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AP Physics I

Kinematics in 2D

2016-10-05

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Kinematics in One Dimension

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Review of 1-D Kinematics

- Kinematics is the description of how objects move with respect to a defined reference frame.

- Displacement is the change in position of an object.

- Average speed is the distance traveled divided by the time it took; average velocity is the displacement divided by the time.

- Instantaneous velocity is the limit as the time becomes infinitesimally short.

- Average acceleration is the change in velocity divided by the time.

Review of 1-D Kinematics

- Instantaneous acceleration is the limit as the time interval becomes infinitesimally small.

- There are four equations of motion for constant acceleration, each requires a different set of quantities.

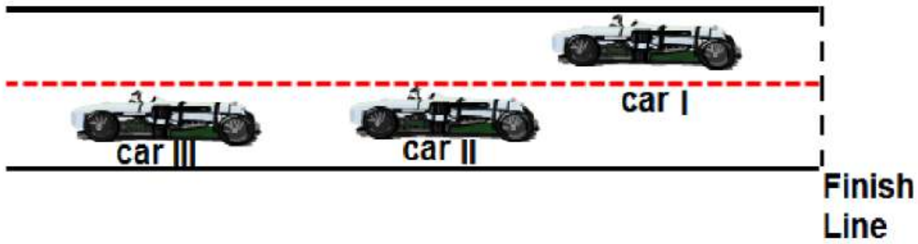
$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$\overline{v} = \frac{v + v_o}{2}$$

1 A snapshot of three racing cars is shown on the diagram. All three cars start the race at the same time, at the same place and move along a straight track. As they approach the finish line, which car has the lowest average speed?



A Car I

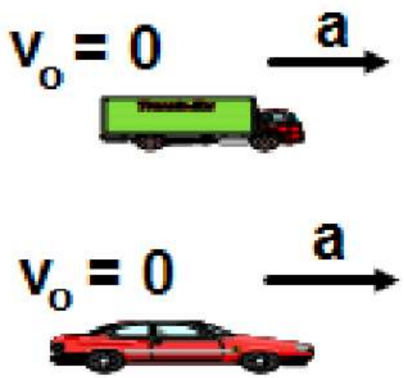
B Car II

C Car III

D All three cars have the same average speed

Answer

2 A car and a delivery truck both start from rest and accelerate at the same rate. However, the car accelerates for twice the amount of time as the truck. What is the final speed of the car compared to the truck?



A Half as much

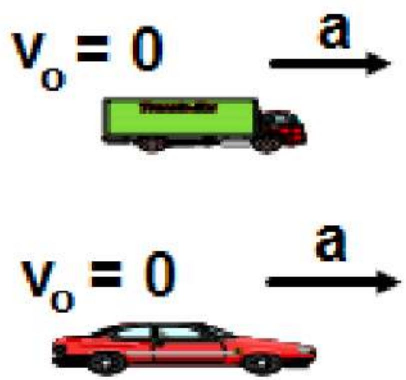
B Twice as much

C Four times as much

D One quarter as much

Answer

3 A car and a delivery truck both start from rest and accelerate at the same rate. However, the car accelerates for twice the amount of time as the truck. What is the traveled distance of the car compared to the truck?



A Half as much

B The same

C Twice as much

D Four times as much

Answer

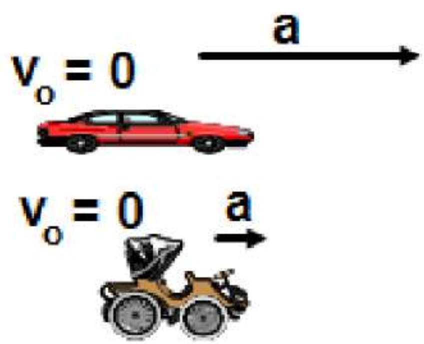
4 A modern car can develop an acceleration four times greater than an antique car like “Lanchester 1800”. If they accelerate over the same distance, what would be the velocity of the modern car compared to the antique car?

A Half as much

B The same

C Twice as much

D Four times as much



Answer

Graphing Motion at Constant Acceleration

In physics, there is another method in addition to algebraic analysis that can be used, called graphical analysis. The formula $v = v_0 + at$ can be interpreted by the graph. We just need to recall our memory from math classes where we already saw a similar formula, $y = mx + b$.

From these two formulas we can make some analogies:

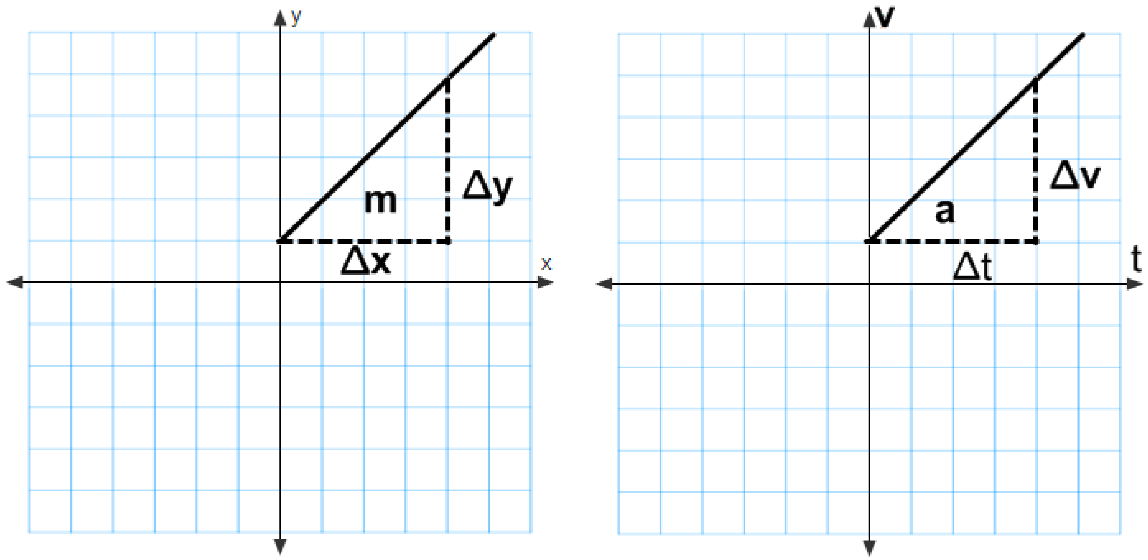
v -> y (dependent variable of x)
v_0 -> b (intersection with vertical axis),
t -> x (independent variable),
a -> m (slope of the graph- the ratio between rise and run $\Delta y/\Delta x$)

Motion at Constant Acceleration

Below we can find the geometric explanation to the acceleration $a = \Delta v/\Delta t$.

If slope is equal to: $m = \Delta y/\Delta x$

Then consider a graph with velocity on the y-axis and time on the x-axis. What is the slope for the graph on the right?

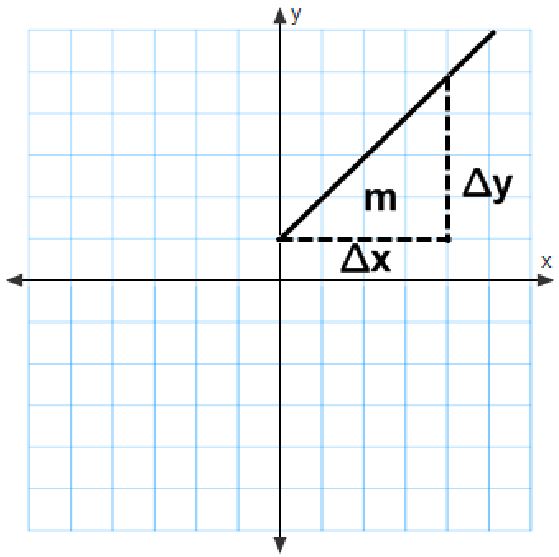


Motion at Constant Acceleration

The graph on the right has a slope of Δv/Δt, which is equal to acceleration. Therefore, the slope of a velocity vs. time graph is equal to acceleration.

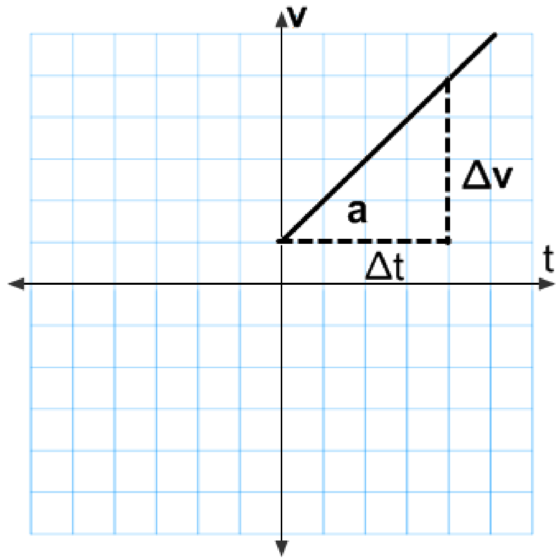
(slope)

$y = \Delta y / \Delta x$

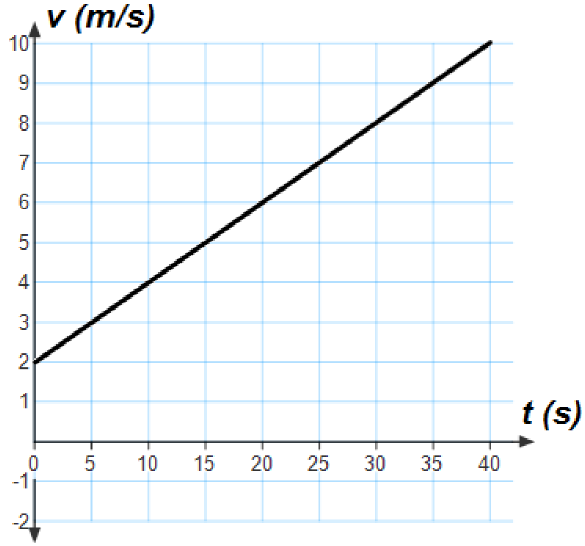


(slope of velocity vs. time)

$a = \Delta v / \Delta t$



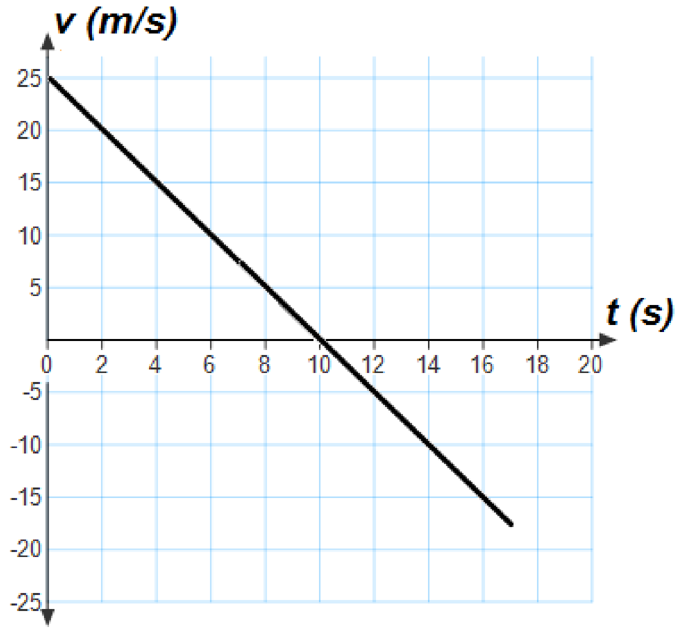
5 The velocity as a function of time is presented by the graph.
What is the acceleration?



Answer

6 The velocity as a function of time is presented by the graph.

Find the acceleration.

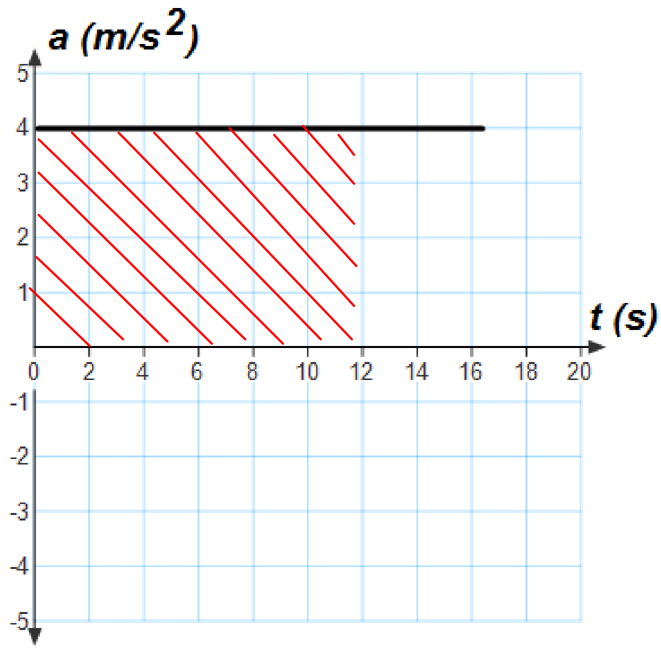


Answer

Motion at Constant Acceleration

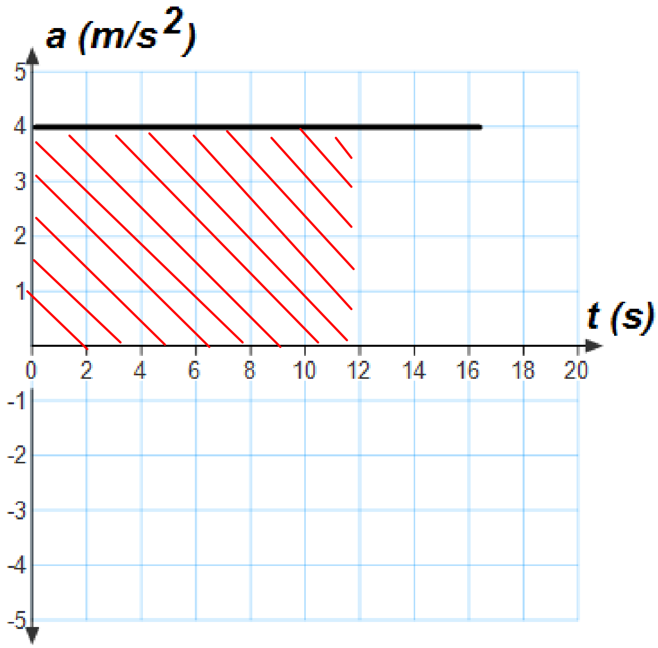
The acceleration graph as a function of time can be used to find the velocity of a moving object.

When the acceleration is constant it can be shown on the graph as a straight horizontal line.



Motion at Constant Acceleration

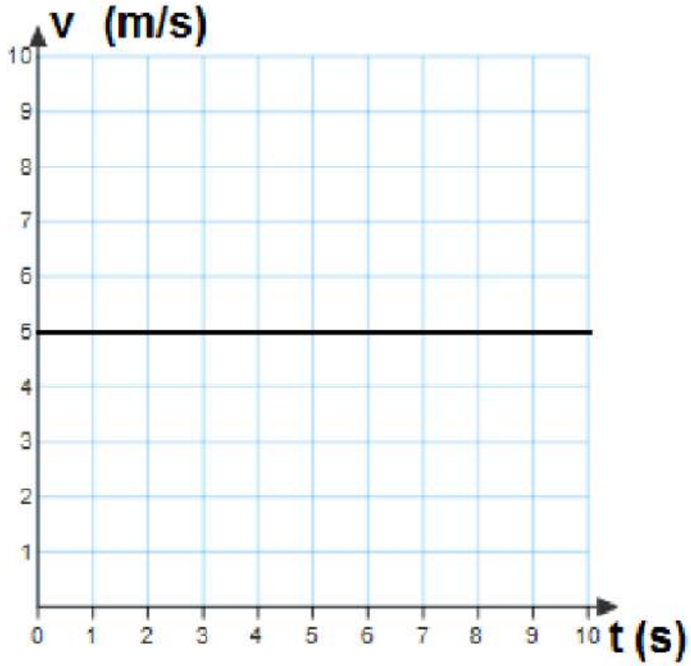
In order to find the change in velocity for a certain limit of time we need to calculate the area under the acceleration versus time graph.



The change in velocity during first 12 seconds is equivalent to the shadowed area (4x12 = 48).

The change in velocity during first 12 seconds is 48 m/s.

7 Which of the following statements is true?



A The object slows down

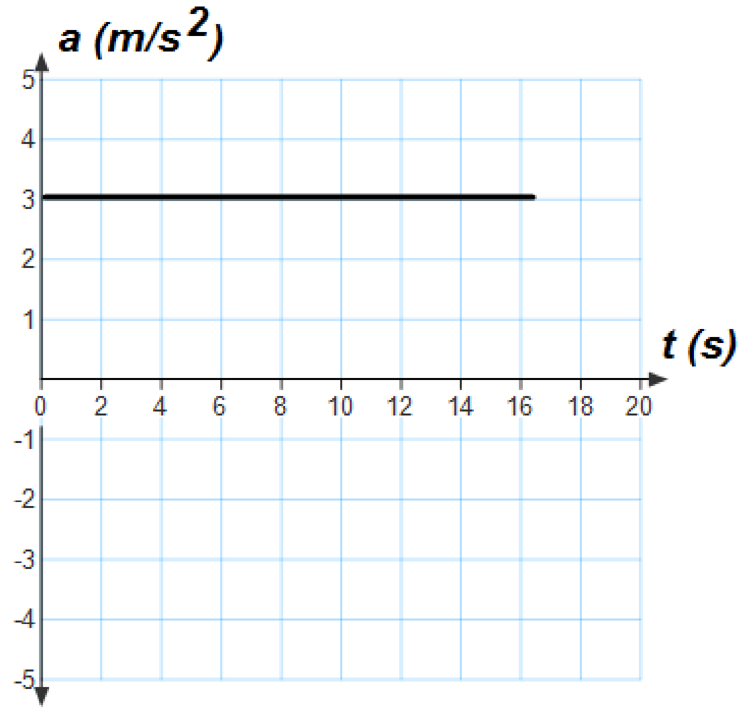
B The object moves with a constant velocity

C The object stays at rest

D The object is in free fall

Answer

8 The following graph shows acceleration as a function of time of a moving object. What is the change in velocity during first 10 seconds?



