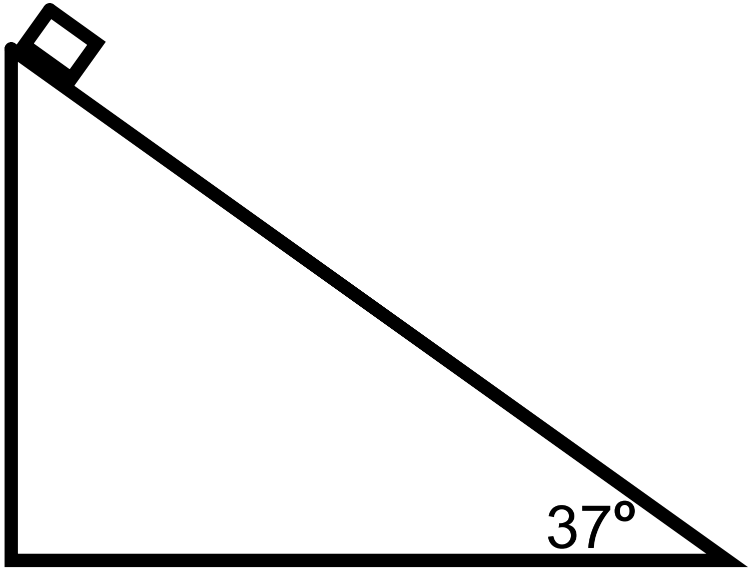
$\Delta GPE = mgd\sin\theta$

27 A 5.0 kg block is at the top of a 6.0 m long frictionless ramp, which is at an angle of 37^0. What is the height of the ramp?



Answer

28 The 5.00 kg block slides to the bottom of the 6.00 m long frictionless ramp, which is at an angle of 37.0°. What is the change in its GPE?



Answer

GPE, Kinetic Energy
and
Elastic Potential Energy

Lab

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KE and EPE

In the Algebra Based Physics course, we used the third kinematics equation, along with the definition of GPE to derive an expression for Kinetic Energy - the energy of an object in motion.

$$KE = \frac{1}{2}mv^2$$

Also, another type of potential energy was derived based on Robert Hooke's Law about the relationship between the force necessary to compress a spring and how much it was compressed. It is called Elastic Potential Energy (EPE).

$$EPE = \frac{1}{2}kx^2$$

And since KE and EPE are forms of energy, they are both expressed in Joules.

GPE, KE and EPE

We can now solve problems with GPE, KE and EPE in two dimensions using what has been covered so far in this chapter.

The key is understanding how these different forms of energy depend on their components along the x and y axis.

Let's look at GPE first. If an object is moving along an incline, does its potential energy depend on the displacement traveled along the incline?

GPE, KE and EPE

No, it does not!

ΔGPE = mgh, where h is the vertical displacement (purely along the y axis) that the object has moved.

The incline displacement is not important - only the vertical displacement.

What about KE?

GPE, KE and EPE

As with all energy, KE is a scalar. However, it relates directly to the velocity, and velocity is a vector.

When we perform calculations of velocity from KE and GPE, we need to be careful to relate the change in GPE only to the change in KE in the y direction - thus it only affects the velocity in the y direction.

Now, what about EPE?

GPE, KE and EPE

In the case of EPE, the amount that the spring is compressed is the important variable - no trigonometry is required.

Kinetic Energy will either use the vertical displacement an object covers (for its relationship to GPE) or the actual displacement of the object from the spring's force (for its relationship to EPE).

Let's work a couple of problems by using the Conservation of Energy to make this more clear.

29 A projectile is fired at an angle of 45^0. Which factor is required to calculate the maximum height the projectile reaches by using the Conservation of Energy?

A The total initial velocity of the projectile.

B The projectile's initial velocity in the x direction.

C The projectile's initial velocity in the y direction.

D The horizontal distance traveled by the projectile.

Answer

30 A spring launcher fires a marble at an angle of 52^0. You want to calculate the energy stored in the compressed spring to find the initial energy of the system. What value of x is used in the equation, $EPE = 1/2\ kx^2$?

A The horizontal displacement of the compressed spring.

B The vertical displacement of the compressed spring.

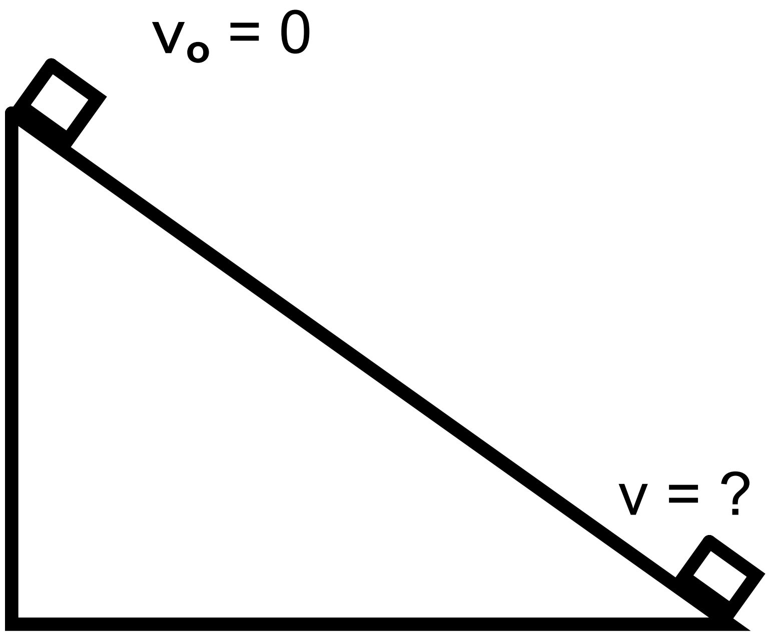
C The straight line displacement of the spring after it is released.

D The straight line displacement of the spring when it is fully compressed.

Answer

Energy Problem Solving

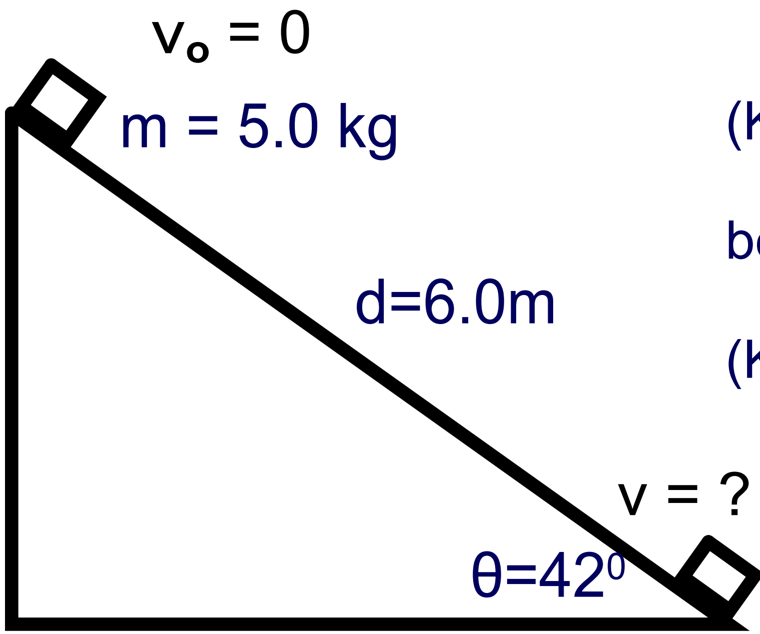
What is the final velocity of a box of mass 5.0 kg that slides 6.0 m down a frictionless incline at an angle of 42^0 to the horizontal?



The system will be the block. Since there is no friction, there are no external forces and we can use the Conservation of Energy (the gravitational force is taken care of by GPE). What types of energy are involved here?

