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NEW JERSEY CENTER
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AP Physics 1

AP Physics 2

Electric Potential and Capacitors

2017-07-26

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Electric Potential and Capacitors

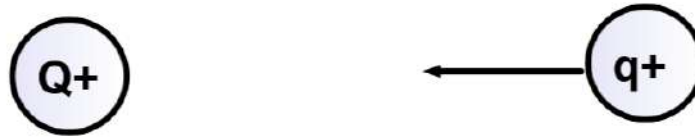
Click on the topic to go to that section

- **Electric Potential Energy**
- **Electric Potential (Voltage)**
- **Electric Potential due to a Uniform Electric Field**
- **Capacitance and Capacitors**

Electric Potential Energy

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Electric Potential Energy



Start with two positive charges initially at rest, with Q at the origin, and q at infinity.

In order to move q towards Q, a force opposite to the Coulomb repulsive force (like charges repel) needs to be applied.

$$\vec{F}_E = \frac{kQq}{r^2} \hat{r}$$

Note that this force is constantly increasing as q gets closer to Q, since it depends on the distance between the charges, r, and r is decreasing.

Work and Potential Energy



Recall that Work is defined as: $W = F \bullet r_{parallel}$

To calculate the work needed to bring q from infinity, until it is a distance r from Q , we need to use calculus, because of the non constant force. Then, use the relationship: $\Delta U_E = -W$

The Work is done by the Electric Field of charge Q (by definition) on charge q . As q moves opposite the Electric Field of charge Q , negative work is done by the field.

Work and Potential Energy



Assume that the potential energy of the Q-q system is zero at infinity, and adding up the incremental force times the distance between the charges at each point, integral calculus finds that the Electric Potential Energy, U_E is:

$$U_E = \frac{kQq}{r}$$

q is moved towards Q at a constant velocity so that there is no change in kinetic energy. The sum of the forces due to the Electric Field and the external force must be zero.

Electric Potential Energy

This is the equation for the potential energy due to two point charges separated by a distance r .

$$U_E = \frac{kQq}{r}$$

This process is similar to how Gravitational Potential Energy was developed.

The benefit of using Electric Potential Energy instead of the Electrical Force is that energy is a scalar, and calculations are much simpler. There is no direction, but the sign matters.

Electric Potential Energy

Again, just like in Gravitational Potential Energy, Electric Potential Energy requires a system - it is not a property of just one object. In this case, we have a system of two charges, Q and q .

Another way to define the system is by assuming that the magnitude of Q is much greater than the magnitude of q , thus, the Electric Field generated by Q is also much greater than the field generated by q (which may be ignored).

Now we have a field-charge system, and the Electric Potential Energy is a measure of the interaction between the field and the charge, q .

Electric Potential Energy

What is this Electric Potential Energy?

It tells you how much energy is stored by work being done on the system, and is now available to return that energy in a different form, such as kinetic energy. Just like Gravitational Potential Energy.

If two positive charges are placed near each other, they are a system, and they have Electric Potential Energy. Once released, they will accelerate away from each other - turning potential energy into kinetic energy. These moving charges can now perform work on another system.

Electric Potential Energy

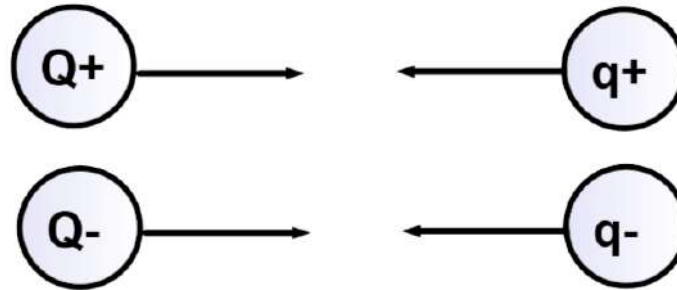


A positive charge and a negative charge have a negative potential energy (positive work is done by Q's electric field on q as the charges are attracted to each other).

$$U_E = \frac{k(Q)(-q)}{r} = -\frac{kQq}{r}$$

It takes negative work by an external force to keep them from accelerating towards each other (the sum of the forces on the charge has to equal zero). The potential energy of the system decreases as the charges move closer to each other.