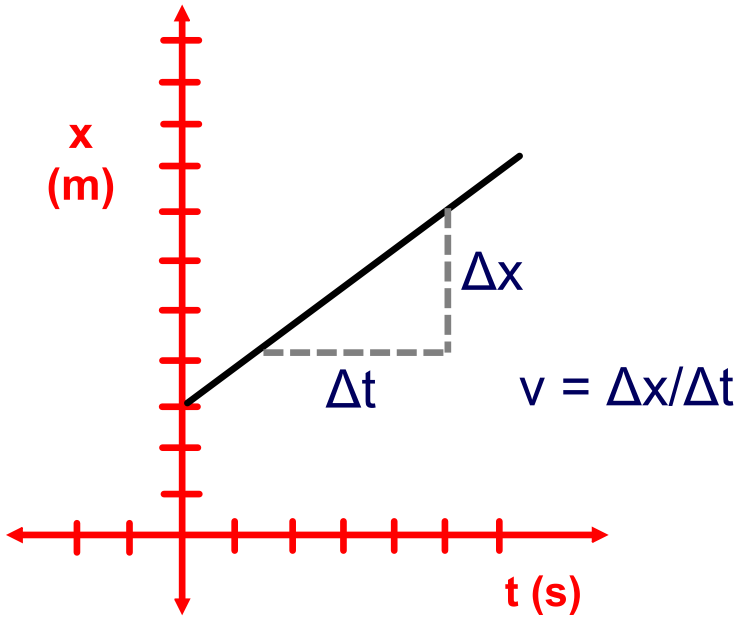
Answer

Analyzing Position vs Time Graphs

Recall earlier in this unit that slope was used to describe motion.

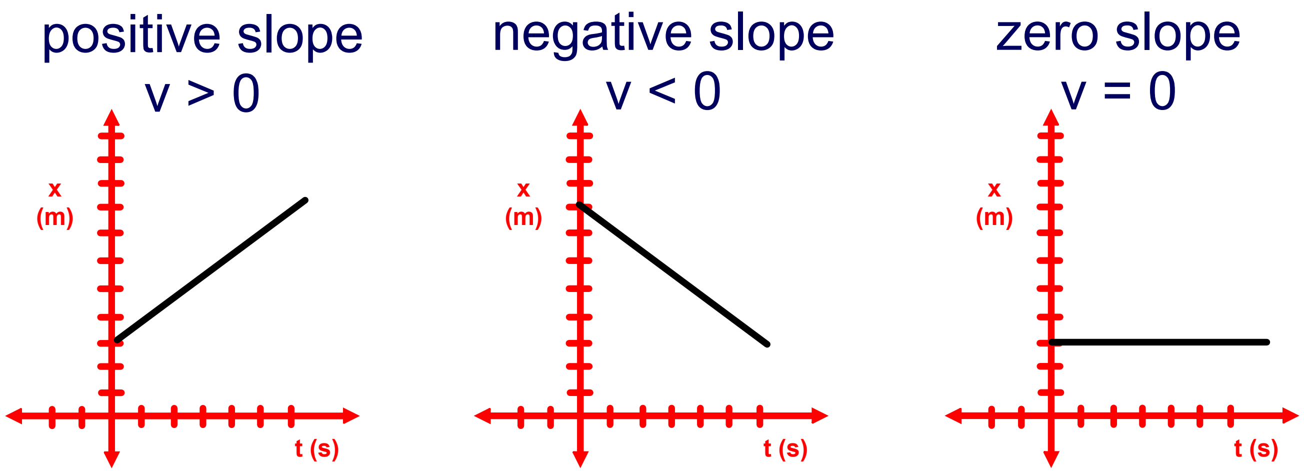
The slope in a position vs. time graph is Δx/Δt, which is equal to velocity.

Therefore, slope is equal to velocity on a position vs. time graph.



Analyzing Position vs Time Graphs

A positive slope is a positive velocity, a negative slope is a negative velocity, and a slope of zero means zero velocity.



A positive velocity means moving in the positive direction, a negative velocity means moving in the negative direction, and zero velocity means not moving at all.

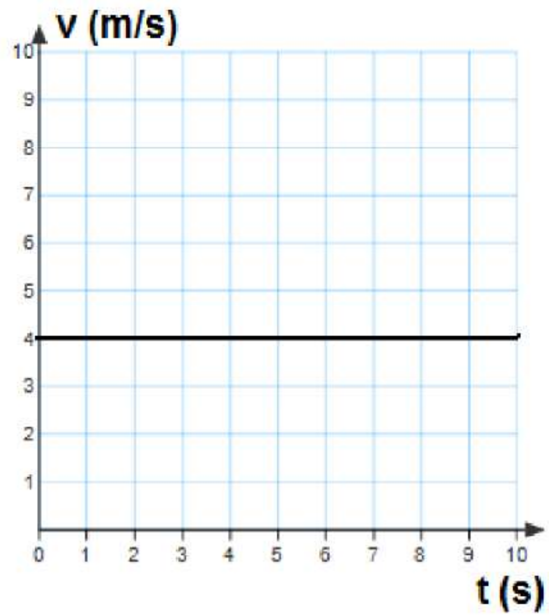
9 The graph represents the relationship between velocity and time for an object moving in a straight line. What is the traveled distance of the object at 9 s?

A 10 m

B 24 m

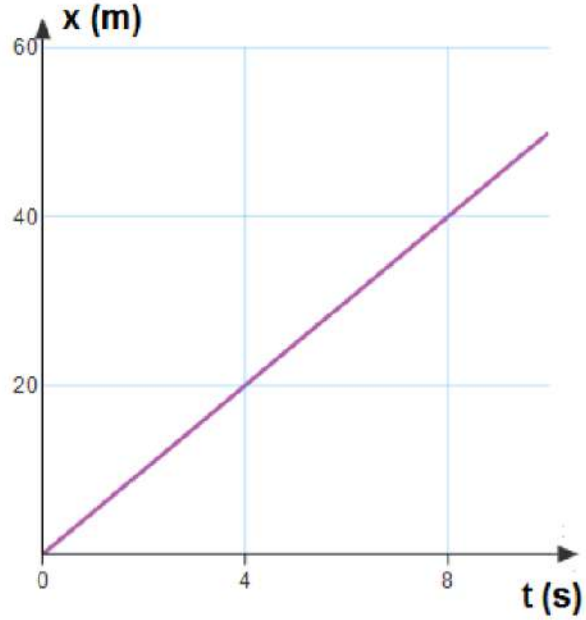
C 36 m

D 48 m



Answer

10 Which of the following is true?



A The object increases its velocity

B The object decreases its velocity

C The object's velocity stays unchanged

D The object stays at rest

Answer

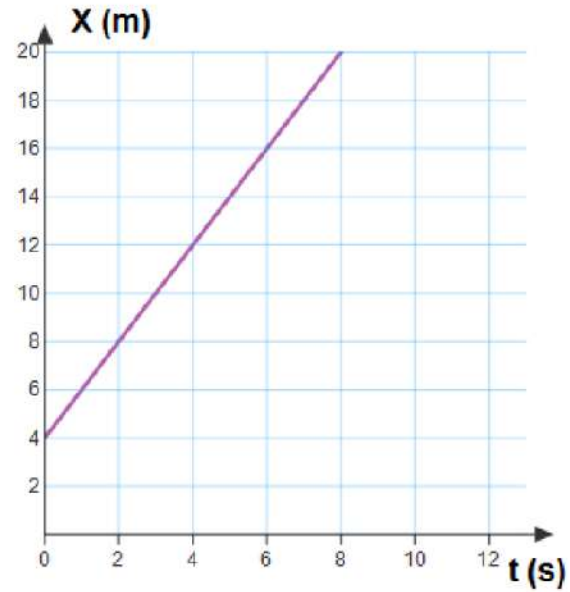
11 What is the velocity of the object?

A 2 m/s

B 4 m/s

C 6 m/s

D 8 m/s



Answer

Free Fall

All unsupported objects fall towards the earth with the same acceleration.

We call this acceleration the "acceleration due to gravity" and it is denoted by g.

$$g = 9.8\ m/s^2$$

Keep in mind, ALL objects accelerate towards the earth at the same rate.

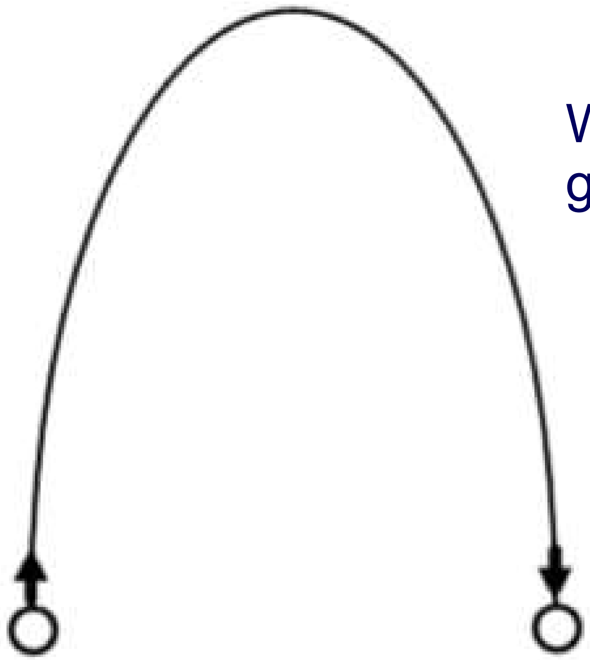
g is a constant!

Free Fall

What happens at the top?

What happens when it goes down?

What happens when it goes up?



An object is thrown upward with initial velocity, v_o

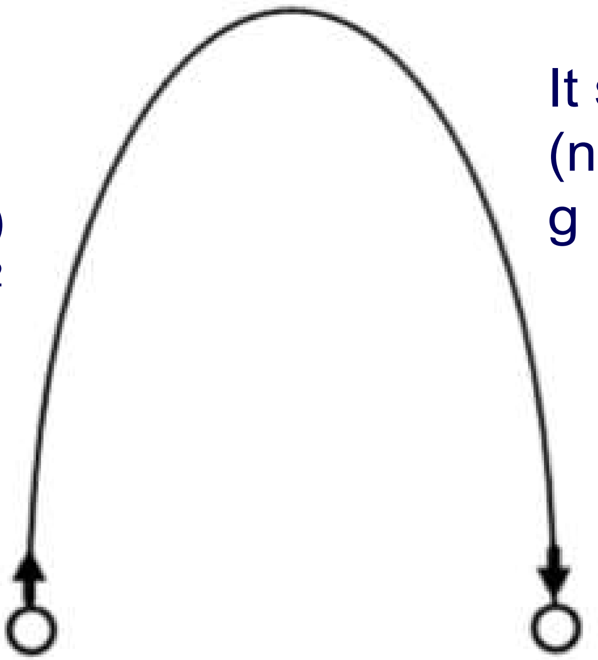
(Click on question for answer.)

What happens when it lands?

Free Fall Answers

It stops momentarily.
$v = 0$
$g = -9.8\ m/s^2$

It slows down.
(negative acceleration)
$g = -9.8\ m/s^2$



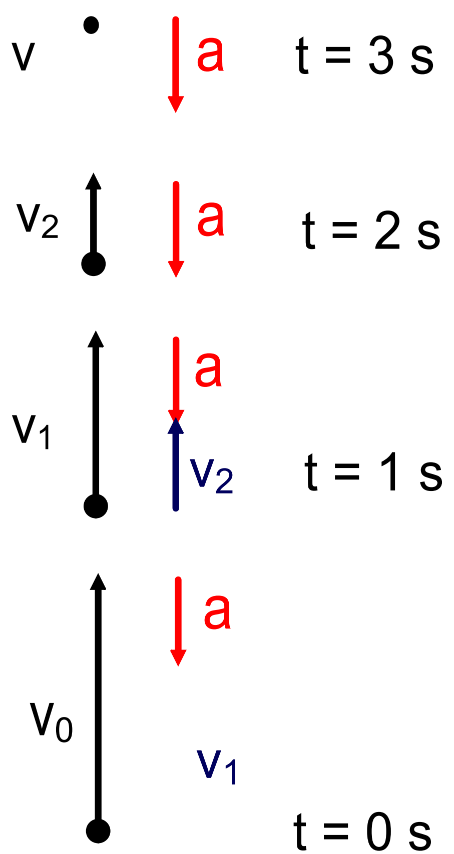
It speeds up.
(negative acceleration)
$g = -9.8\ m/s^2$

An object is thrown upward
with initial velocity, v_o

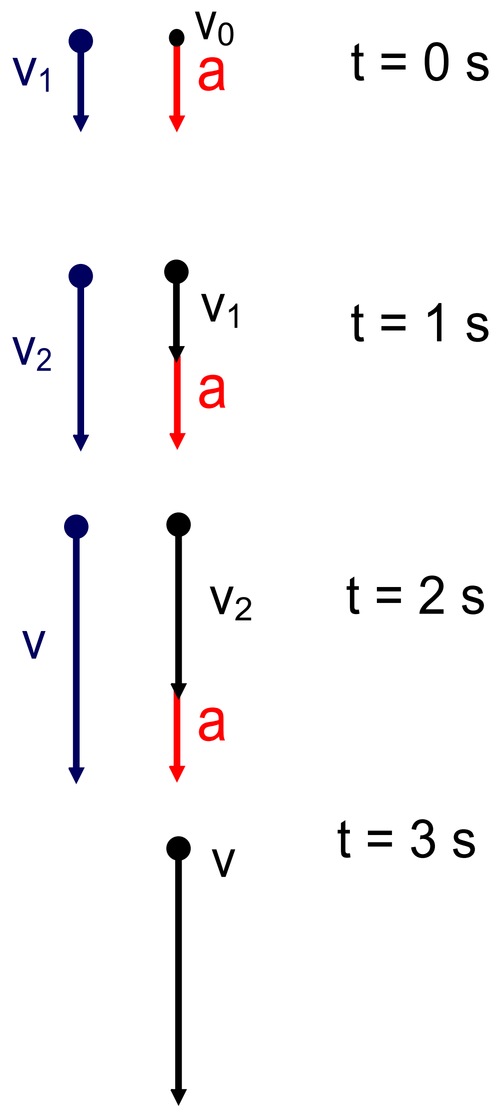
It returns with its
original velocity.

Free Fall

On the way up:



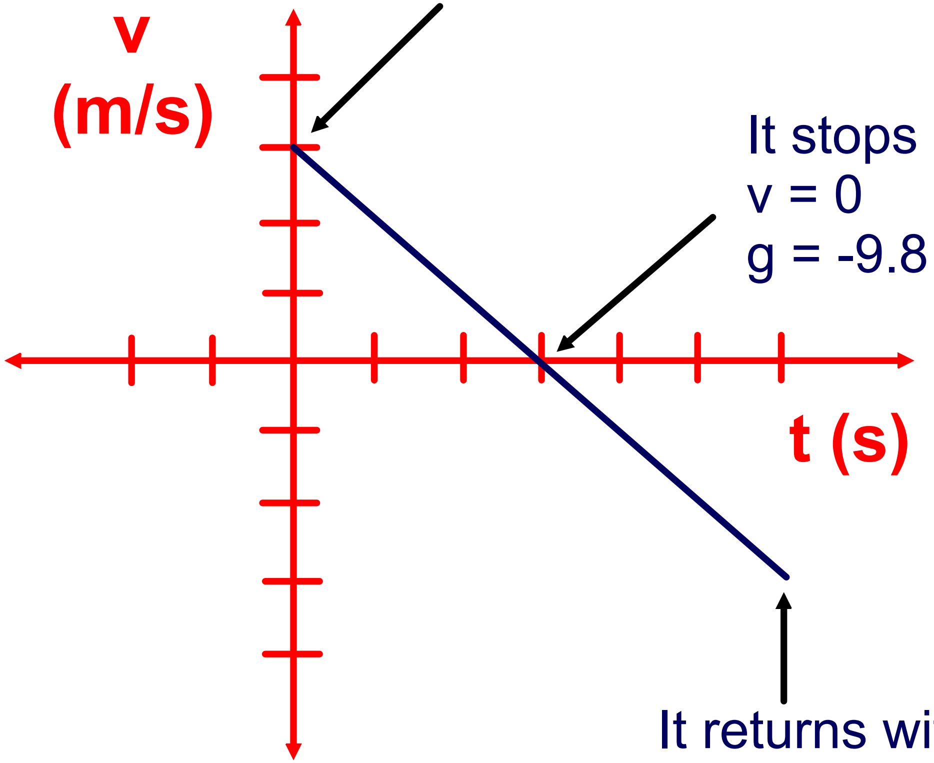
On the way down:



Free Fall

For any object thrown straight up into the air, what does the velocity vs time graph look like?

An object is thrown upward with initial velocity, v_0



It stops momentarily.
$v = 0$
$g = -9.8\ m/s^2$

It returns with its original velocity but in the opposite direction.

12 A ball is thrown straight up from point A it reaches a maximum height at point B and falls back to point C. Which of the following is true about the direction of the ball's velocity and acceleration between A and B?

A v↓ ↑a

B v↑ ↓a

C v↑ ↑a

D v↓ ↓a

Answer

13 A ball is thrown straight up from point A it reaches a maximum height at point B and falls back to point C. Which of the following is true about the direction the ball's velocity and acceleration between B and C?