Energy Problem Solving

Only three types of energy have been discussed so far. And in this case, there is only GPE and KE:



$(KE + EPE + GPE)_0 = (KE + EPE + GPE)$

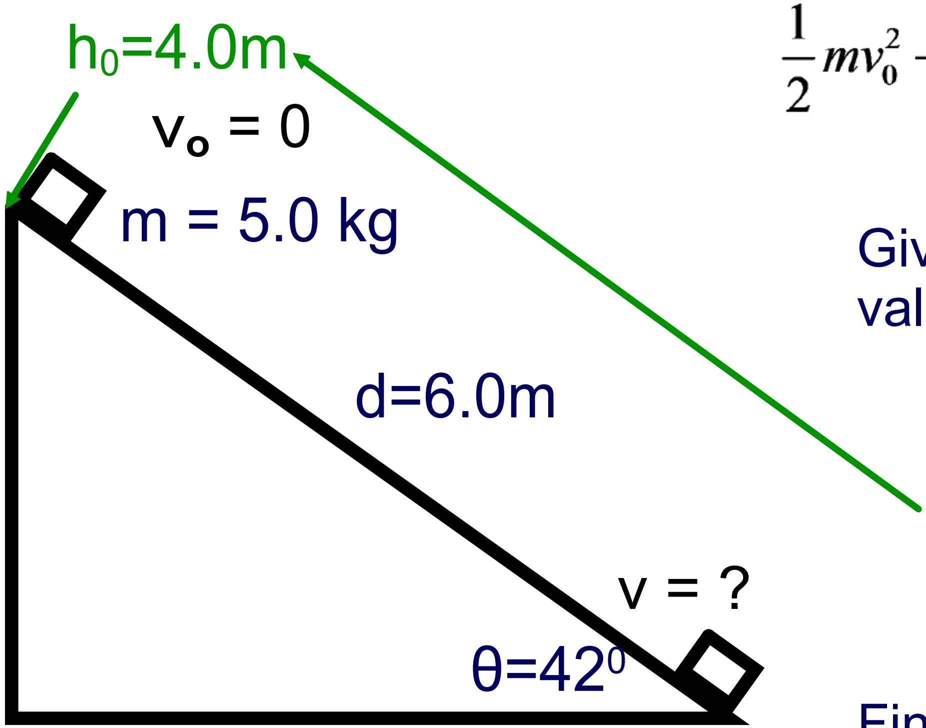
becomes:

$(KE + GPE)_0 = (KE + GPE)$

to save subscripts, we'll assume that no subscript implies a final quantity ($KE = KE_f$)

Energy Problem Solving

Let's put in the equations now:



$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv^2 + mgh$$

Given (and using trig to find the value of h_0 from d):

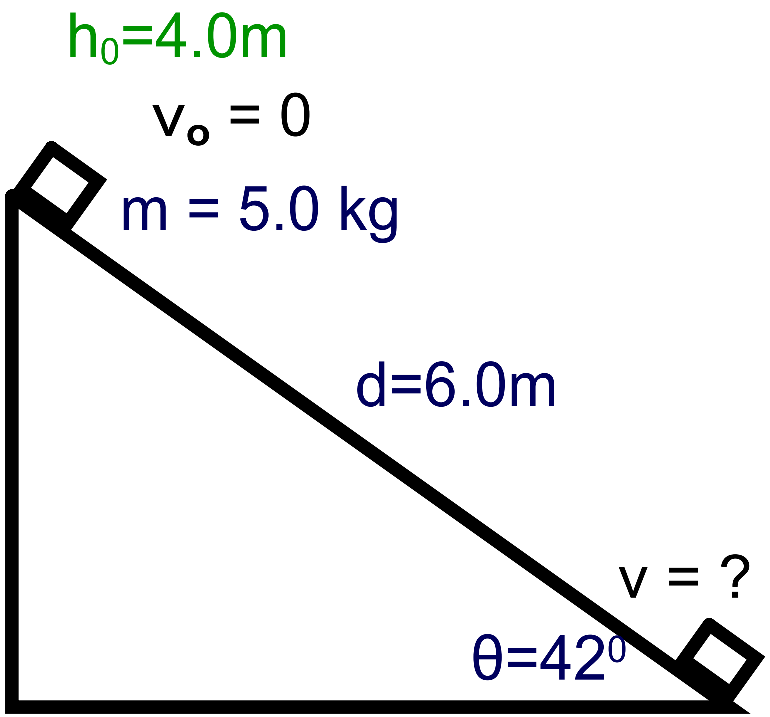
$$v_0 = 0\,m/s$$

$$h_0 = d\sin\theta = (6.0m)\sin 42^0 = 4.0m$$

$$h = 0m$$

Find: v

Energy Problem Solving



$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv^2 + mgh$$

$$0 + mgh_0 = \frac{1}{2}mv^2 + 0$$

$$gh_0 = \frac{1}{2}v^2$$

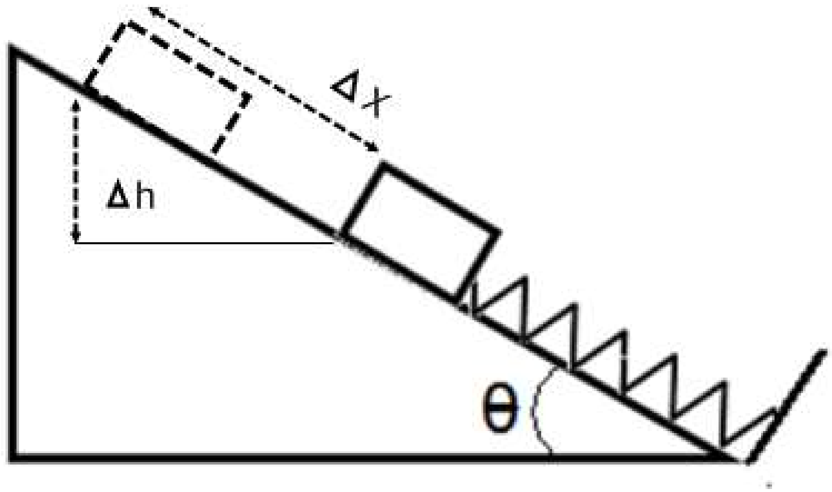
$$v^2 = 2gh_0$$

$$v = \sqrt{2gh_0}$$

$$v = \sqrt{2(9.8\,m/s^2)(4.0m)} = 8.9\,m/s$$

The velocity at the bottom of the incline is 8.9 m/s.

Energy Problem Solving

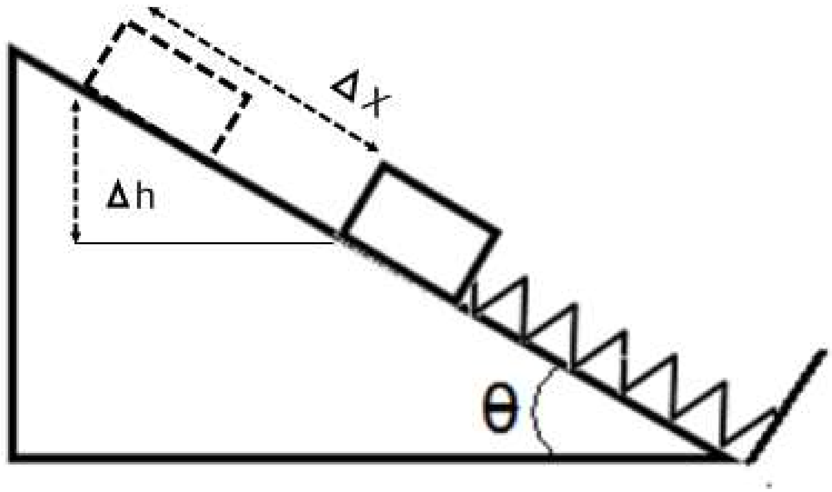


Consider the inclined plane problem that was just worked, but add a spring at the bottom of the incline.