A v↓ ↑a

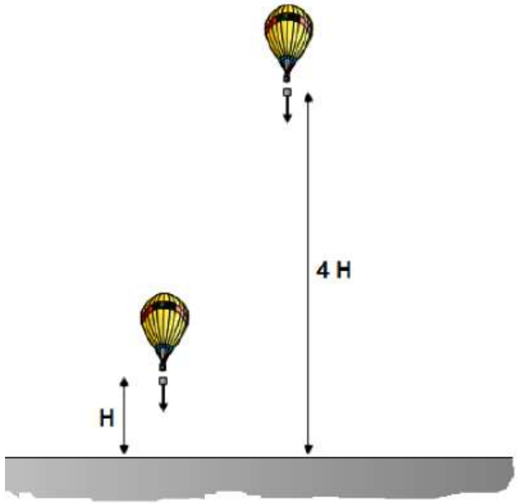
B v↑ ↓a

C v↑ ↑a

D v↓ ↓a

Answer

14 A package is dropped from an air balloon two times. In the first trial the distance between the balloon and the surface is H and in the second trial4H. Compare the time it takes for the package to reach the surface in the second trial to that in the first trial?



A The time in the second trial is four times greater.

B The time in the second trial is two times greater.

C The time in the second trial is four times less.

D The time in the second trial is two times less.

Answer

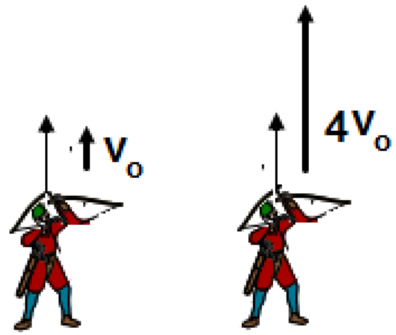
15 An archer practicing with an arrow bow shoots an arrow straight up two times. The first time the initial speed is v_0 and second time he increases the initial sped to $4v_0$. How would you compare the maximum height in the second trial to that in the first trial?

A Two times greater

B Four times greater

C Eight times greater

D Sixteen times greater



Answer

Vectors and Scalars

Scalar - a quantity that has only a magnitude (number or value)

Vector - a quantity that has both a magnitude *and a direction*

Which of the following are vectors? Scalars?

Quantity	Vector	Scalar
Time		
Distance		
Displacement		
Speed		

Answer

16 A runner runs halfway around a circular path of radius 10 m. What is the displacement of the runner?

A 0 m

B 10 m

C 20 m

D 31.4 m

Answer

17 A runner runs halfway around a circular path of radius 10 m. What is the total traveled distance of the jogger?

A 0 m

B 10 m

C 20 m

D 31.4 m

Answer

Adding Vectors in Two Dimensions

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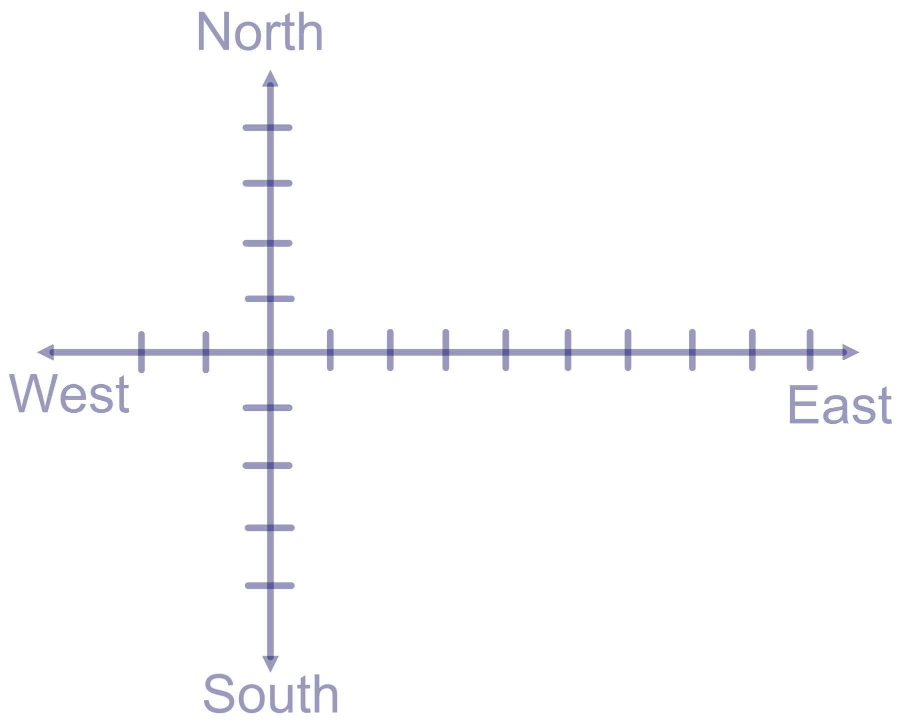
Adding Vectors

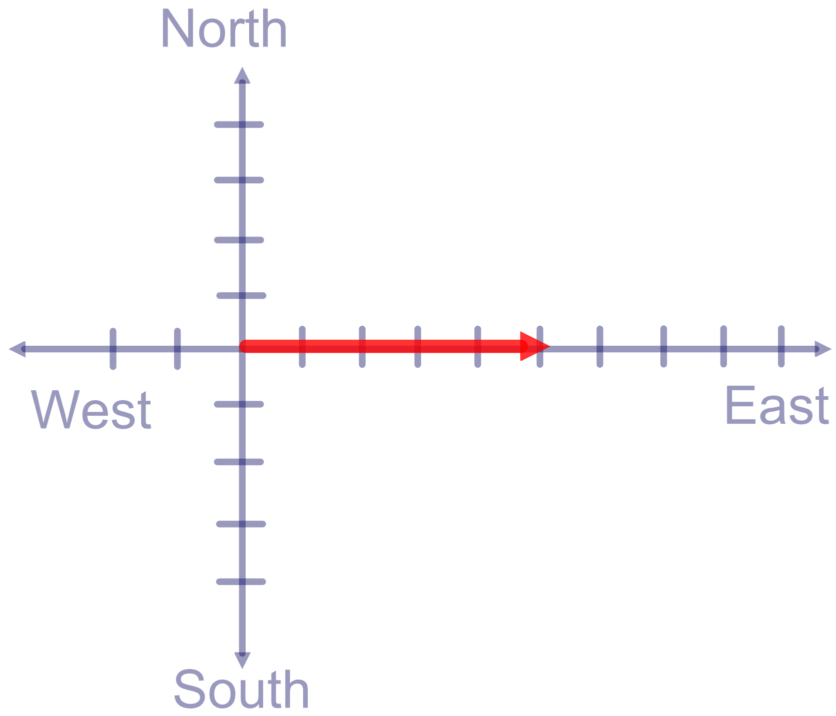
Last year, we learned how to add vectors along a single axis. The example we used was for adding two displacements.



Adding Vectors

Last year, we learned how to add vectors along a single axis. The example we used was for adding two displacements.

1. Draw the first vector, beginning at the origin, with its tail at the origin.

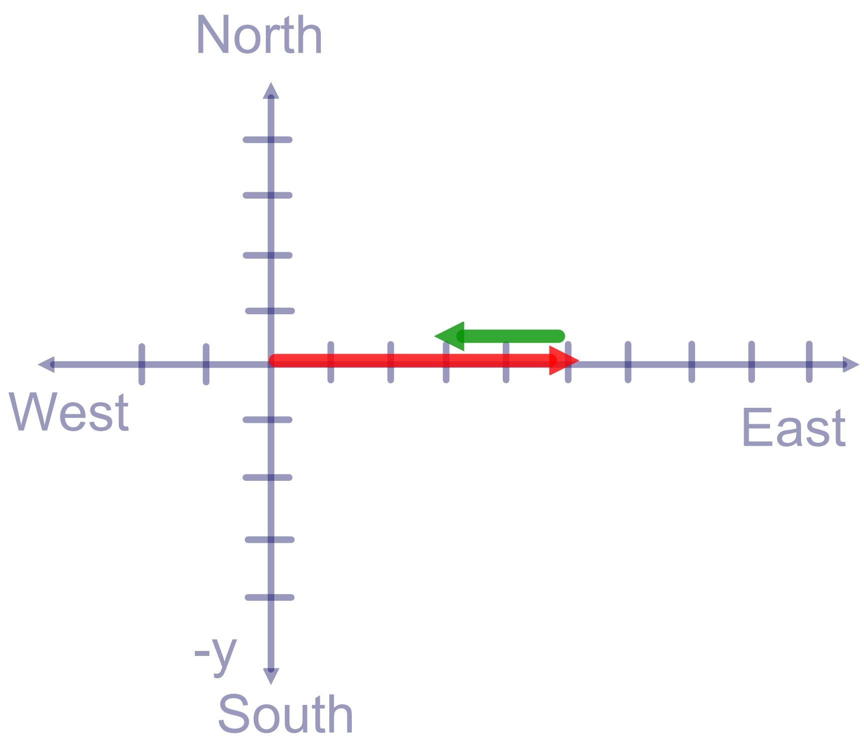


Adding Vectors

Last year, we learned how to add vectors along a single axis. The example we used was for adding two displacements.

1. Draw the first vector, beginning at the origin, with its tail at the origin.

2. Draw the second vector with its tail at the tip of the first vector.



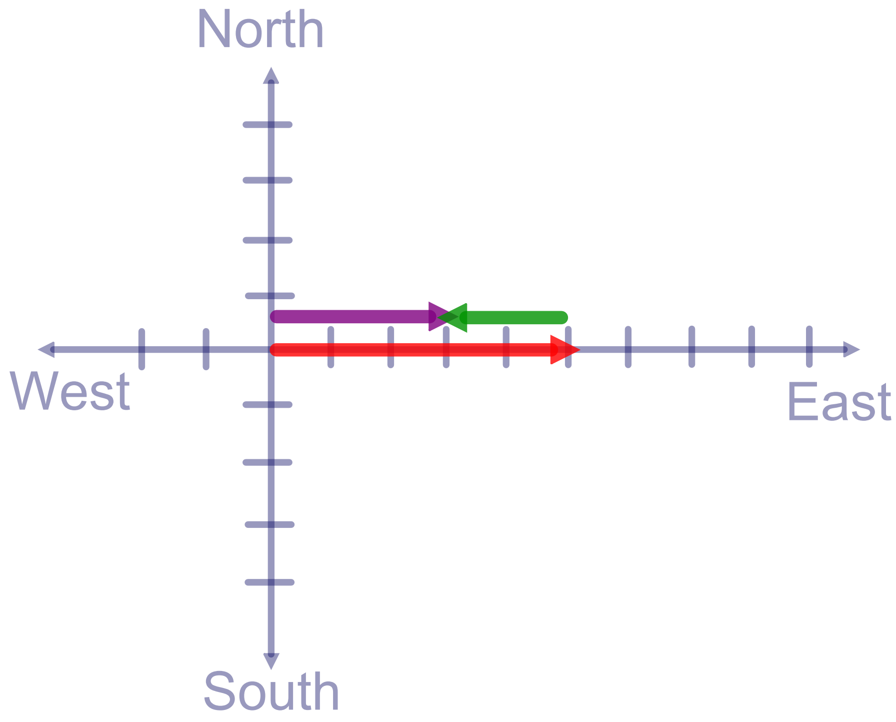
Adding Vectors

Last year, we learned how to add vectors along a single axis. The example we used was for adding two displacements.

1. Draw the first vector, beginning at the origin, with its tail at the origin.

2. Draw the second vector with its tail at the tip of the first vector.

3. Draw the Resultant (the answer) from the tail of the first vector to the tip of the last.



Adding Vectors

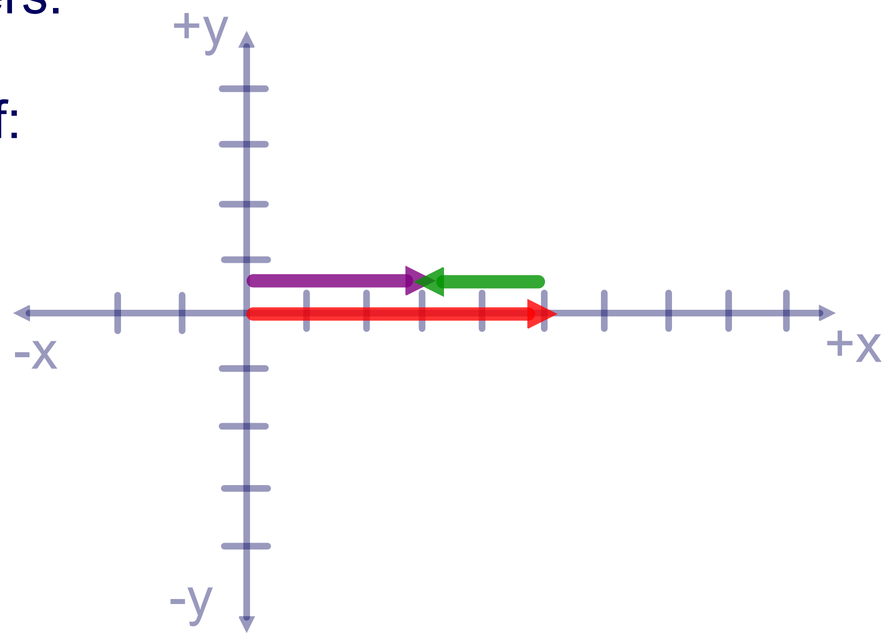
The direction of each vector matters.

In this first case, the vector sum of:

5 units to the East plus

2 units to West is

3 units to the East.



Adding Vectors

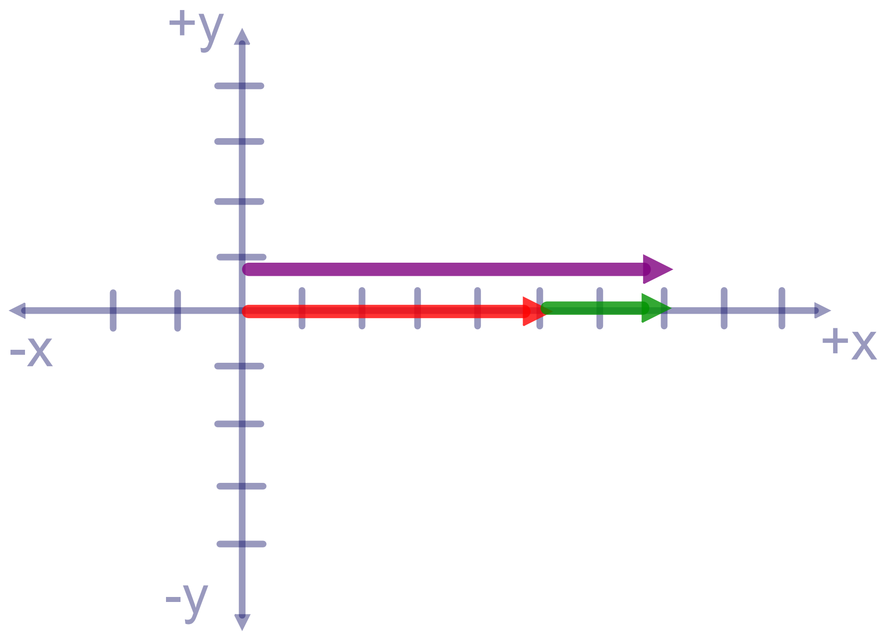
The direction of each vector matters. For instance, if the second vector had been 2 units to the EAST (not west), we get a different answer.

In this second case, the vector sum of:

5 units to the East plus

2 units to EAST is

7 units to the East .



Adding Vectors in 2-D

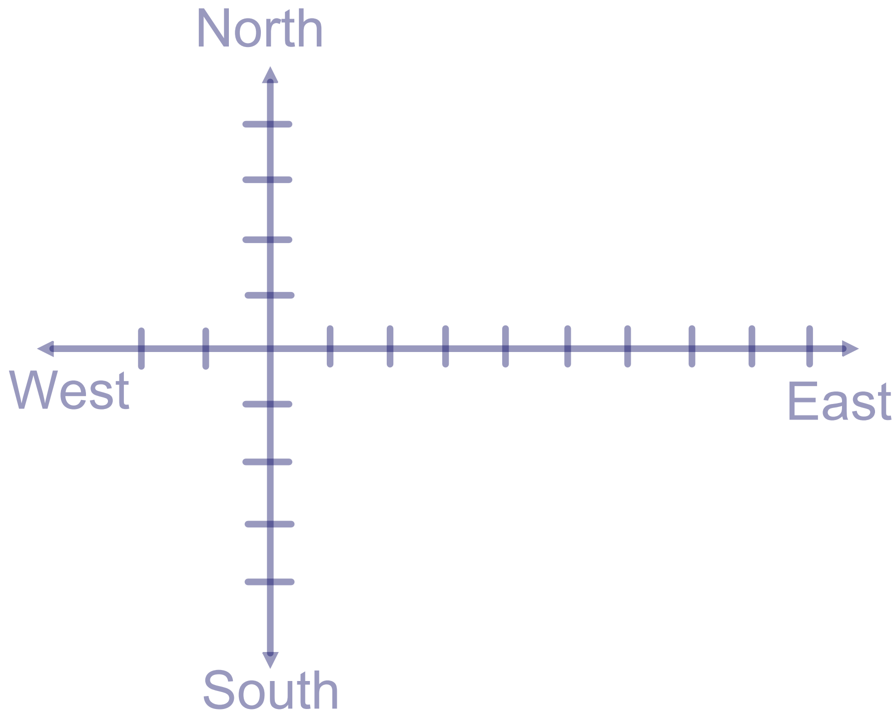
But how about if the vectors are along different axes.

For instance, let's add vectors of the same magnitude, but along different axes.

What is the vector sum of:

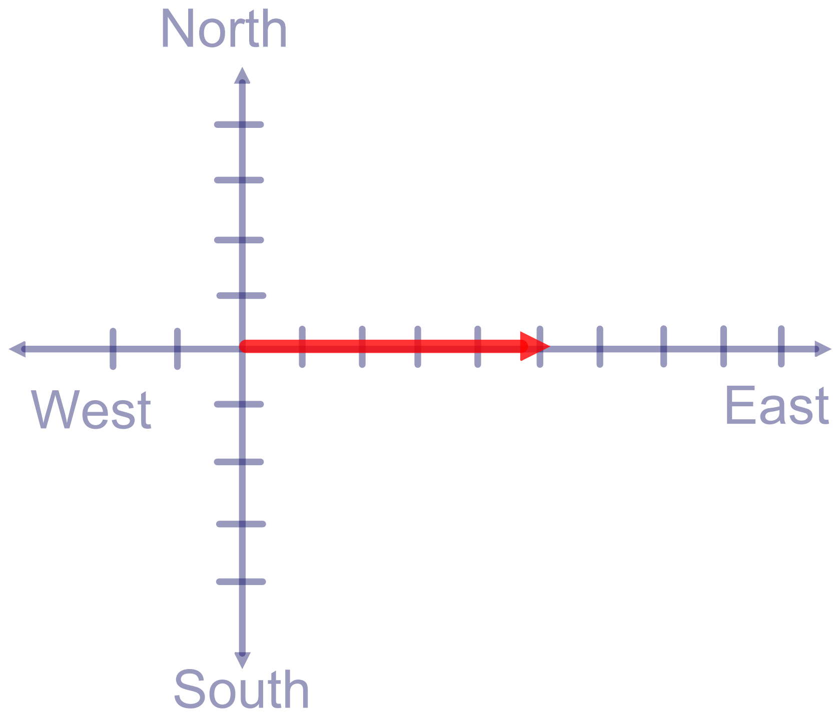
5 units East plus

2 units North



Adding Vectors

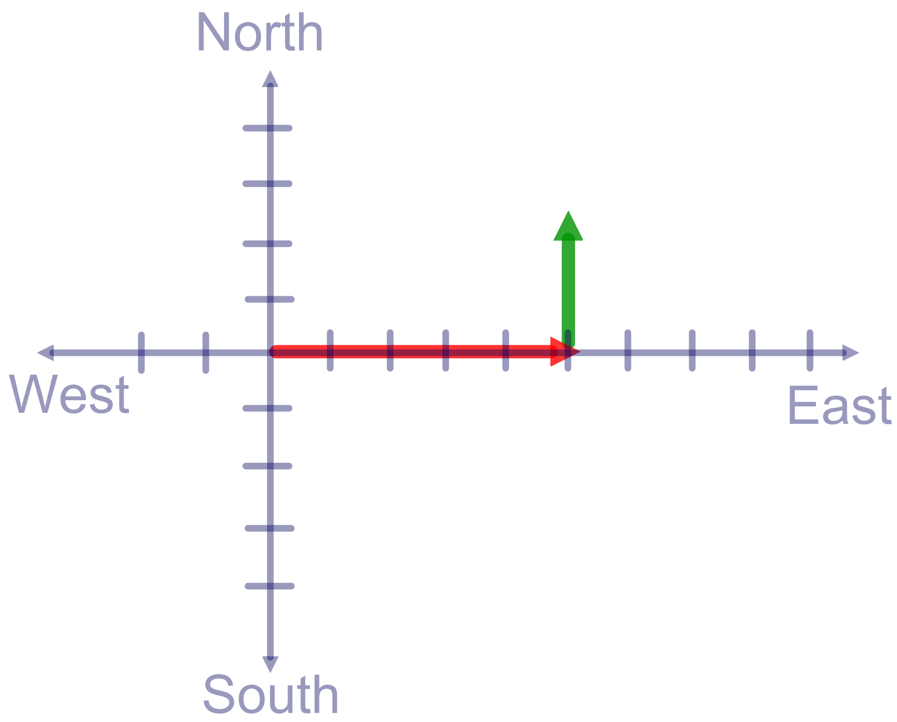
1. Draw the first vector, beginning at the origin, with its tail at the origin.



Adding Vectors

1. Draw the first vector, beginning at the origin, with its tail at the origin.

2. Draw the second vector with its tail at the tip of the first vector.

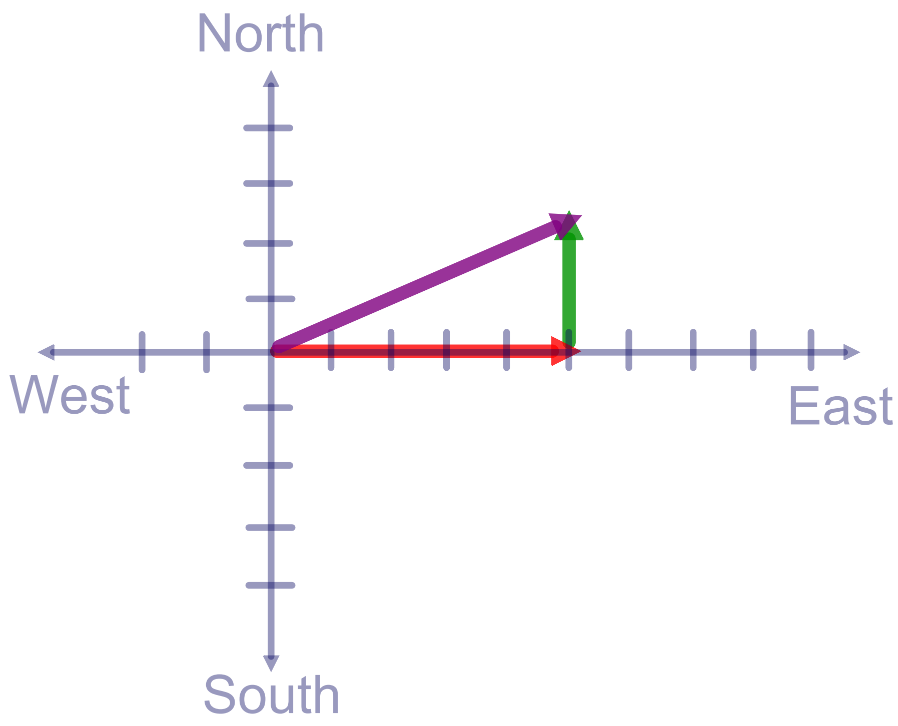


Adding Vectors

1. Draw the first vector, beginning at the origin, with its tail at the origin.

2. Draw the second vector with its tail at the tip of the first vector.

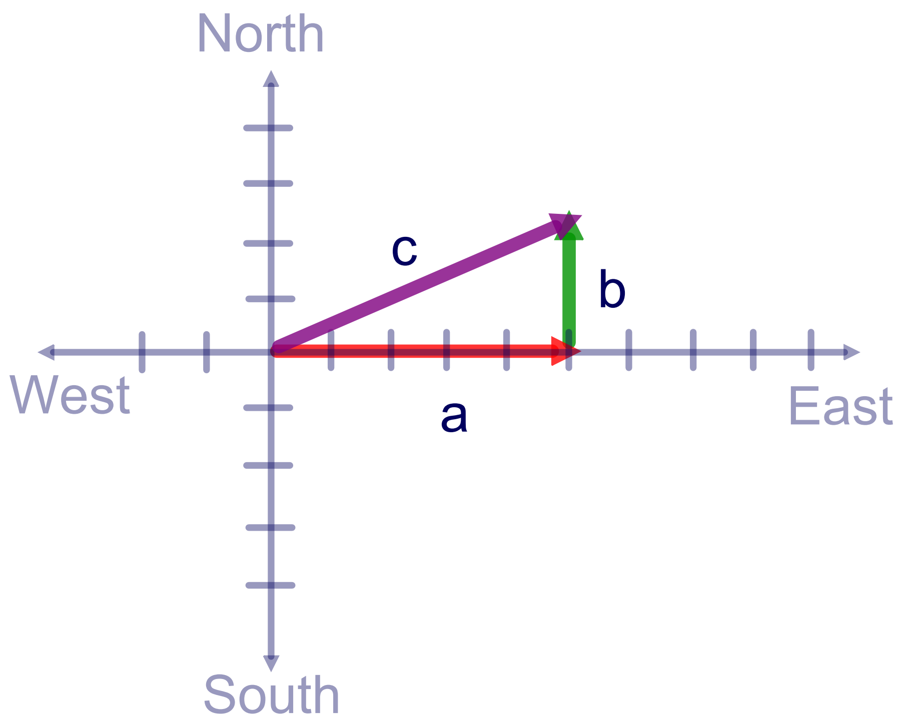
3. Draw the Resultant (the answer) from the tail of the first vector to the tip of the last.



Adding Vectors

Drawing the Resultant is the same as we did last year.

But calculating its *magnitude* and *direction* require the use of *right triangle mathematics.*



We know the length of both SIDES of the triangle (a and b), but we need to know the length of the HYPOTENUSE (c).

Magnitude of a Resultant

The magnitude of the resultant is equal to the length of the vector.

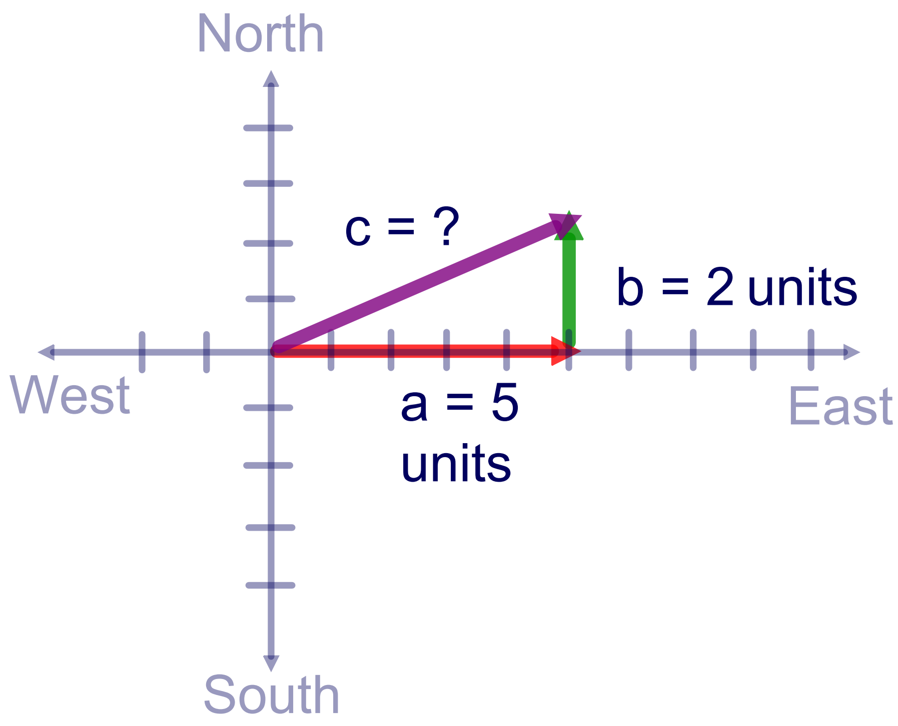
We get the magnitude of the resultant from the Pythagorean Theorem:

$c^2 = a^2 + b^2$

or in this case:

$R^2 = 5^2 + 2^2$
$R^2 = 25 + 4$
$R^2 = 29$

$R = \sqrt{(29)} = 5.4$ units



18 What is the magnitude of the Resultant of two vectors A and B, if A = 8.0 units north and B = 4.5 units east?

Answer

19 What is the magnitude of the Resultant of two vectors A and B, if A = 24.0 units east and B = 15.0 units south?

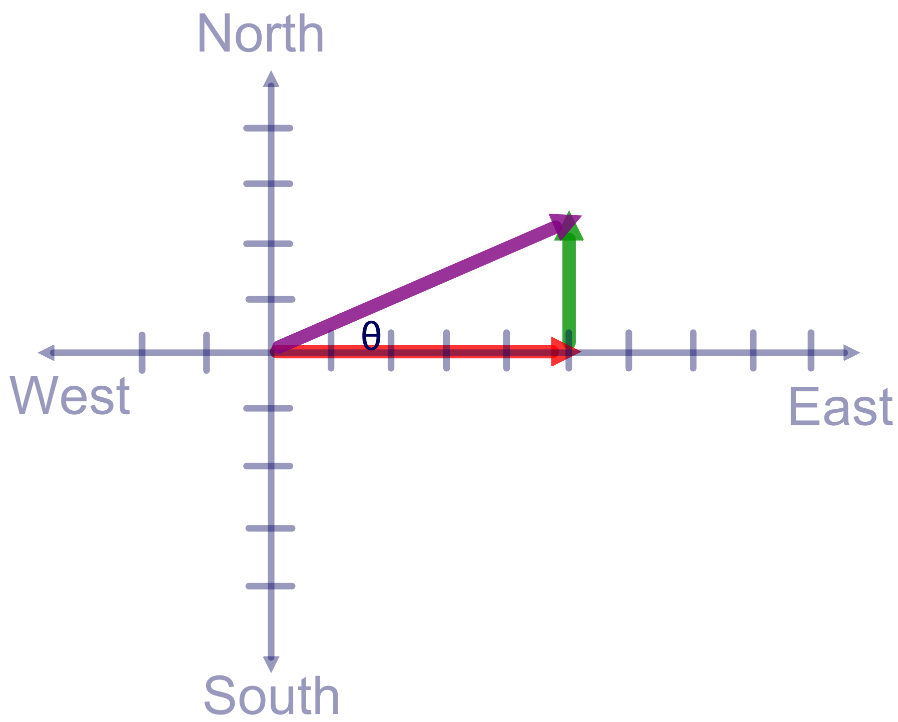
Answer

Adding Vectors

In physics, we say the direction of a vector is equal to the angle θ between a chosen axis and the resultant.

In Kinematics, we will primarily use the x-axis to measure θ.

However, if we were to change the axis we used and apply the proper mathematical techniques, we should get the same result!



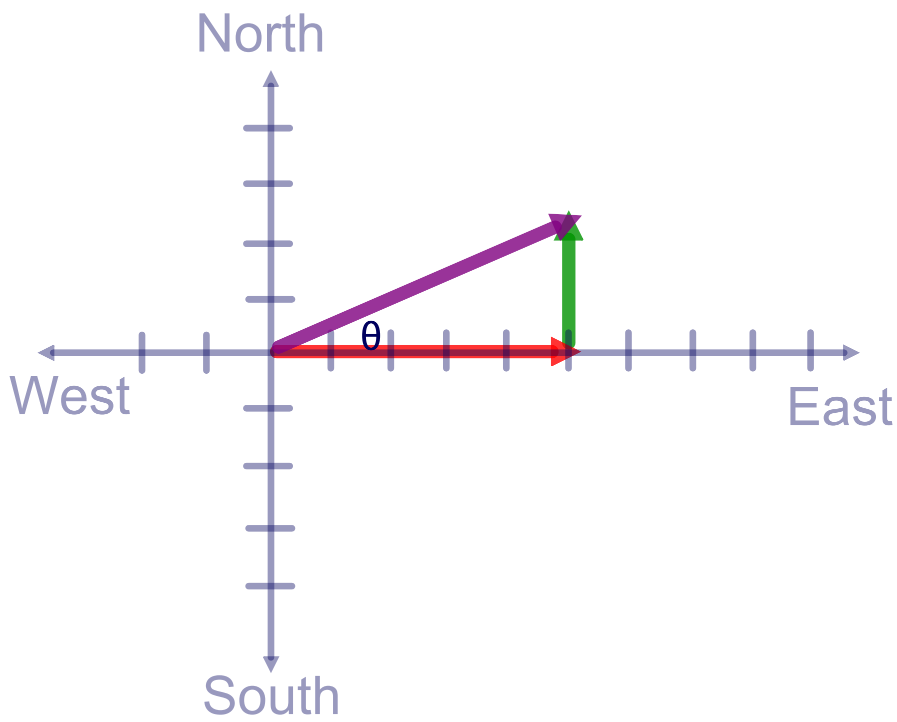
Adding Vectors

To find the value of the angle θ, we need to use what we already know:

the length of the two sides opposite and adjacent to the angle.