The spring will be compressed a distance Δx and then released. Find the velocity of the box when it rises back to where it was first compressed - a height of Δh.

What energies do we have to consider?

Energy Problem Solving



Once compressed, the box has EPE, GPE and zero KE.

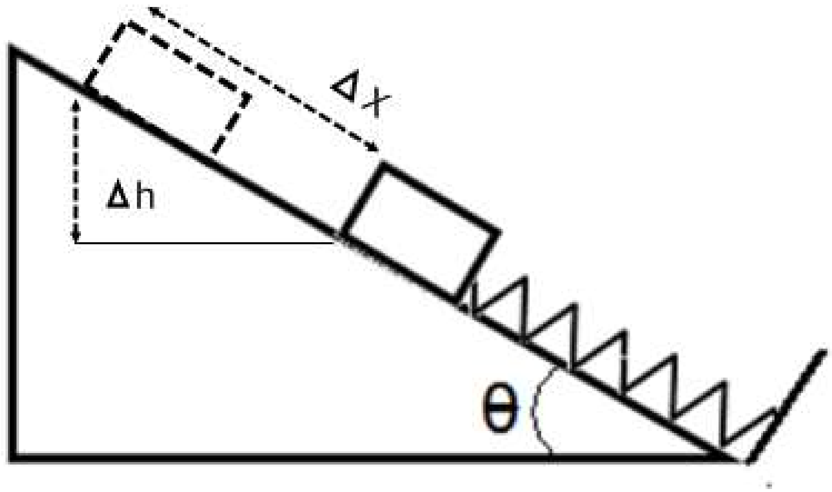
When it loses touch with the spring at Δh above its fully compressed point, it will have KE, GPE and zero EPE.

$(KE + EPE + GPE)_0 = (KPE + EPE + GPE)$

becomes:

$(EPE + GPE)_0 = (KE + GPE)$

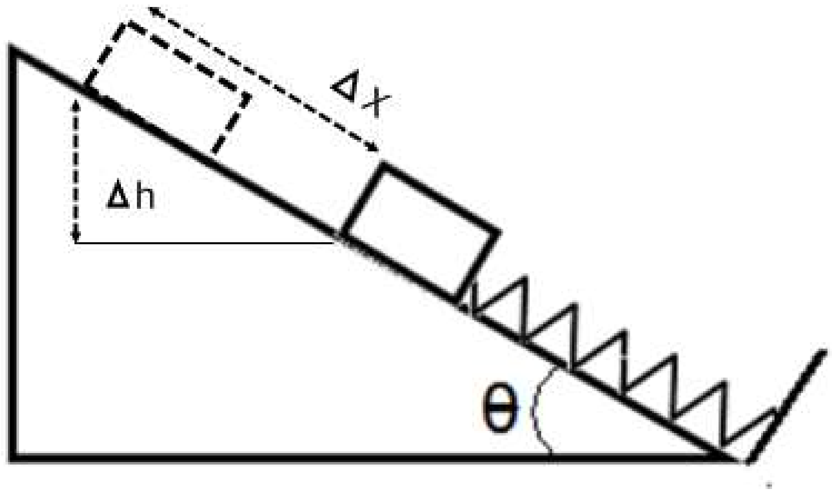
Energy Problem Solving



Before we proceed further with the solution, think how hard this problem would be to solve without using Conservation of Energy.

Once the object is released and the spring starts moving away from its compressed state, the force is no longer constant - it will require mathematical integration (calculus) to solve. Free body diagrams are not the best way to find the velocity of the object.

Energy Problem Solving



Let's put in the equations and rearrange them to solve for v.

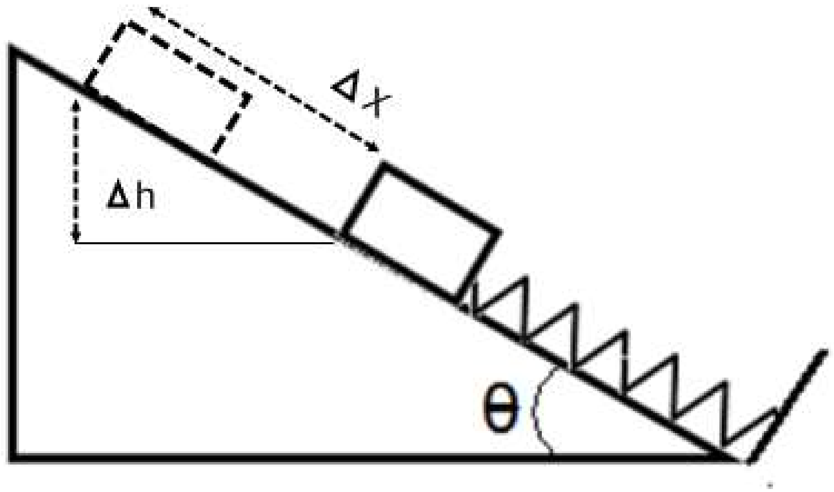
$$\frac{1}{2}k\Delta x^2 + mgh_0 = \frac{1}{2}mv^2 + mgh$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta x^2 + mgh_0 - mgh$$

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}k\Delta x^2 - mg(h - h_0)\right)}$$

$$v = \sqrt{\frac{k}{m}\Delta x^2 - 2g\Delta h}$$

Energy Problem Solving



We now have the equation for the velocity when the block rises a vertical displacement of Δh.

$$v = \sqrt{\frac{k}{m}\Delta x^2 - 2g\Delta h}$$

But if we're only given Δx, how do we find Δh?

Use trigonometry and recognize that $\Delta h = \Delta x \sin\theta$.

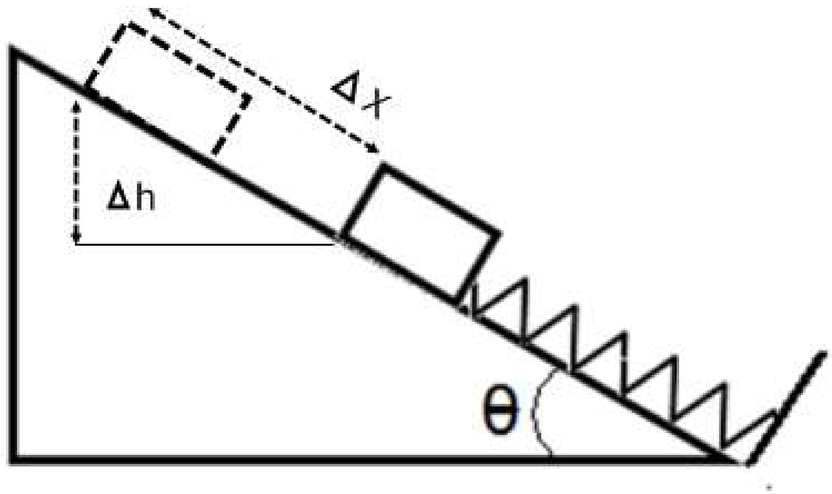
31 A box on an inclined plane is in contact with a spring. The box is released, compressing the spring. For every increment Δx, the box moves down the incline, how much does its height, Δh, change?