(remember SOH CAH TOA)

$$\tan\theta = \frac{opposite}{adjacent}$$



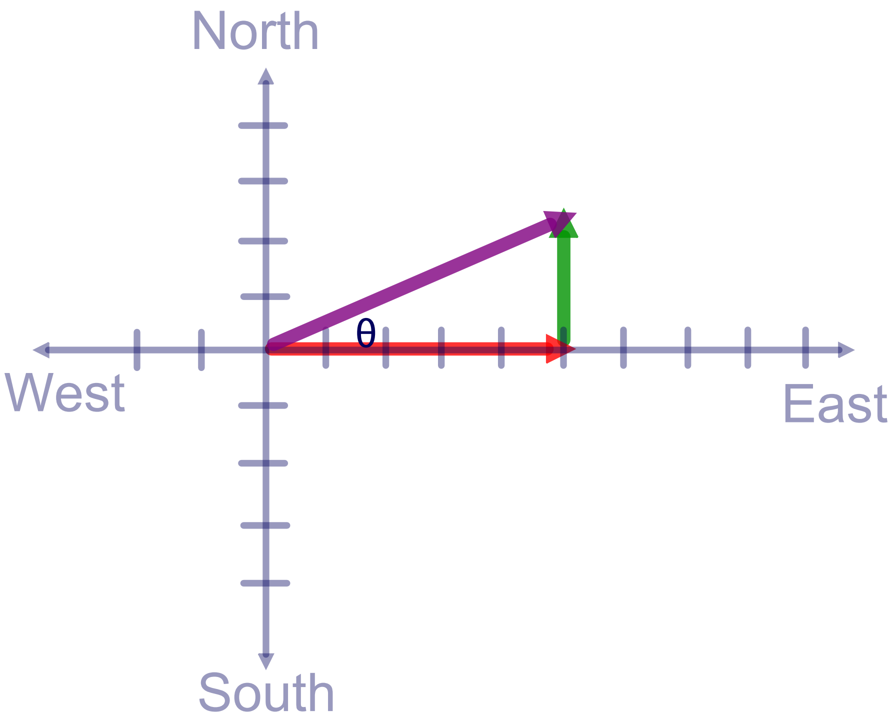
Adding Vectors

$\tan(\theta) = opp / adj$

$\tan(\theta) = (2 \text{ units}) / (5 \text{ units})$

$\tan(\theta) = 2/5$

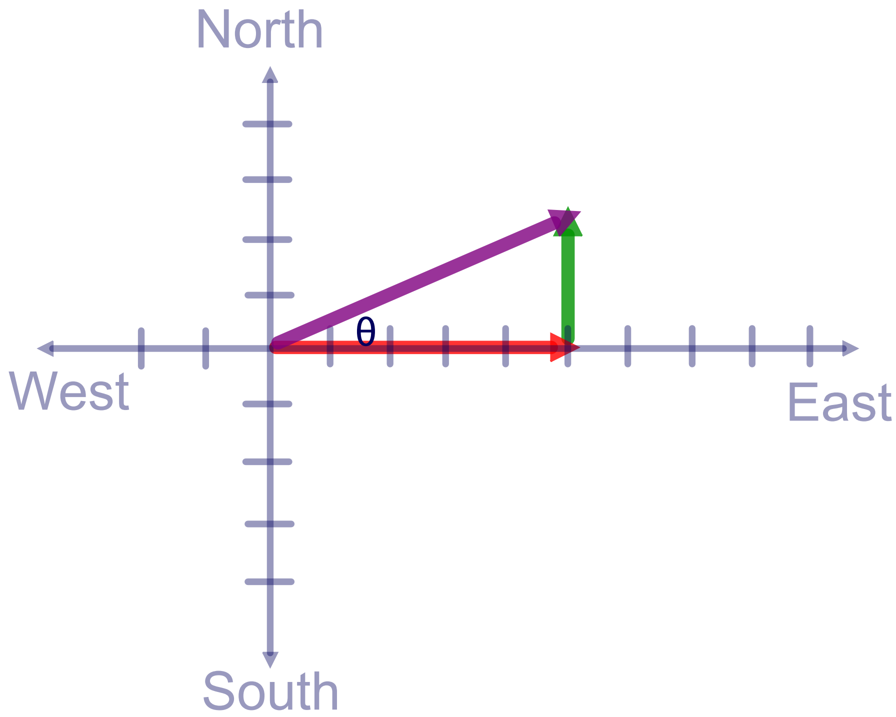
$\tan(\theta) = 0.40$



To find the value of θ, we must take the *inverse tangent:*

$\theta = \tan^{-1}(0.40) = 22^{0}$

Adding Vectors



The Resultant is 5.4 units in the direction of 22^{0} North of East

magnitude

direction

20 What is the direction of the Resultant of the two vectors A and B if: A = 8.0 units north and B = 4.5 units east if East is 0^o and North is 90^o?

Answer

21 What is the direction (from East) of the Resultant of the two vectors A and B if: A = 24.0 units east and B = 15.0 units south?

Answer

22 Find the magnitude and direction of the resultant of two vectors A and B if:

A = 400 units north
B = 250 units east

Magnitude = ?

Answer

23 Find the magnitude and direction of the resultant of two vectors A and B if:

A = 400 units north
B = 250 units east

Direction = ?

Answer

24 A student walks a distanceof 300 m East, then walks 400 m North. What is the magnitude of the net displacement?

A 300 m
B 400 m
C 500 m
D 700 m

Answer

25 A student walks a distance of 300 m East, then walks 400 m North. What is the total traveled distance?

A 300 m
B 400 m
C 500 m
D 700 m

Answer

26 Two displacement vectors have magnitudes of 5.0 m and 7.0 m, respectively. When these two vectors are added, the magnitude of the sum:

A is 2.0 m.

B could be as small as 2.0 m, or as large as 12 m.

C is 12 m.

D is larger than 12 m.

Answer

27 The resultant of two vectors is the largest when the angle between them is

A 0°
B 45°
C 90°
D 180°

Answer

28 The resultant of two vectors is the smallest when the angle between them is:

A $0°$
B $45°$
C $90°$
D $180°$

Answer

Basic Vector Operations

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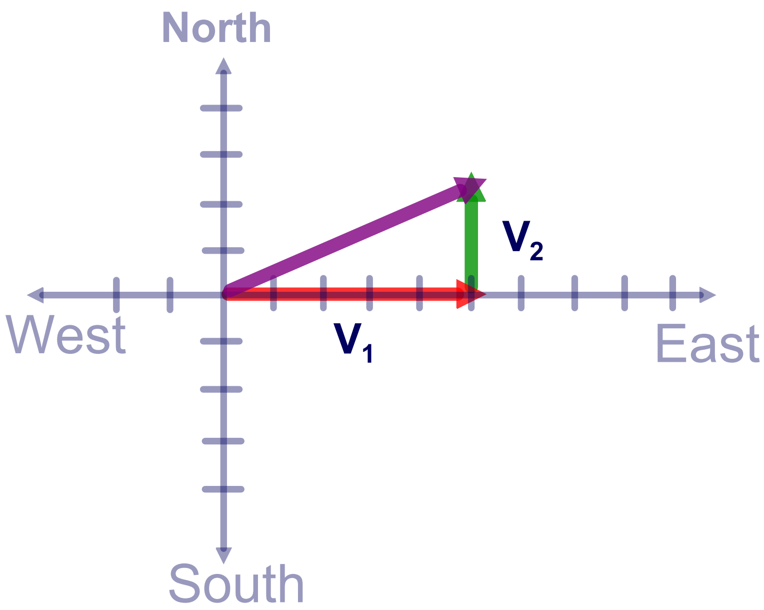
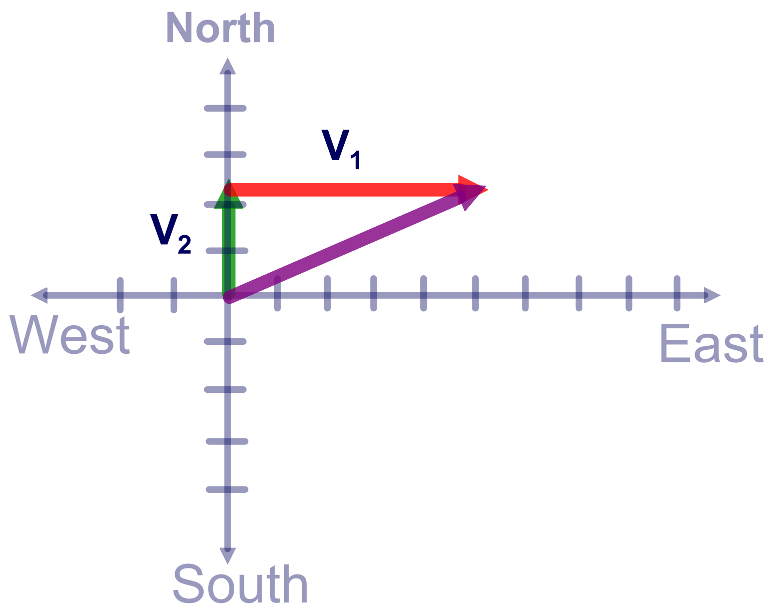
Adding Vectors

Adding Vectors in the opposite order gives the same resultant.

$$V_1 + V_2 = V_2 + V_1$$

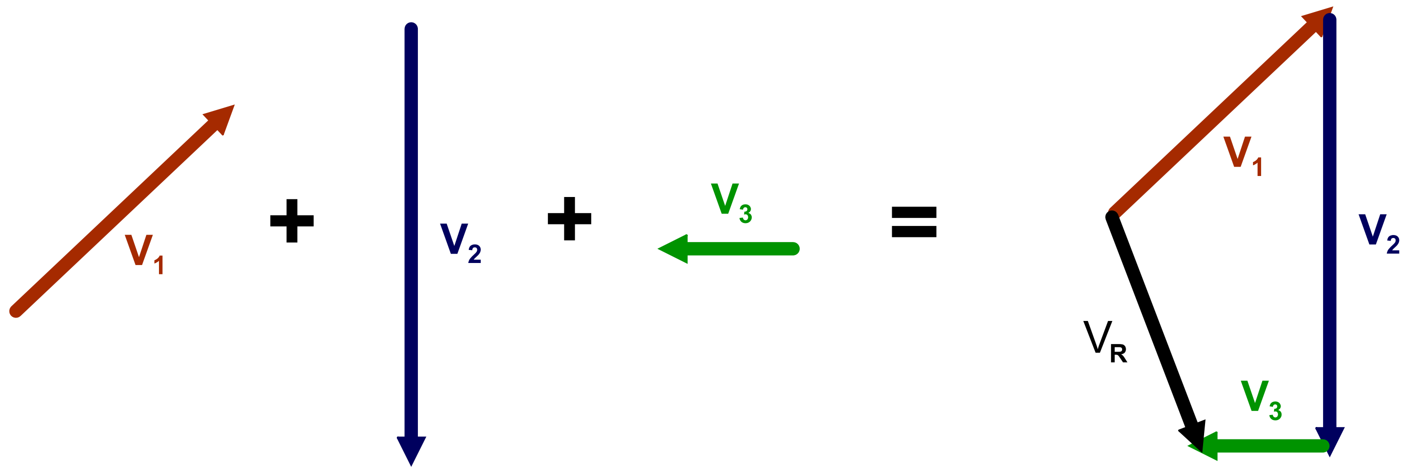




Adding Vectors

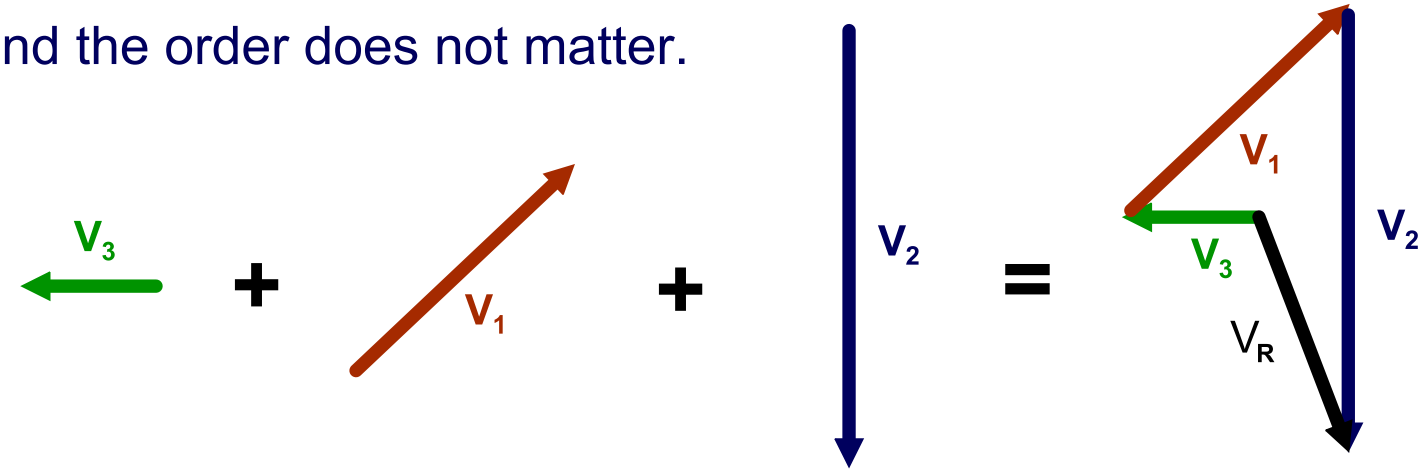
Even if the vectors are not at right angles, they can be added graphically by using the "tail to tip" method.

The resultant is drawn from the tail of the first vector to the tip of the last vector.

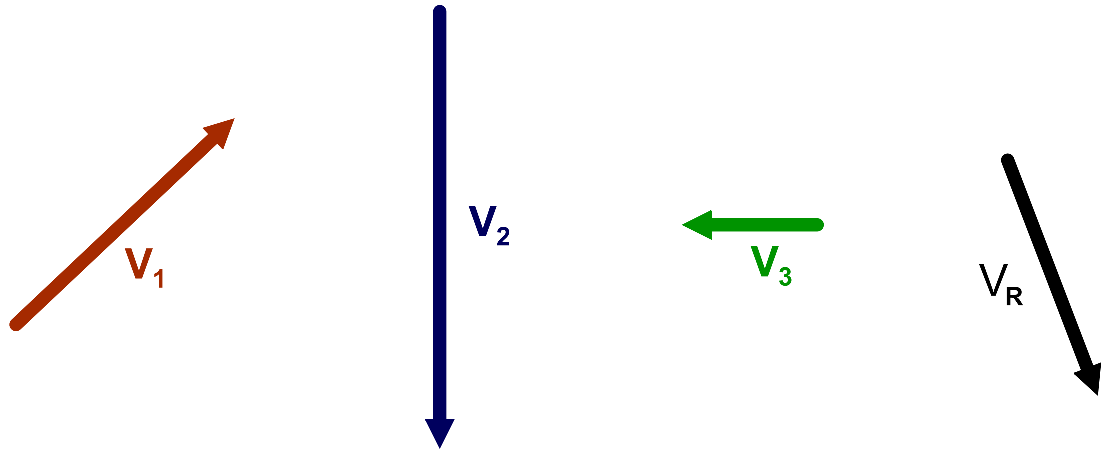


Adding Vectors

...and the order does not matter.

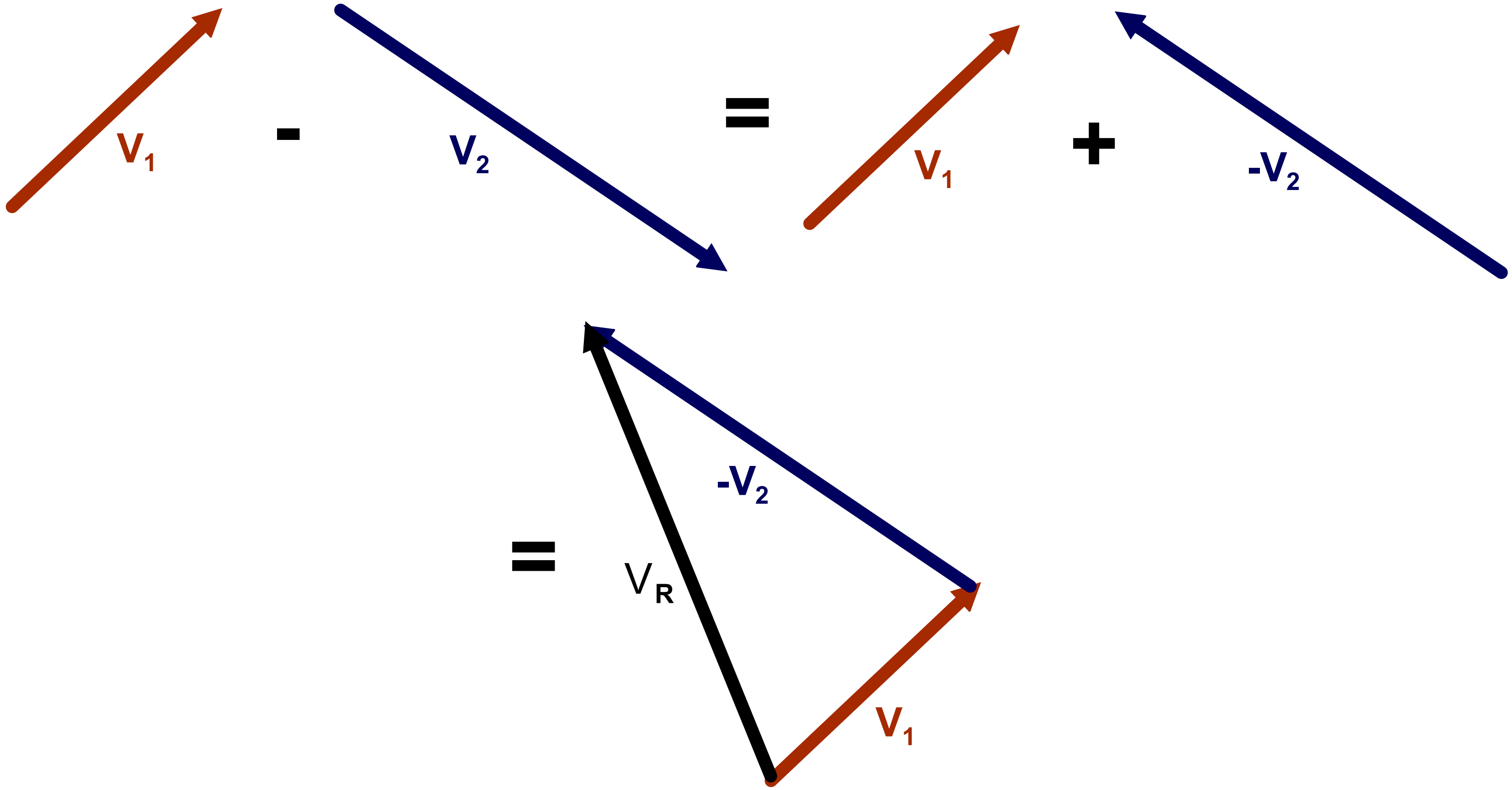


Try it.



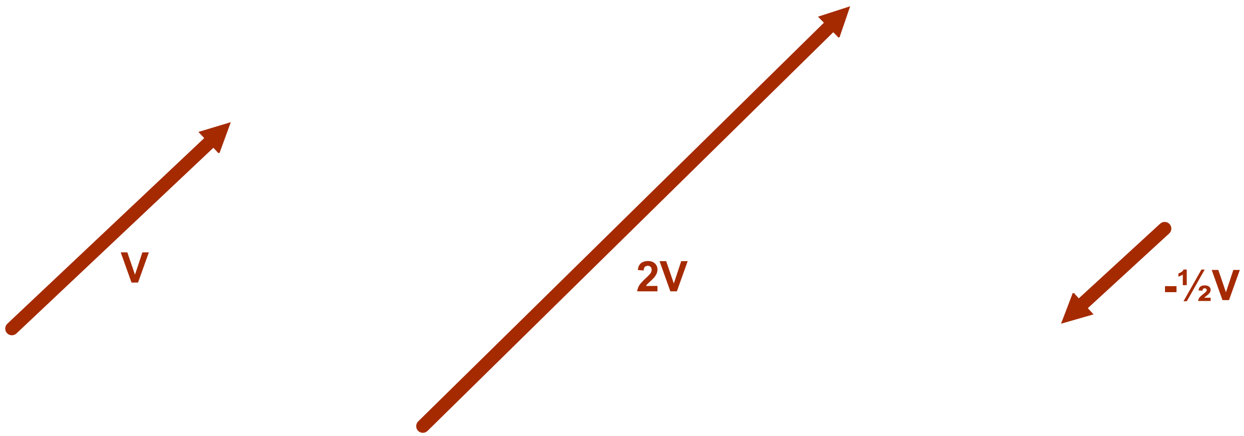
Subtracting Vectors

In order to subtract a vector, we add the negative of that vector. The negative of a vector is defined as that vector in the opposite direction.



Multiplication of Vectors by Scalars

A vector V can be multiplied by a scalar c. The result is a vector cV which has the same direction as V. However, if c is negative, it changes the direction of the vector.



29 Which of the following operations will not changea vector?

- A Translate it parallel to itself
- B Rotate it
- C Multiply it by a constant factor
- D Add a constant vector to it

Answer

Vector Components

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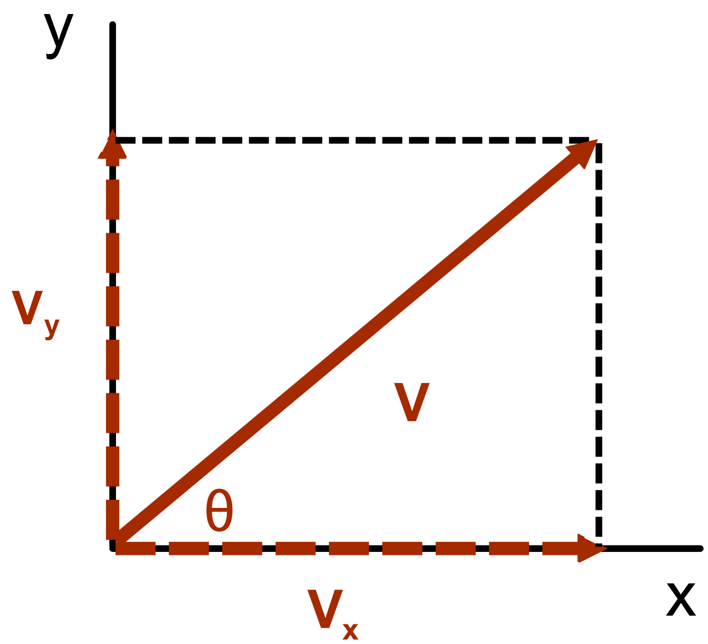
Adding Vectors by Components

Any vector can be described as the sum as two other vectors called components. These components are chosen perpendicular to each other and can be found using trigonometric functions.



$$\sin\theta = \frac{V_y}{V}$$

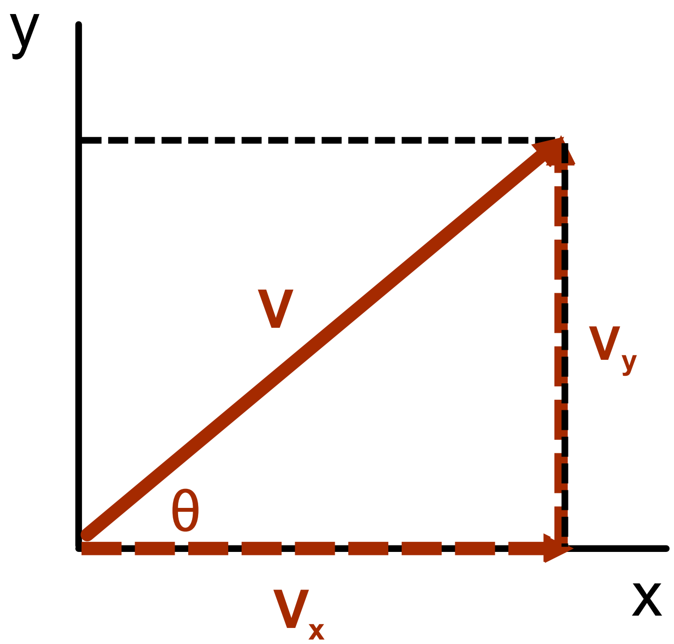
$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^{\ 2} + V_y^{\ 2}$$

Adding Vectors by Components

In order to remember the right triangle properties and to better identify the functions, it is often convenient to show these components in different arrangements (notice v_y below).



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^{\ 2} + V_y^{\ 2}$$

30 If a ball is thrown with a velocity of 25 m/s at an angle of 37° above the horizontal, what is the vertical component of the velocity?

A 12 m/s

B 15 m/s

C 20 m/s

D 25 m/s

Answer

31 If a ball is thrown with a velocity of 25 m/s at an angle of 37° above the horizontal, what is the horizontal component of the velocity?

A 12 m/s

B 15 m/s

C 20 m/s

D 25 m/s

Answer

32 If you walk 6.0 km in a straight line in a direction north of east and you end up 2.0 km north and several kilometers east. How many degrees north of east have you walked?

A 19°

B 45°

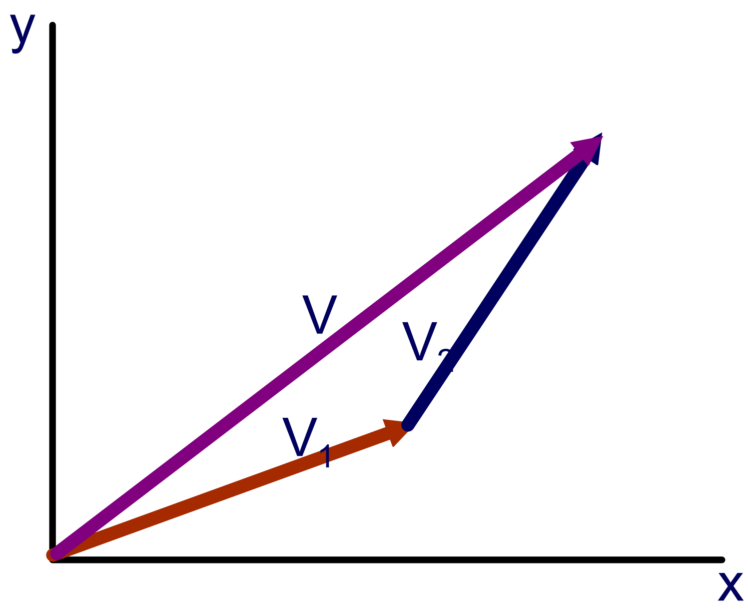
C 60°

D 71°

Answer

Adding Vectors by Components

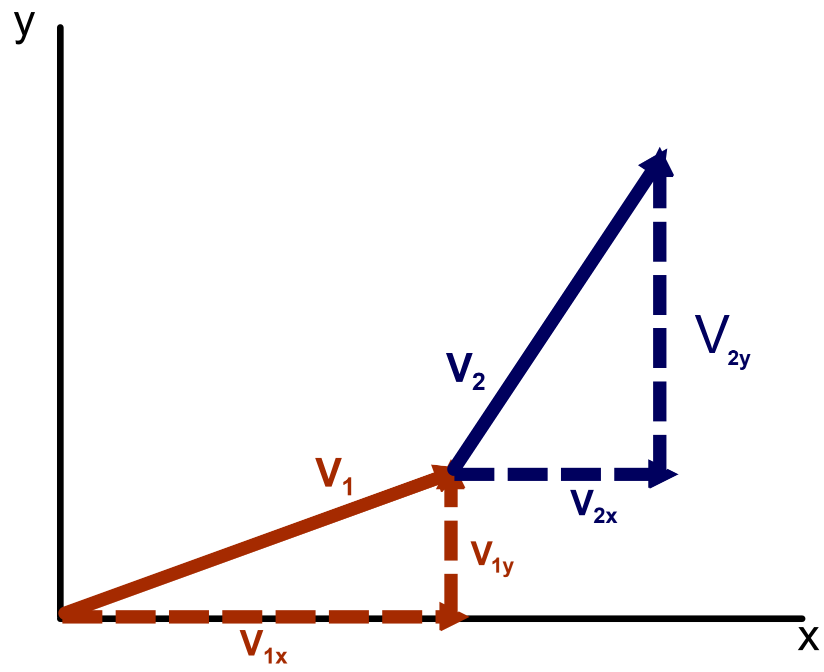
Using the tip-to-tail method, we can sketch the resultant of any two vectors.



But we cannot find the magnitude of 'v', the resultant, since v_1 and v_2 are two-dimensional vectors

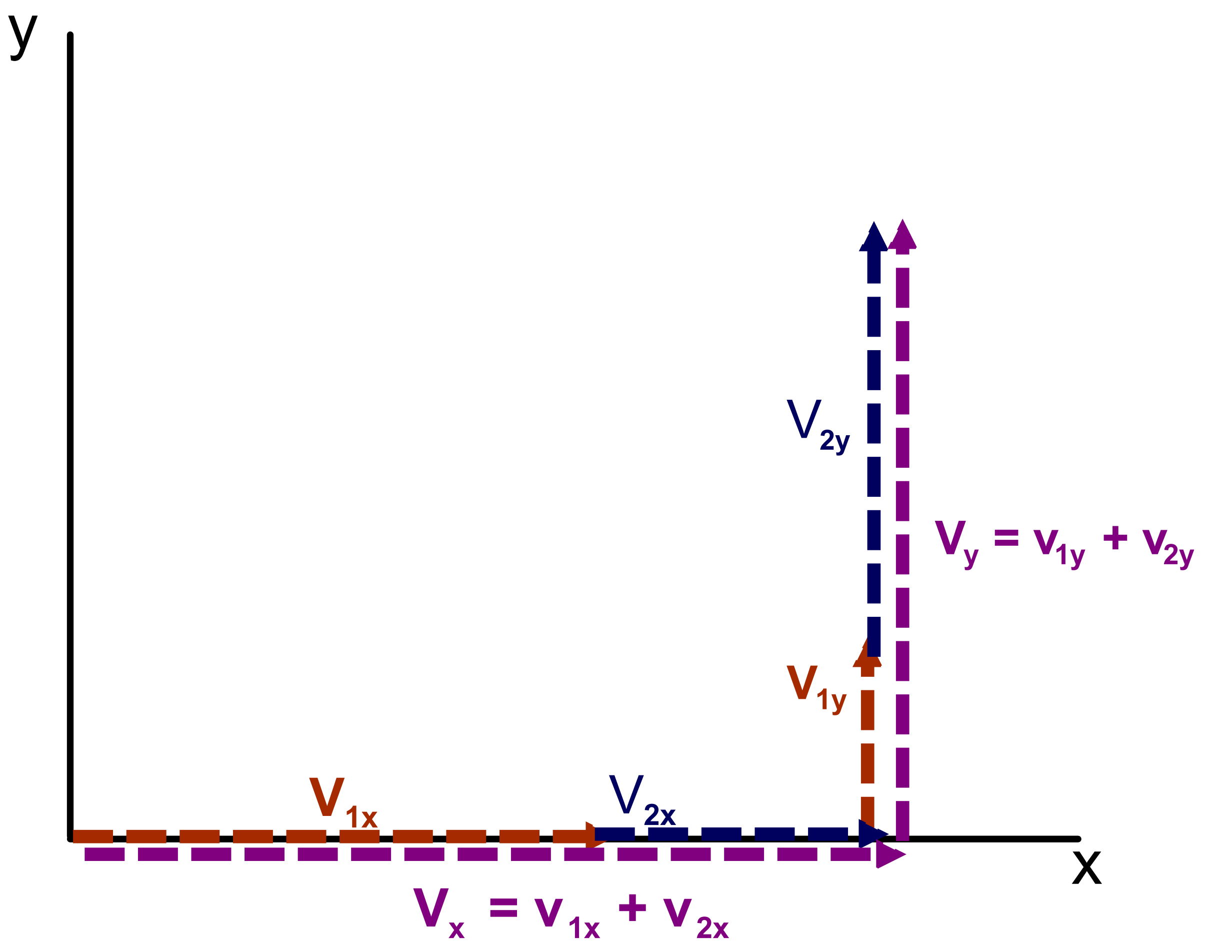
Adding Vectors by Components

We now know how to break v_1 and v_2 into components...

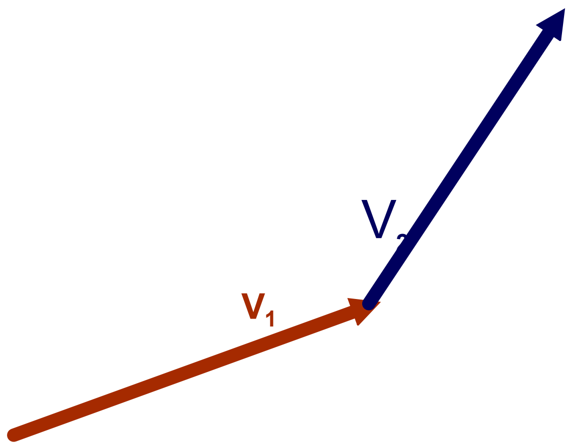


Adding Vectors by Components

And since the x and y components are one dimensional, they can be added as such.

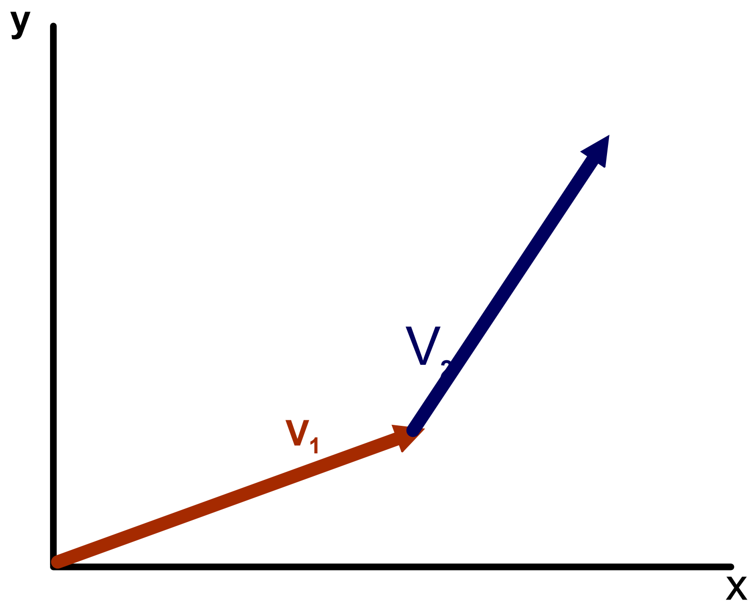


Adding Vectors by Components



1. Draw a diagram and add the vectors graphically.

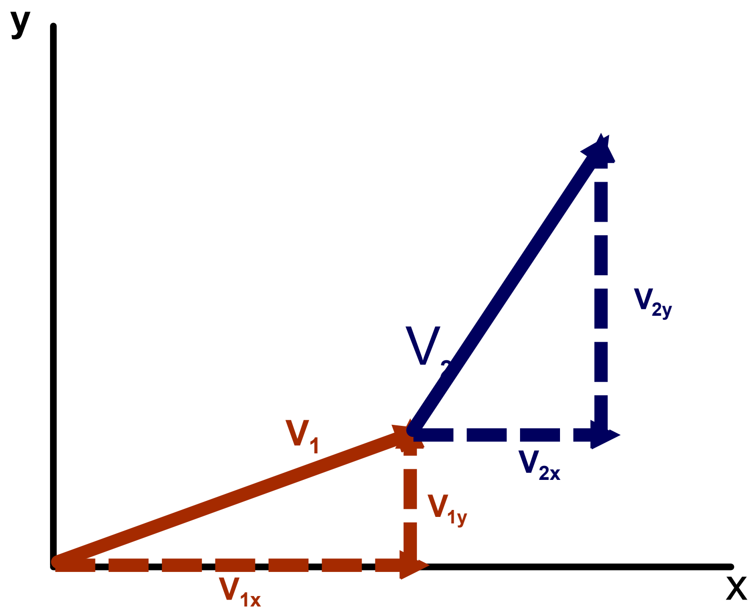
Adding Vectors by Components



1. Draw a diagram and add the vectors graphically.

2. Choose x and y axes.

Adding Vectors by Components

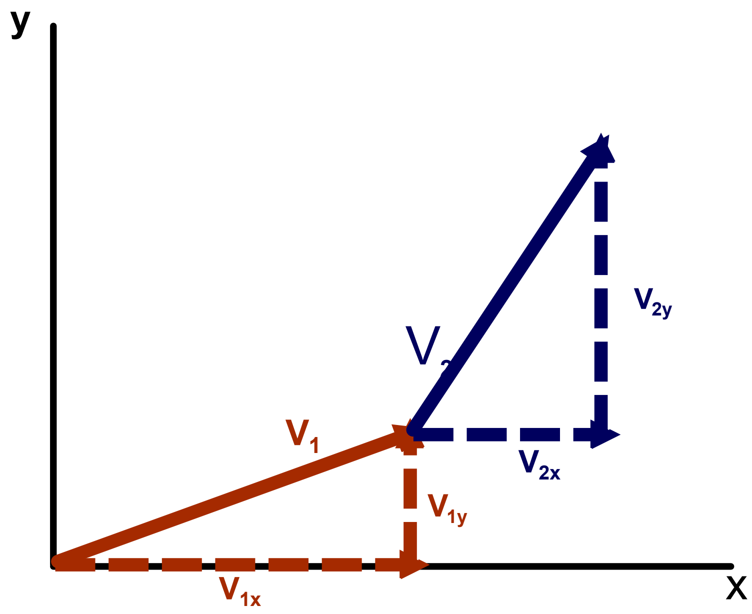


1. Draw a diagram and add the vectors graphically.

2. Choose x and y axes.

3. Resolve each vector into x and y components.

Adding Vectors by Components

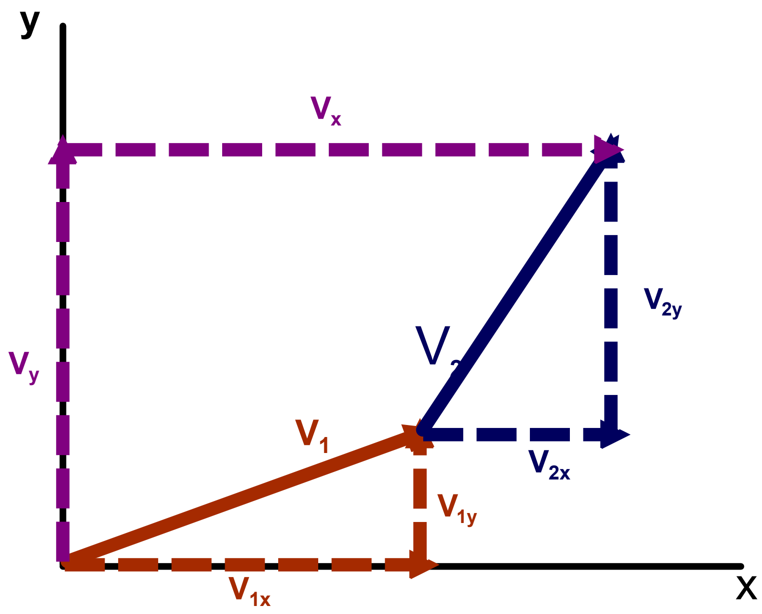


1. Draw a diagram and add the vectors graphically.
2. Choose x and y axes.
3. Resolve each vector into x and y components.
4. Calculate each component.