A $(\Delta x)^2$

B $\Delta x \sin\theta$

C $\Delta x \cos\theta$

D $\Delta x \tan\theta$



Answer

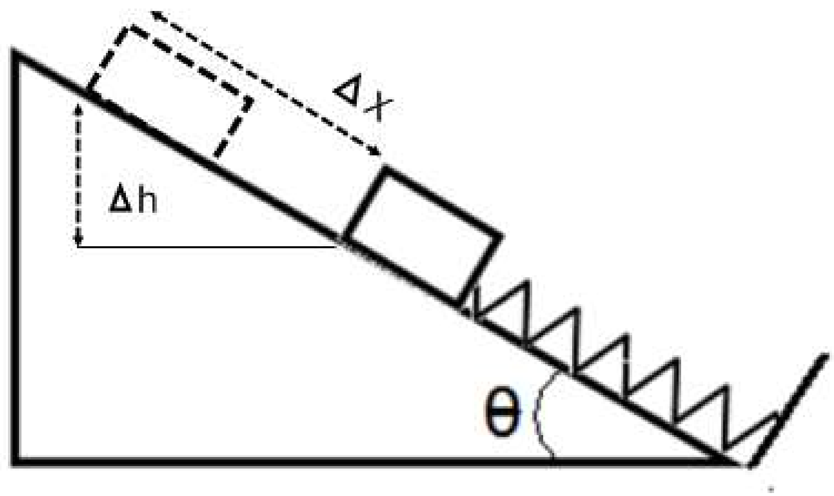
32 A box is held on top of a spring on an inclined plane of angle θ = 31⁰. The box is released, compressing the spring. If the spring moves 7.0 m down the plane, how much does its height, Δh, change?

A 7.2 m

B 6.5 m

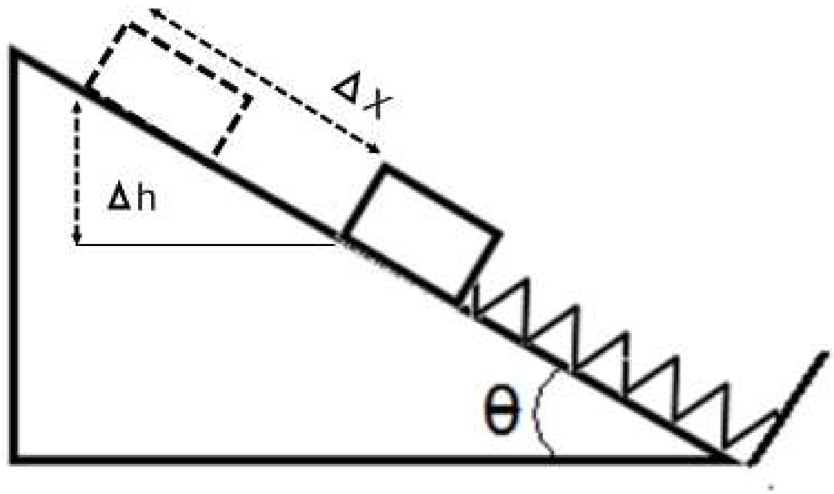
C 6.0 m

D 3.6 m



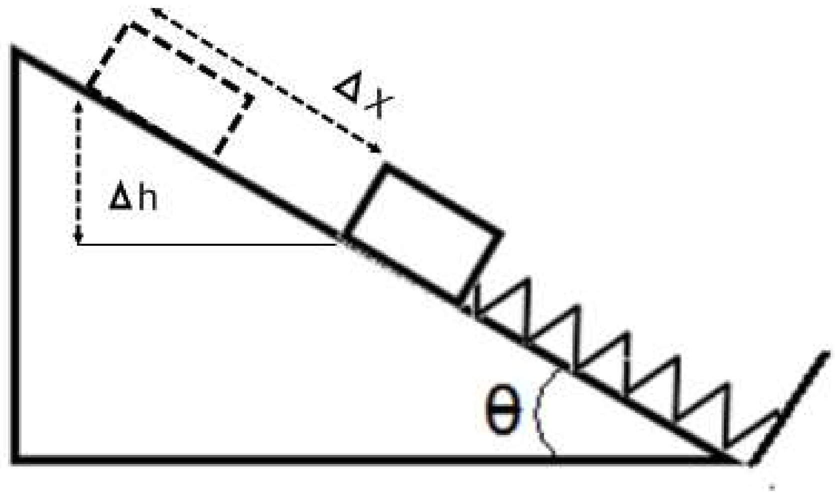
Answer

33 A box of mass m is on an inclined plane that makes an angle of θ with the horizontal and is in contact with a spring of spring constant k. The box is released, and it compresses the spring an amount Δx before rebounding. In terms of m, g, k and θ, what is the value of Δx?



Answer

34 A spring (k = 150 N/m) on an incline of $\theta = 54^0$ is compressed a distance of $\Delta x = .060$ m along the incline by a mass of 0.042 kg and then released. What is its velocity when it passes the point where it was first compressed and loses touch with the spring?



Answer

35 A marble launcher shoots a marble vertically and then shoots a marble in the horizontal direction. Describe why the exit velocity of the marble in the two cases is different. Which exit velocity is greater?

Answer

Conservation of Energy Problem Solving

Lab

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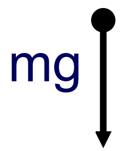
Falling Objects - the Energy way

An object, at rest, falls from a height, h_0, to the ground, and you want to find out what its velocity is right before it hits the ground (assume no air friction).

Before you learned the Conservation of Energy, you would draw a free body diagram and then use a Kinematics equation to find the velocity. Review this with your group and then remove the screen below to check:



$$\Sigma F_y = -mg = ma$$

$$a = -g$$

$$v^2 = v_0^2 + 2a(h - h_0)$$

$$v^2 = v_0^2 + 2gh_0$$

$$v = \sqrt{2gh_0}$$

Falling Objects - the Energy way

Now, let's use the Conservation of Energy to solve this problem. Define the system as the object and GPE at the ground as zero. Since there is no friction, the work on the system will be zero.

$$E_0 + W = E_f$$

$$E_0 = E_f$$

$$GPE_0 = (KE + GPE)_f = KE_f$$

$$mgh_0 = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh_0}$$

In this case, both methods take about the same amount of effort. But, in cases where the motion, or the forces acting on the object are more complex or are changing, the Energy approach is easier.

36 A ball of mass 0.45 kg falls from a building of height = 21 m. What is the ball's speed right before it hits the ground?

A 14 m/s

B 20 m/s

C 210 m/s

D 410 m/s

Answer

37 Three objects, of masses 0.43 kg, 12 kg and 43 kg, fall from a height of 31 m. Which object has the greater velocity right before it hits the ground (assume no air friction)?

A All three have the same velocity.

B The 0.43 kg mass.

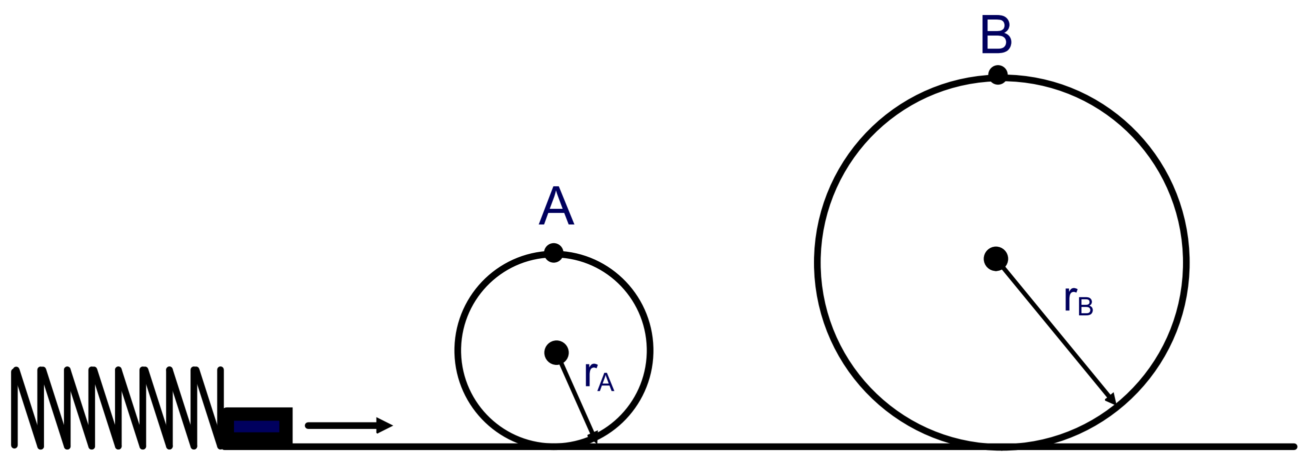
C The 12 kg mass.

D The 43 kg mass.

Answer

The Spring and the Roller Coaster

Look at the below diagram. A block of mass m, is ejected by a compressed spring and spins around a couple of loops before exiting to the right. What is the velocity of the block just as it leaves the spring and at points A and B (assume no friction)? Could you use free body diagrams and Newton's Laws?



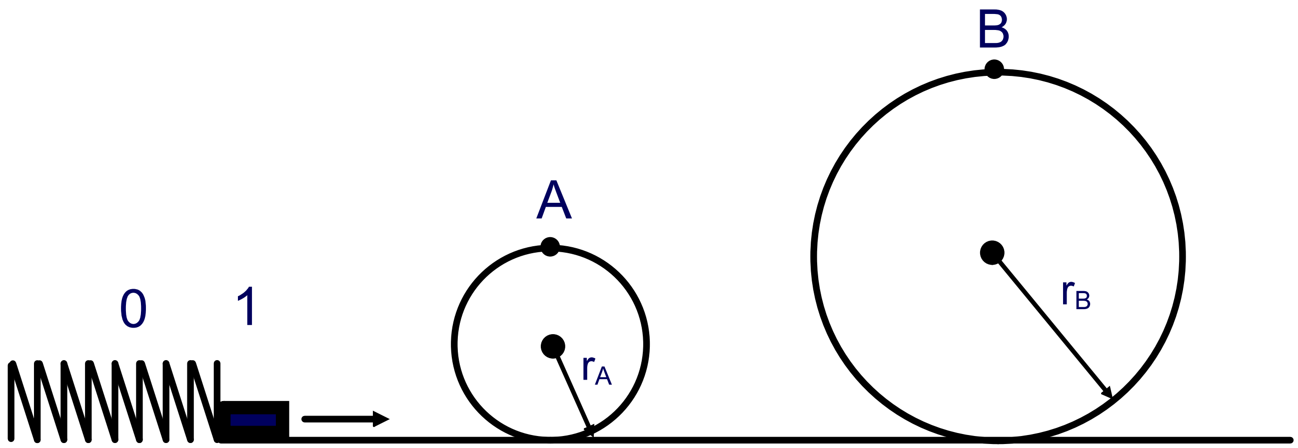
The Spring and the Roller Coaster

That would be very complicated! How would Conservation of Energy work? The system is the spring, the block and the loops. Assume no friction, hence there is no work done on this system:

$E_0 = E_1 = E_A = E_B$

E_0 is defined as the energy of the compressed spring right before it is released, E_1 is the energy of the block right after it leaves the spring, and E_A and E_B are the energies at points A and B.

Demo



The Spring and the Roller Coaster

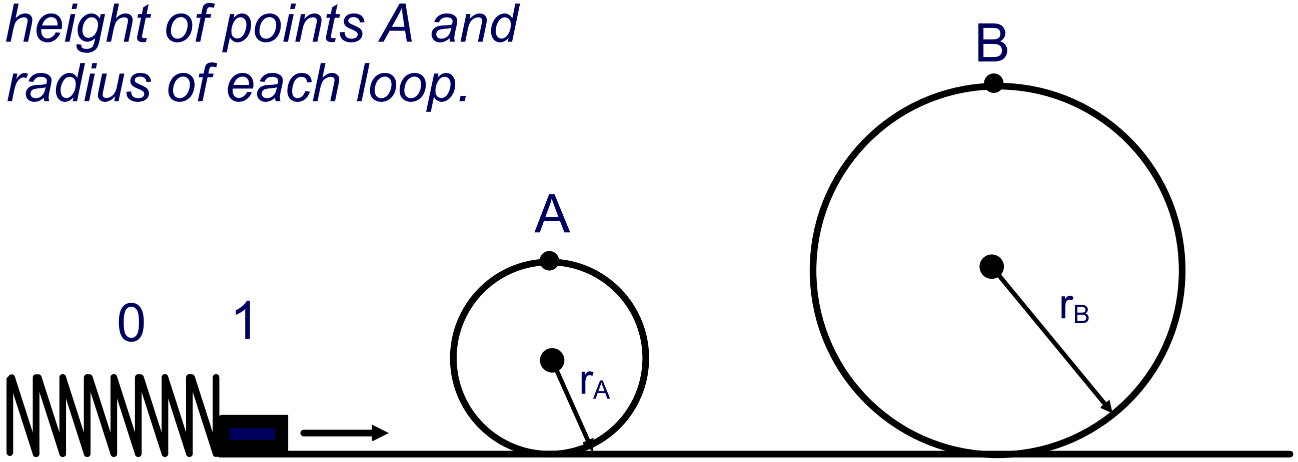
$E_0 = E_1 = E_a = E_f$

Assume the spring has a spring constant, k, was compressed Δx and released, and GPE = 0 along the base of the loops.

$EPE = KE_1 = (KE + GPE)_A = (KE + GPE)_B$

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_1^2 = \frac{1}{2}mv_A^2 + mg(2r_A) = \frac{1}{2}mv_B^2 + mg(2r_B)$$

Note how the height of points A and B is twice the radius of each loop.

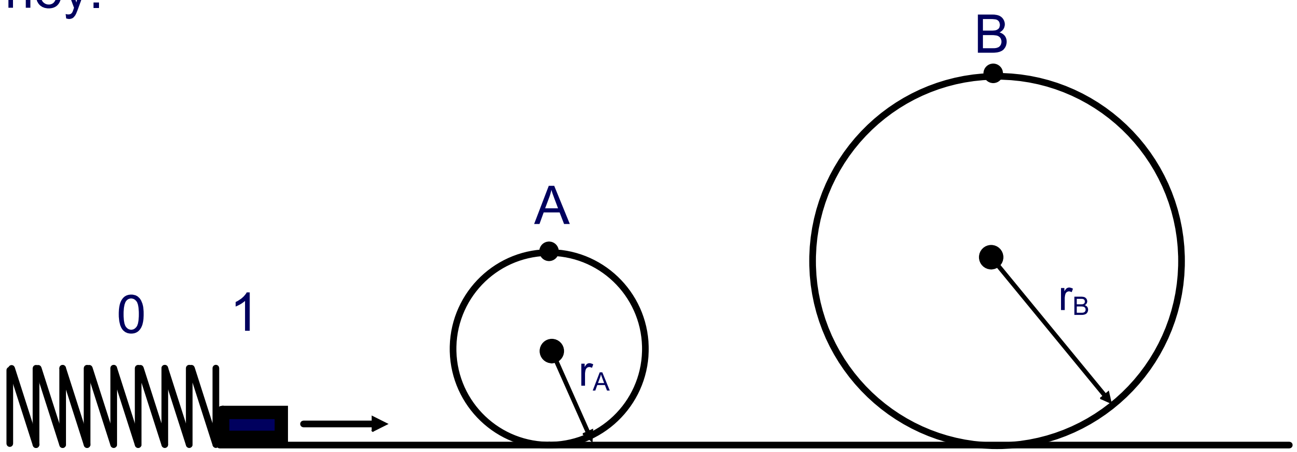


The Spring and the Roller Coaster

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_1^2 = \frac{1}{2}mv_A^2 + mg(2r_A) = \frac{1}{2}mv_B^2 + mg(2r_B)$$

What's nice about the Conservation of Energy, and working out the algebra in advance, is we can solve for different quantities in the above equation depending on what we were given.

The total Energy stays the same, no matter where the block is on its journey.