$v_{1x} = v_1 \cos(\theta_1)$ $\quad$ $v_{2x} = v_2 \cos(\theta_2)$

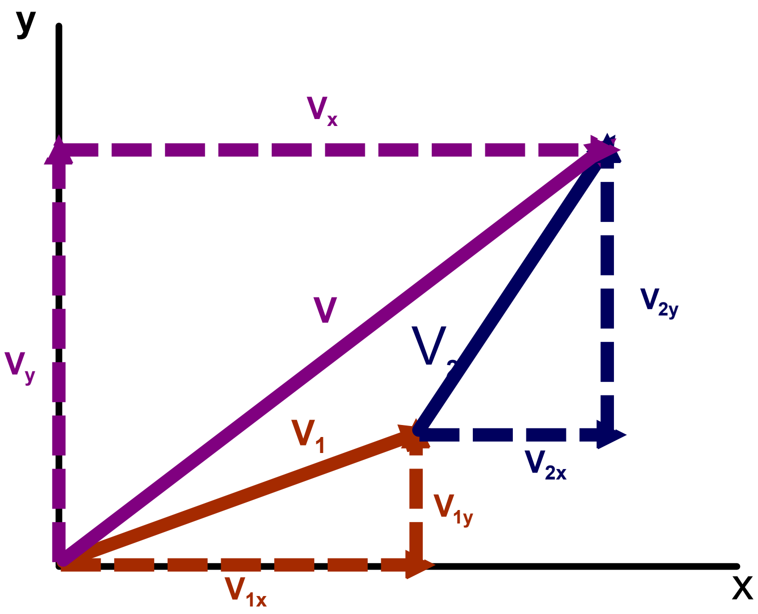
$v_{1y} = v_1 \sin(\theta_1)$ $\quad$ $v_{2y} = v_2 \sin(\theta_2)$

Adding Vectors by Components



1. Draw a diagram and add the vectors graphically.
2. Choose x and y axes.
3. Resolve each vector into x and y components.
4. Calculate each component.
5. Add the components in each direction.

Adding Vectors by Components



1. Draw a diagram and add the vectors graphically.

2. Choose x and y axes.

3. Resolve each vector into x and y components.

4. Calculate each component.

5. Add the components in each direction.

6. Find the length and direction of the resultant vector.

Example:

Graphically determine the resultant of the following three vector displacements:

1. 24m, 30º north of east

2. 28m, 37º east of north

3. 20m, 50º west of south

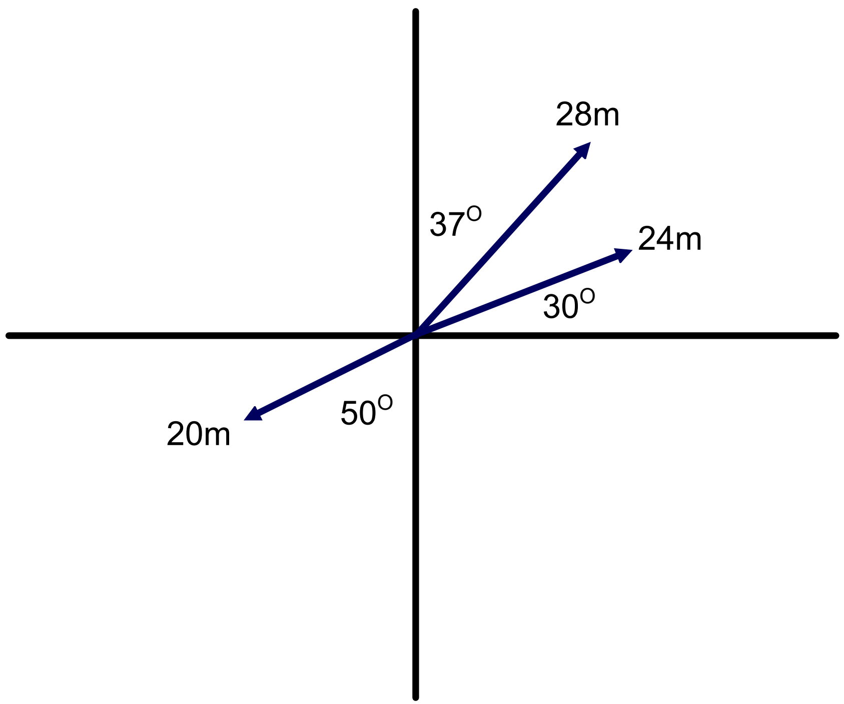
Example:

Graphically determine the resultant of the following three vector displacements:

1. 24m, 30º north of east

2. 28m, 37º east of north

3. 20m, 50º west of south

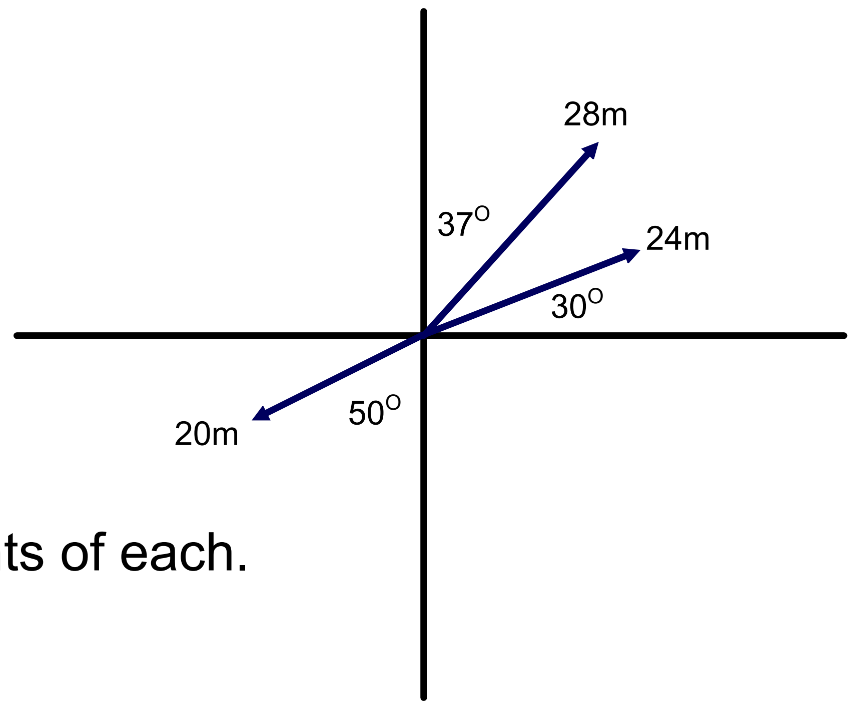


First, draw the vectors.

Example:

Graphically determine the resultant of the following three vector displacements:

1. 24m, 30º north of east
2. 28m, 37º east of north
3. 20m, 50º west of south



Next, find the x and y components of each.

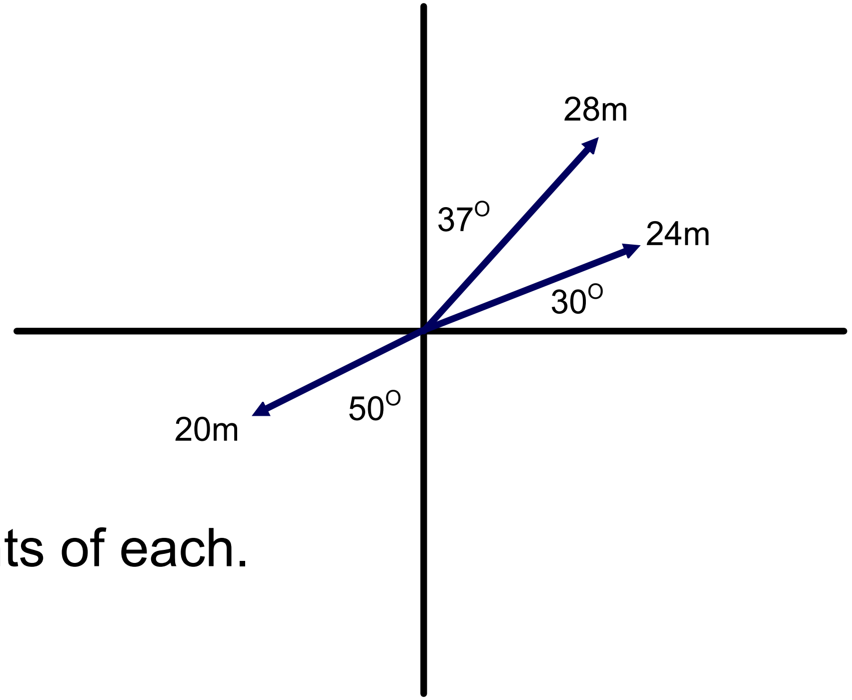
$$d_{1x} = 24m\cos 30^\circ = 20.7m$$

$$d_{1y} = 24m\sin 30^\circ = 12m$$

Example:

Graphically determine the resultant of the following three vector displacements:

1. 24m, 30º north of east
2. 28m, 37º east of north
3. 20m, 50º west of south



Next, find the x and y components of each.

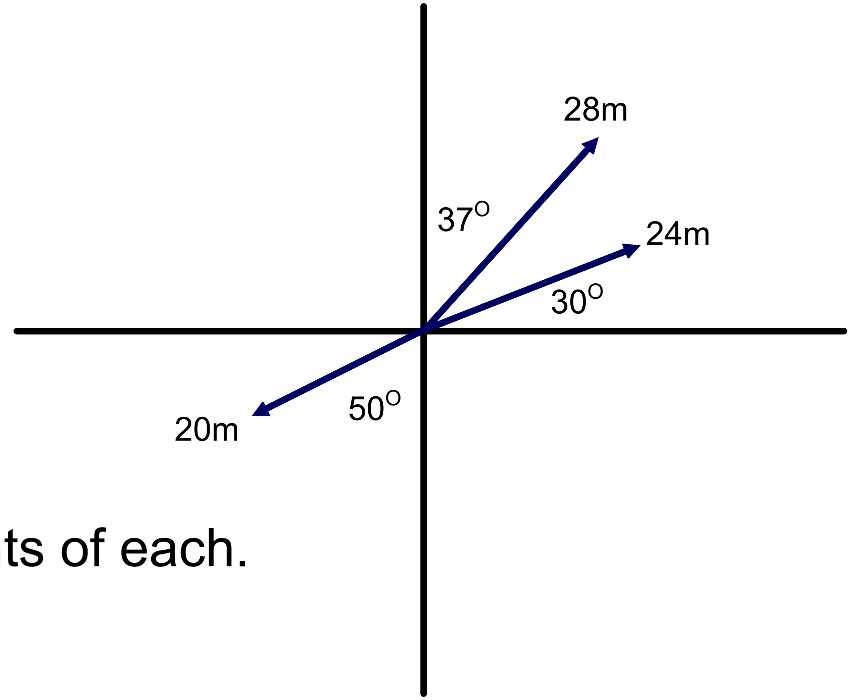
$$d_{2x} = 28m \sin 37° = 16.9m$$

$$d_{2y} = 28m \cos 37° = 22.4m$$

Example:

Graphically determine the resultant of the following three vector displacements:

1. 24m, 30º north of east
2. 28m, 37º east of north
3. 20m, 50º west of south



Next, find the x and y components of each.

$$d_{3x} = 20m\sin 50° = -15.3m$$

$$d_{3y} = 20m\cos 50° = -12.9m$$

Note: these are both negative because south and west are negative directions.

Now we can put the components in a chart and solve for the resultant vector.

	x (m)	y (m)
d_1	20.8	12.0
d_2	16.9	22.4
d_3	-15.3	-12.9
Σ	22.4	21.5

To find the magnitude of the resultant, use the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

$$c = \sqrt{(22.4m)^2 + (21.5m)^2} = 31m$$

To find the direction of the resultant, use inverse tangent.

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\theta = \tan^{-1}\frac{21.5}{22.4} = 43.8°$$

33 Graphically determine the magnitude and direction of the resultant of the following three vector displacements:

1. 15 m, 30° north of east
2. 20 m, 37° north of east
3. 25 m, 45° north of east

Magnitude = ?

Answer

34 Graphically determine the magnitude and direction of the resultant of the following three vector displacements:

1. 15 m, 30º north of east
2. 20 m, 37º north of east
3. 25 m, 45º north of east

Direction = ?

Answer

35 Which of the following is an accurate statement?

A. A vector cannot have a magnitude of zero if one of its components is not zero.

B. The magnitude of a vector can be equal to less than the magnitude of one of its components.

C. If the magnitude of vector A is less than the magnitude of vector B, then the x-component of A must be less than the x-component of B.

D. The magnitude of a vector can be either positive or negative.

Answer

Projectile Motion

Demo

Demo

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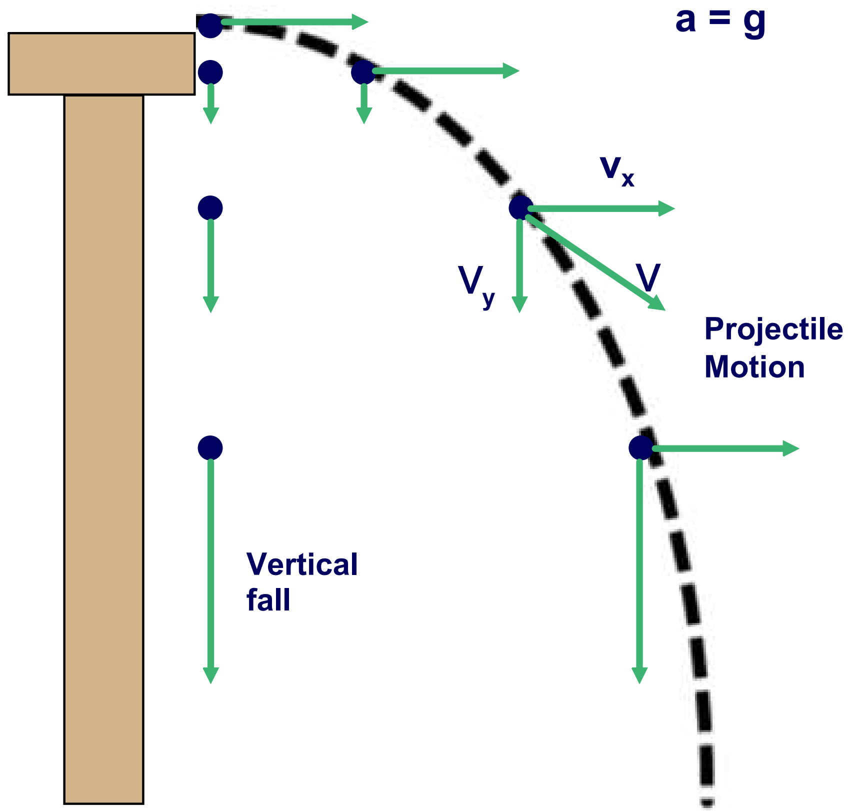
Projectile Motion: 1st Type

A projectile is an object moving in two dimensions under the influence of Earth's gravity. Its path is a parabola.