The Spring and the Roller Coaster

Find the velocity at point A first:

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_1^2 = \frac{1}{2}mv_A^2 + mg(2r_A) = \frac{1}{2}mv_B^2 + mg(2r_B)$$

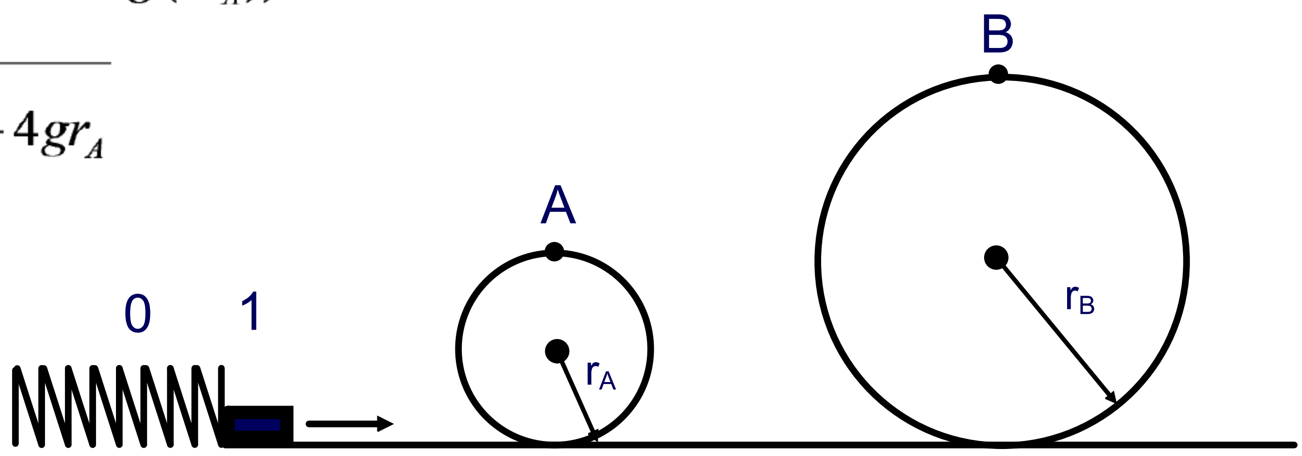
$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_A^2 + mg(2r_A)$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kx^2 - mg(2r_A)$$

$$v_A = \sqrt{\frac{2}{m}(\frac{1}{2}kx^2 - mg(2r_A))}$$

$$v_A = \sqrt{\frac{k}{m}x^2 - 4gr_A}$$

Now we just need to substitute in the values. What else can we solve for by modifying this solution a little?



The Spring and the Roller Coaster

The velocity at point B is found the same way!

$$v_A = \sqrt{\frac{k}{m}x^2 - 4gr_A}$$

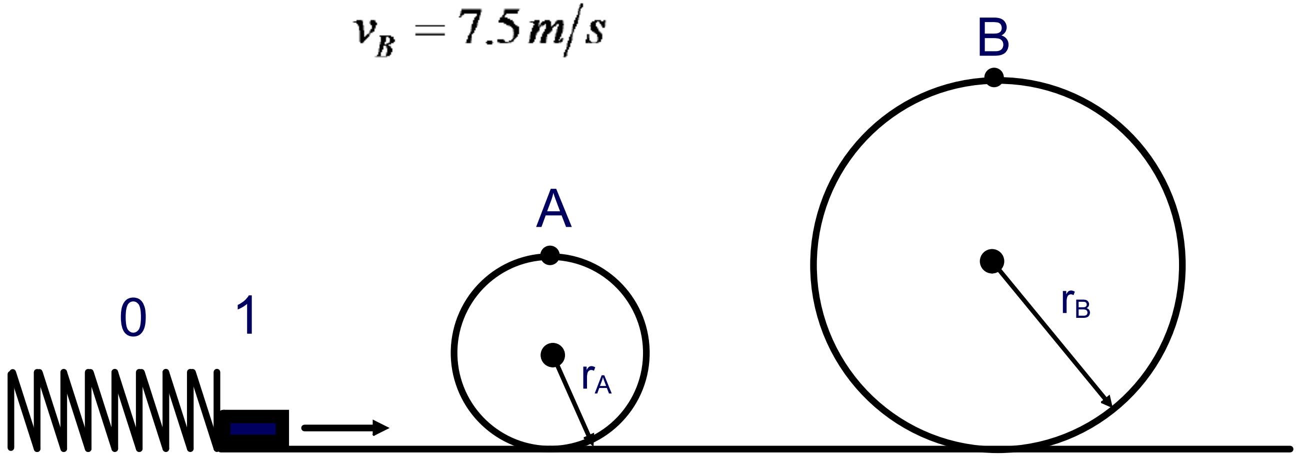
$$v_A = \sqrt{\frac{92\,N/m}{0.25kg}(0.5m)^2 - 4(9.8\,m/s^2)(0.51m)}$$

$$v_A = 8.5\,m/s$$

$$v_B = \sqrt{\frac{k}{m}x^2 - 4gr_B}$$

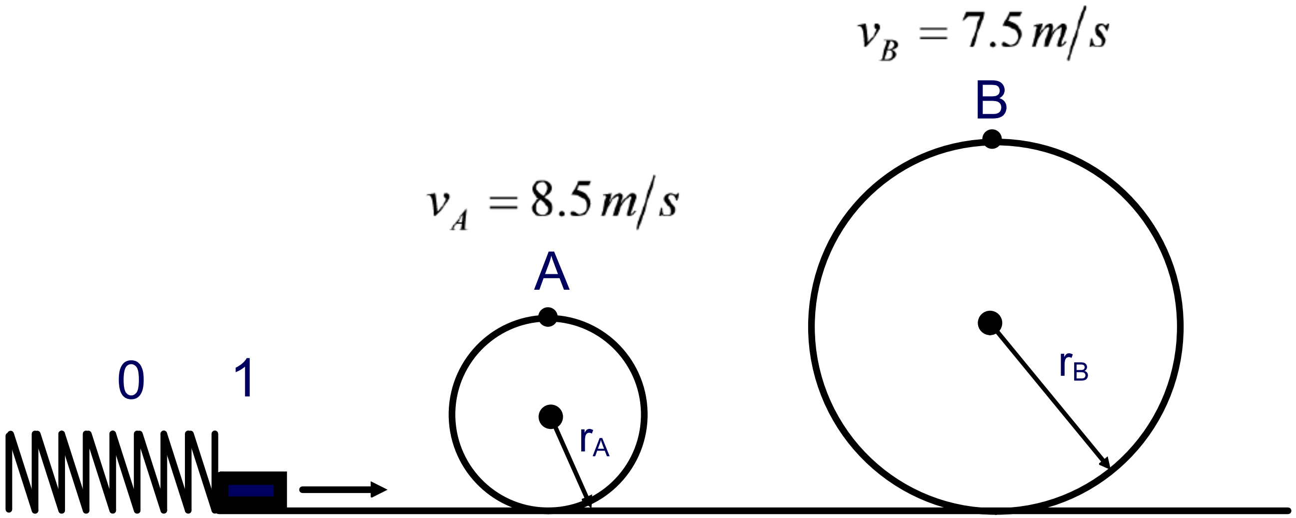
$$v_B = \sqrt{\frac{92\,N/m}{0.25kg}(0.5m)^2 - 4(9.8\,m/s^2)(0.90m)}$$

$$v_B = 7.5\,m/s$$



The Spring and the Roller Coaster

Intuitively, why is the velocity at point B less than point A?



The block has a greater GPE at point B - so since Energy is conserved, it has a smaller KE - hence, a smaller velocity. Note how we don't care how it got there - an elliptical loop would give the same velocity.

38 A spring gun, aimed in the horizontal direction with k = 250 N/m is compressed 0.050 m and released. How fast will a 0.025 kg dart go when it exits the gun?

Answer

39 A student uses a spring gun, with k = 180 N/m, to launch a marble vertically into the air. The mass of the marble is 0.0040 kg and the spring is compressed 0.030 m. How high will the marble go above the spring's initially compressed position?

Answer

40 A student uses a spring gun (k = 120 N/m) to launch a marble at an angle of 52^0 to the horizontal (m = .0020 kg, Δx = 0.041 m). What is the maximum height that the marble will reach above its initially compressed position?

Answer

41 A roller coaster car is pulled up to a height of 50 m (A), where it then goes down the other side of the track. It traverses two other loops, one at a height of 40 m (B), and the second at a height of 30 m (C). Rank the velocities of the car at the three heights from greatest to least.

A B > A > C

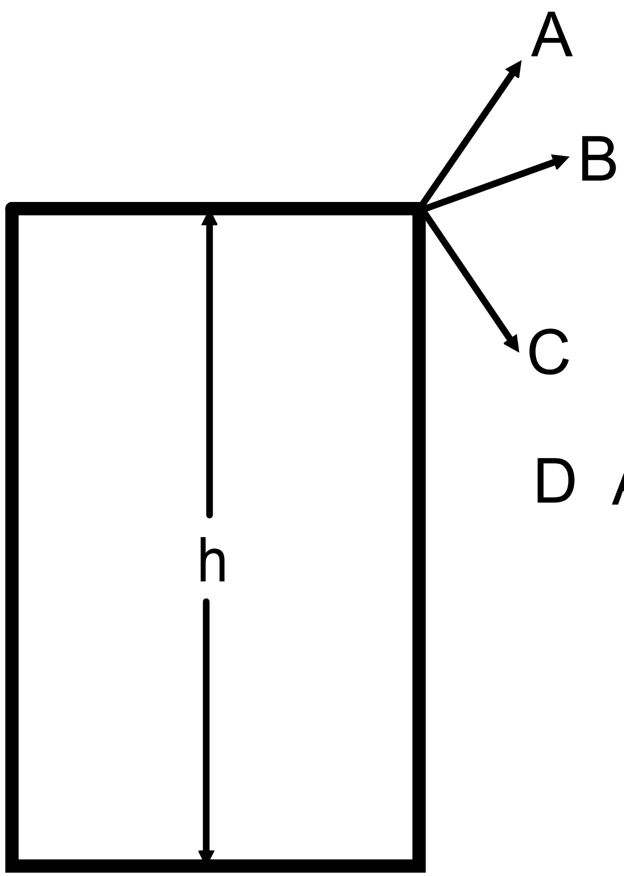
B C > B > A

C C > A > B

D B > C > A

Answer

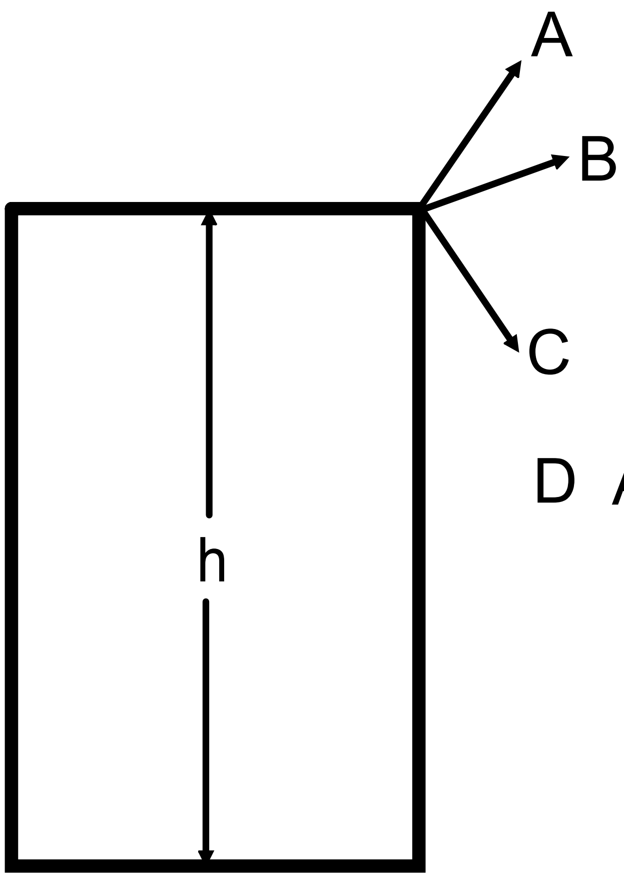
42 Three identical objects are thrown with the same initial speeds in different directions from the top of a building. Which will have the greatest speed when it strikes the ground?



D All have the same speed.

Answer

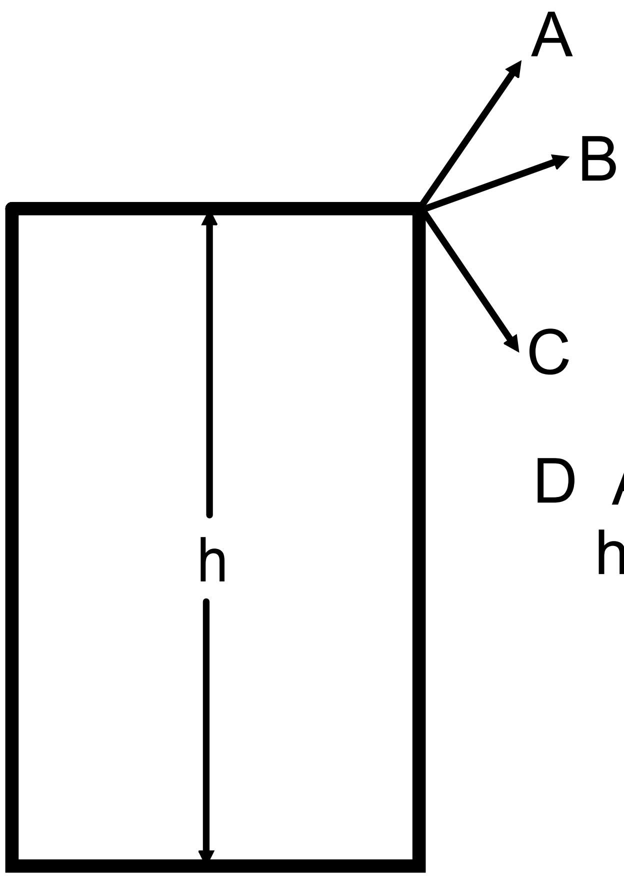
43 Three identical objects are thrown with the same initial speeds in different directions from the top of a building. Which will hit the ground first?



D All hit at the same time.

Answer

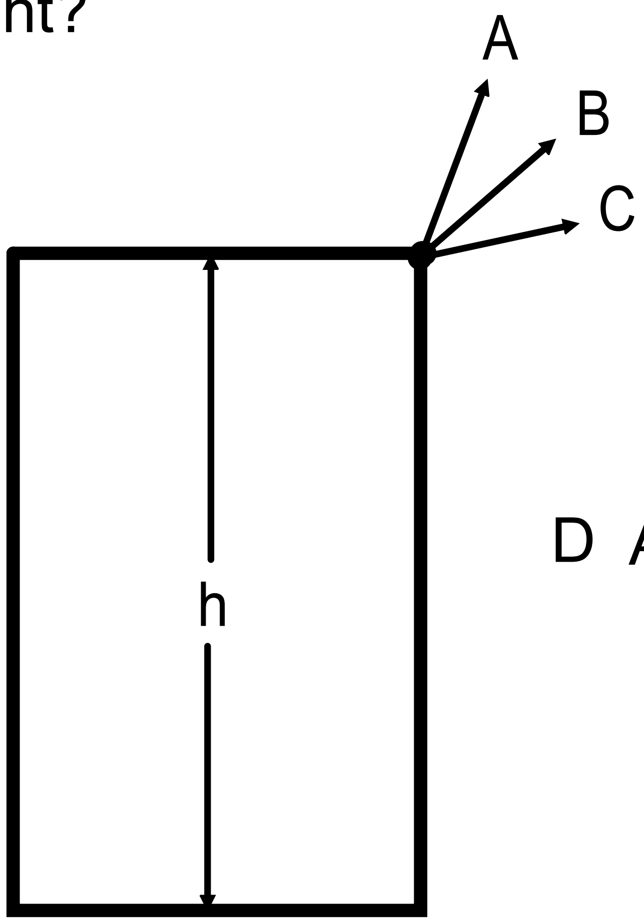
44 Three identical objects are thrown with the same initial speeds in different directions from the top of a building. Which will go the highest?