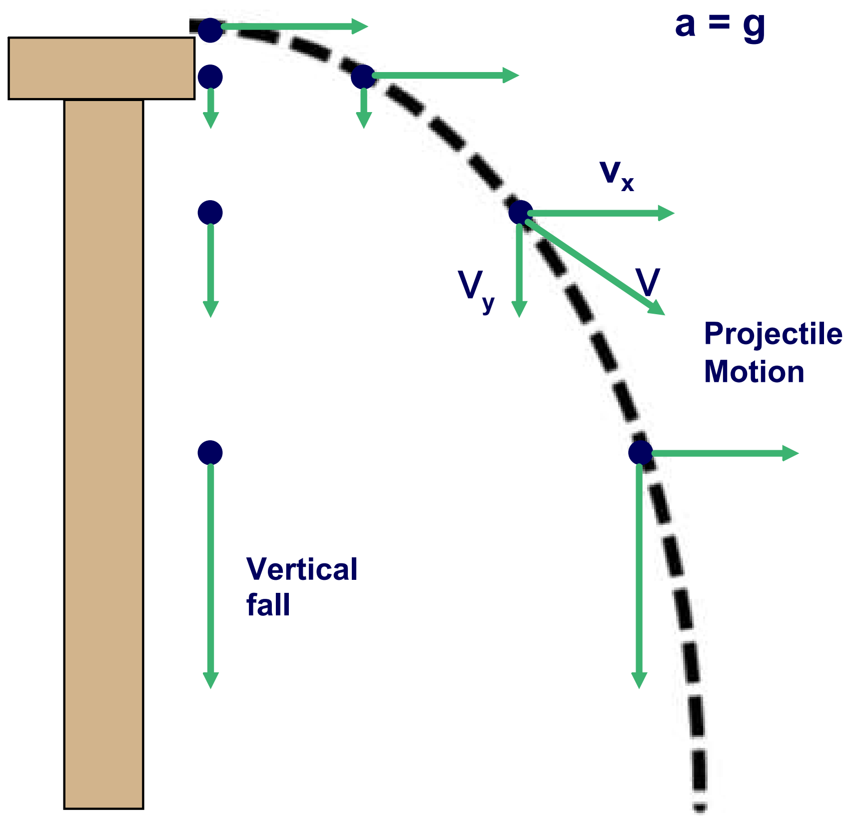


Projectile Motion: 1st Type

Projectile motion can be understood by analyzing the vertical and horizontal motion separately.

The speed in the x-direction is constant.

The speed in the y-direction is changing.



A mountain lion leaps horizontally from a 7.5 m high rock with a speed of 4.5 m/s. How far from the base of the rock will he land?

A mountain lion leaps horizontally from a 7.5 m high rock with a speed of 4.5 m/s. How far from the base of the rock will he land?

First, determine the time it will take for the lion to reach the ground.

$$v_{0y} = 0$$

$$a_y = -9.8\frac{m}{s^2}$$

$$y_0 = 7.5m$$

$$y = 0$$

$$t = ?$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = y_0 + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{-2y_0}{a_y}} = \sqrt{\frac{-2(7.5m)}{\left(-9.8\frac{m}{s^2}\right)}} = 1.24s$$

A mountain lion leaps horizontally from a 7.5 m high rock with a speed of 4.5 m/s. How far from the base of the rock will he land?

Then, determine how far from the base he will land.

$$v_{0x} = 4.5\frac{m}{s}$$

$$a_x = 0$$

$$x_0 = 0$$

$$x = ?$$

$$t = 1.24s$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = v_{0x}t = \left(4.5\frac{m}{s}\right)(1.24s) = 5.6m$$

36 A cannon ball is shot from a cannon at a height of 15 m with a velocity of 20 m/s. How far away will the cannon ball land?

Answer

37 A marble rolls off a table from a height of 0.8 m with a velocity of 3 m/s. Then another marble rolls off the same table with a velocity of 4 m/s. Which values are the same for both marbles?

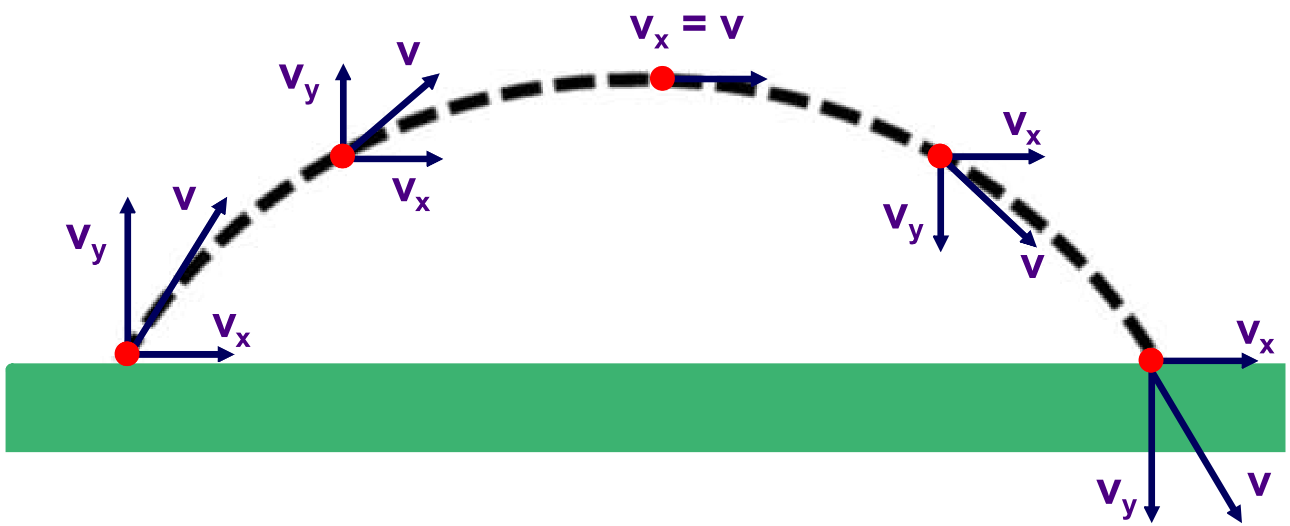
Justify your answer qualitatively, with no equations or calculations.

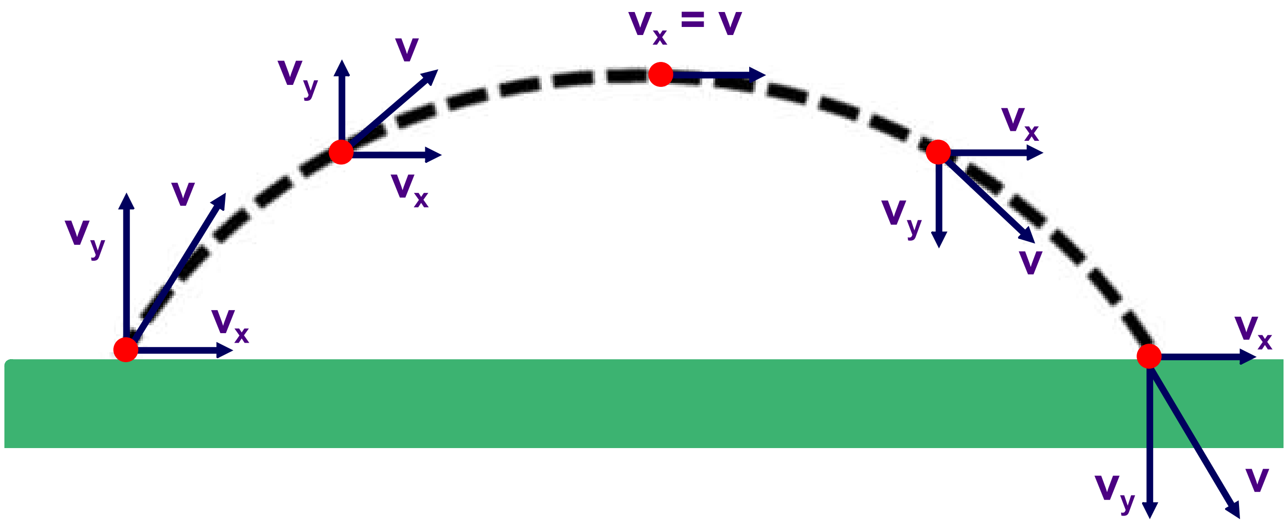
A The final speeds of the marbles.
B The time each takes to reach the ground.
C The distance from the base of the table where each lan

Answer

Projectile Motion: 2nd Type

If an object is launched at an angle with the horizontal, the analysis is similar except that the initial velocity has a vertical component.





Projectile motion can be described by two kinematics equations:

horizontal component:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = v_0 \cos\theta t$$

$$v_x = v_{0x}t + a_x t = v_0 \cos\theta$$

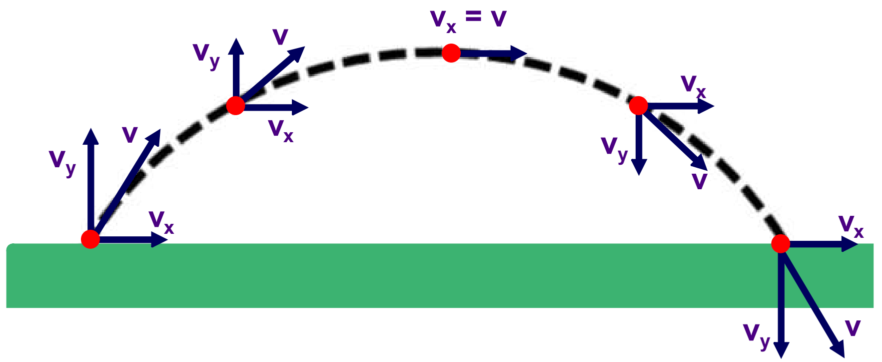
$$a_x = 0$$

vertical component:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = v_o \sin\theta t - \frac{1}{2}gt^2$$

$$v_y = v_{0y}t + a_y t = v_0 \sin\theta - gt$$

$$a_y = g$$



Flying time:

$$y = v_o \sin\theta t - \frac{1}{2} g t^2$$

$$y_0 = 0$$

$$0 = v_o \sin\theta t - \frac{1}{2} g t^2$$

$$\boxed{t = \frac{2 v_0 \sin\theta}{g}}$$

Horizontal range:

$$x = v_0 \cos\theta t$$

$$x = v_0 \cos\theta \frac{2 v_0 \sin\theta}{g}$$

$$\boxed{x = \frac{v_0^{\,2} \sin 2\theta}{g}}$$

Maximum Height:

$$y = v_o \sin\theta t - \frac{1}{2} g t^2$$

$$t = \frac{t_{total}}{2}$$

$$y = v_o \sin\theta \frac{v_0 \sin\theta}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g} \right)^2$$

$$y = \frac{v_0^{\,2} \sin^2\theta}{g} - \frac{v_0^{\,2} \sin^2\theta}{2g}$$

$$\boxed{y = \frac{v_0^{\,2} \sin^2\theta}{2g}}$$

Lab

38 Ignoring air resistance, thehorizontal component of a projectile's velocity:

A is zero.
B remains constant.
C continuously increases.
D continuously decreases.

Answer

39 A ball is thrown with a velocity of 20 m/s at an angle of 60° above the horizontal. What is the horizontal component of its instantaneous velocity at the exact top of its trajectory?

A 10 m/s
B 17 m/s
C 20 m/s
D zero

Answer

40 Ignoring air resistance, the magnitude of the horizontal component of a projectile's acceleration:

A is zero.
B remains a non-zero constant.
C continuously increases.
D continuously decreases.

Answer

41 At what angle should a water-gun be aimed in order for the water to land with the greatest horizontal range?

A 0°

B 30°

C 45°

D 60°

Answer

42 (Mulitple Answer) An Olympic athlete throws a javelin at six different angles above the horizontal, each with the same speed: 20°, 30°, 40°, 60°, 70° and 80°. Which two throws cause the javelin to land the same distance away? Be prepared to justify your answer.

A 30° and 80°

B 20° and 70°

C 30° and 70°

D 30° and 60°

Answer

43 You are throwing a ball for the second time. If the ball leaves your hand with twice the velocity it had on your first throw, its horizontal range R (compared to your first serve) would be

A 1.4 R
B R/2
C 2R
D 4R

Answer

44 When a football in a field goal attempt reaches its maximum height, how does its speed compare to its initial speed? (Justify your answer.)

A It is zero.

B It is less than its initial speed.

C It is equal to its initial speed.

D It is greater than its initial speed.

Answer

45 A stone is thrown horizontally from the top of a tower at the same instant a ball is dropped vertically. Which object is traveling faster when it hits the level ground below? (Justify your answer.)

A It is impossible to tell from the information given.

B the stone

C the ball

D Neither, since both are traveling at the same speed.

Answer

46 A plane flying horizontally at a speed of 50.0 m/s and at an elevation of 160 m drops a package. Two seconds later it drops a second package. How far apart will the two packages land on the ground?

A 100 m
B 170 m
C 180 m
D 210 m

Answer

47 An arrow is fired at an angle of θ above the horizontal with a speed of v. Explain how you can calculate the horizontal range and maximum height. (List the steps you would take and the equations you would use so that another student could solve this problem but do not solve the problem.)

Answer

48 A projectile is fired with an initial speed of 30 m/s at an angle of 30° above the horizontal.

Determine the total time in the air.

Answer

49 A projectile is fired with an initial speed of 30 m/s at an angle of 30° above the horizontal.

Determine the maximum height reached by the projectile.

Answer

50 A projectile is fired with an initial speed of 30 m/s at an angle of 30° above the horizontal.

Determine the maximum horizontal distance covered by the projectile.

Answer

51 A projectile is fired with an initial speed of 30 m/s at an angle of 30° above the horizontal.

Determine the velocity of the projectile 2s after firing.

Answer

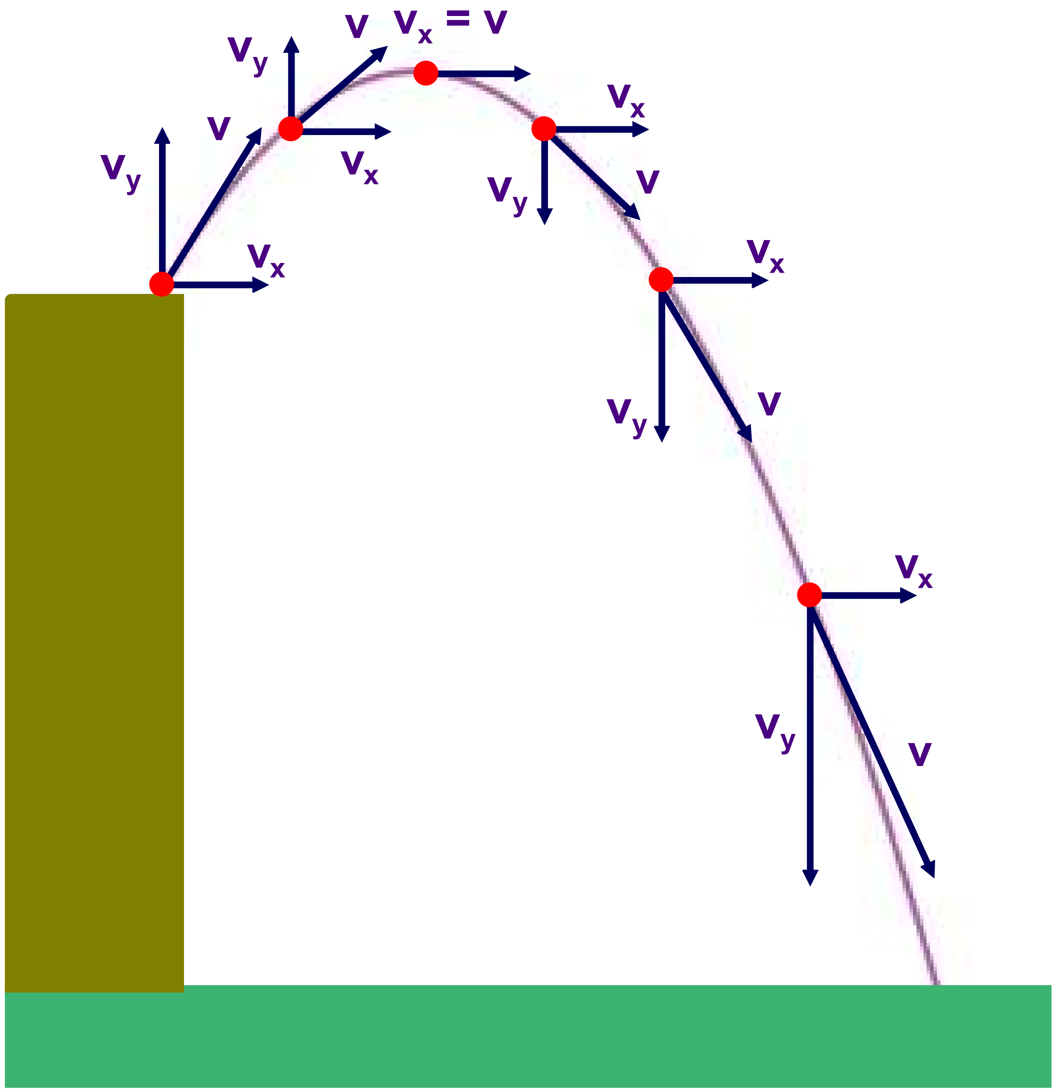
Projectile Motion: 3rd Type

If an object is launched at an angle with the horizontal and from an initial height, the analysis is similar except when finding the total time in the air. You will need to use the quadratic formula.

$$0 = y_0 + v_o \sin\theta t - \frac{g}{2}t^2$$

$$0 = c + bx + ax^2$$

$$t_{total} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



52 A projectile is fired from the edge of a cliff 200 m high with an initial speed of 30 m/s at an angle of 45° above the horizontal.

Determine the total time in the air.

Answer

53 A projectile is fired from the edge of a cliff 200 m high with an initial speed of 30 m/s at an angle of 45° above the horizontal.

Determine the maximum horizontal range.

Answer

54 A projectile is fired from the edge of a cliff 200 m high with an initial speed of 30 m/s at an angle of 45° above the horizontal.

Determine the magnitude of the velocity just before impact.

Answer

55 A projectile is fired from the edge of a cliff 200 m high with an initial speed of 30 m/s at an angle of 45° above the horizontal.

Determine the angle the velocity makes with the horizontal just before impact.

Answer