D All reach the same height.

Answer

45 Three identical objects are thrown with the same speed in different directions from the top of a building. Which will have the greatest kinetic energy at its maximum height?



D All will have the same KE.

Answer

GPE and Escape Velocity

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GPE - expanded

The expression GPE = mgh only works near the surface of and is specific to the planet. Recall that:

$$g = \frac{GM_{planet}}{R^2_{planet}}$$

R_{planet} = radius of planet
M_{planet} = mass of planet

As an object's height above the surface of the planet increases, its distance from the center of the planet is not just the radius of the planet. Thus, a more general expression is needed based on Newton's Universal Gravitational Force equation:

$$F_g = \frac{GM_{planet}m_{object}}{r^2_{planet-object}}$$

$r_{planet\text{-}object}$ = distance from center of the planet to the object

GPE - expanded

Deriving a more general formula for the potential energy of an object that moves further away from the earth requires calculus, because the gravitational force is not constant - it must be "integrated (calculus)" over the distance the object moves.

We'll just present the result, and use a more common physics term for gravitational potential energy, U_G.

$$U_G = -\frac{GMm}{r}$$

What do you think the negative sign in this expression means?

GPE - expanded

$$U_G = -\frac{GMm}{r}$$

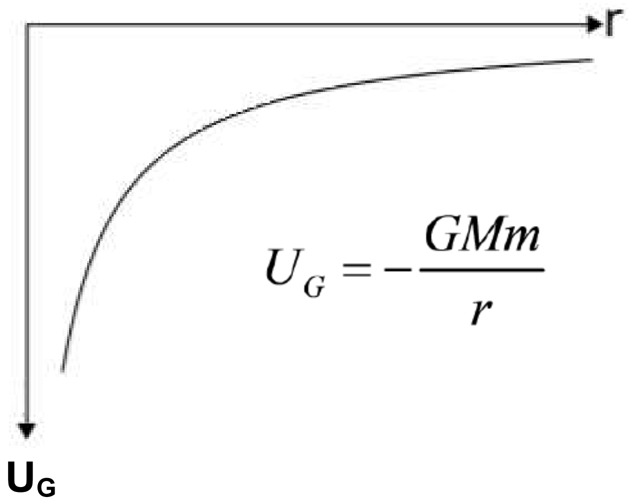
In solving the integral, an assumption was made that the gravitational potential energy between two objects an infinite distance apart is zero.

Add this assumption to the fact that the gravitational force is always an attractive force, and the negative sign pops up.

How can we reconcile this negative sign with the fact that we normally think of Gravitational Potential Energy as a positive number?

Think of what GPE really means....

GPE - expanded



$$U_G = -\frac{GMm}{r}$$

GPE measured for objects near the surface of the earth is really $mg(h-h_0)$, and h_0 is normally set to zero.

So, it's the change in height above the earth's surface that is important - GPE is not an absolute number.

On the graph above, notice as r (the distance from the center of the earth) increases (the object moves above the surface of the earth), U_G becomes less negative. The difference between the final U_G and the initial U_G will be positive! So, mgh will still be a positive number.

46 Why can't the expression (GPE = mgh) be used to determine the GPE between the earth and a weather satellite in geosynchronous orbit above the earth (35.8 km)? **Select two answers.**

A The satellite's orbital period is twice that of the earth's rotational period.

B The satellite is moving at a great speed.

C The value of g is different at the earth's surface and at the satellite.

D The distances between the surface of the earth and the satellite, and from the center of the earth to the satellite are mathematically significant.

Answer

47 Describe how the negative sign in the generalized potential energy equation was determined.

Answer

Escape Velocity

Let's apply this new definition of GPE between two masses to the problem of escape velocity.

Escape velocity is the minimum velocity an object (such as a spaceship) needs to attain to "escape" the earth's gravitational pull. Actually, since the force of gravity has an infinite range, this is never really possible, but the force will be extremely tiny at great distances.

We're going to use conservation of energy, and our system is going to be the earth-spaceship system. *If we're looking for the minimum velocity, what will the magnitude of the potential energy and kinetic energy of the spaceship when it is an infinite distance from the earth?*

Escape Velocity

Since we're looking for the minimum required velocity to escape the earth, we want it to get to infinity with zero kinetic energy. Potential energy was already defined to be zero at infinity. Using the Conservation of Energy:

$$(KE + GPE)_0 = (KE + GPE)$$

$$\frac{1}{2}mv_{escape}^2 - \frac{GMm}{r} = 0$$

$$mv_{escape}^2 = \frac{GMm}{r^2}$$

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Escape Velocity

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Note that escape velocity is independent of the mass of the escaping object. It only depends on the mass and radius of the object being escaped from.

Using this fact, can you explain why despite the fact that Hydrogen and Helium are the most abundant elements in the universe, there are only trace amounts of each element in our atmosphere?

Escape Velocity

When the earth was first formed, the atmosphere contained significant amounts of both elements.

However - Hydrogen and Helium are the lightest elements, so for a given atmospheric temperature, they would move faster than the heavier elements.

Over time, their average speeds exceeded the escape velocity of earth, thus they left the planet.

48 A rocket of mass 5,000 kg will have an escape velocity _______________ the escape velocity of a rocket with mass 10,000 kg.

A one quarter of

B one half of

C the same as

D twice

Answer

49 What is the escape velocity on the planet Earth?
(M_{earth} = 5.97x10^{24} kg, r_e = 6.38x10^{6} m,
G = 6.67x10^{-11} Nm^2/kg^2)

Answer

Power

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Power

It is often important to know not only if there is enough energy available to perform a task but also how much time will be required.

Power is defined as the rate that work is done:

$$P = W/\Delta t$$

Since work is measured in Joules (J) and time is measured in seconds (s) the unit of power is Joules per second (J/s).

In honor of James Watt, who made critical contributions in developing efficient steam engines, the unit of power is known as a Watt (W).

Power

Let's derive another expression for Power:

$P = W/\Delta t$

Since $W = Fd_{parallel}$

$P = (Fd_{parallel}) / \Delta t$

Regrouping, this becomes

$P = F(d_{parallel} / \Delta t)$

Since $v = d/\Delta t$

$P = Fv_{parallel}$

Power can be defined as the product of the force applied and the velocity of the object parallel to that force.

Power

A third useful expression for power can be derived from our original statement of the conservation of energy principle.

$P = W/\Delta t$

Since $W = E_f - E_0$

$P = (E_f - E_0)/\Delta t$

The power absorbed by a system can be thought of as the rate at which the energy in the system is changing.

50 A steam engine does 52 J of work in 12 s. What is the power supplied by the engine?

A 0.23 W

B 4.3 W

C 23 W

D 624 W

Answer

51 How long must a 350.0 W engine run in order to produce 720.0 kJ of work?

A 4.861×10^{-4} s

B 2.057 s

C 2057 s

D 4861 s

Answer

52 A 12.0 kW motor runs a vehicle at a constant speed of 8.00 m/s. What is the force supplied by the motor (assume the power is delivered in the direction of the vehicle's motion)?

A 6.7×10^{-4} N

B 0.67 N

C 1.5 N

D 1500 N

Answer