Electric Potential Energy



Two positive charges or two negative charges have a positive potential energy (negative work is done by Q's electric field on q).

$$U_E = \frac{k(Q)(q)}{r} = \frac{kQq}{r}$$

$$U_E = \frac{k(-Q)(-q)}{r} = \frac{kQq}{r}$$

It takes positive work by an external force to move the charges towards each other. The potential energy of the system increases as the charges move towards each other.

1 Compute the potential energy of the two charges in the following configuration:

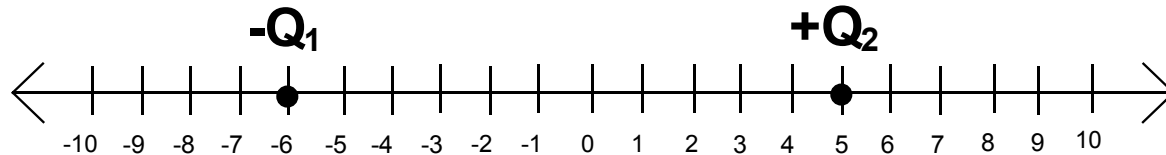


A positive charge, $Q_1 = 5.00 \text{ mC}$ is located at $x_1 = -8.00 \text{ m}$, and
a positive charge $Q_2 = 2.50 \text{ mC}$ is located at $x_2 = 3.00 \text{ m}$.

- ☐ A $1.02 \times 10^4 \text{ J}$
- ☐ B $2.89 \times 10^4 \text{ J}$
- ☐ C $4.17 \times 10^4 \text{ J}$
- ☐ $6.46 \times 10^4 \text{ J}$
- ☐ I need help

Answer

2 Compute the potential energy of the two charges in the following configuration:

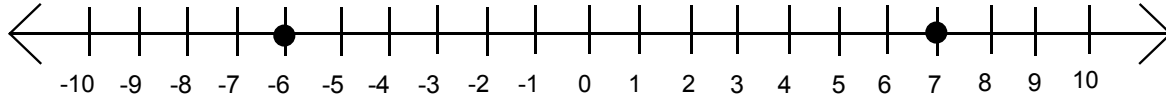


A negative charge, $Q_1 = -3.00 \text{ mC}$ is located at $x_1 = -6.00 \text{ m}$, and a positive charge $Q_2 = 4.50 \text{ mC}$ is located at $x_2 = 5.00 \text{ m}$.

Answer

- ☐ $-7.88 \times 10^4 \text{ J}$
- ☐ $-5.34 \times 10^4 \text{ J}$
- ☐ $-3.02 \times 10^4 \text{ J}$
- ☐ $-1.10 \times 10^4 \text{ J}$
- ☐ i need help

3 Compute the potential energy of the two charges in the following configuration:



A negative charge, $Q_1 = -3.00 \text{ mC}$ is located at $x_1 = -6.00 \text{ m}$, and a negative charge $Q_2 = -2.50 \text{ mC}$ is located at $x_2 = 7.00 \text{ m}$.

- ☐ $2.07 \times 10^3 \text{ J}$
- ☐ $4.58 \times 10^3 \text{ J}$
- ☐ $5.19 \times 10^3 \text{ J}$
- ☐ $7.36 \times 10^3 \text{ J}$
- ☐ I need help

Answer

- 4 A student is given the below values for a system of charges. What can the student say about this configuration? **Select two answers.**

$$U_E = \frac{kQq}{r} = \frac{kq_1q_2}{r}$$

$$r = 0.0034m$$

$$q_1 = 5.4mC$$

$$q_2 = -3.8mC$$

- ☐ If there is no external force acting on the charges, they will move apart.
- ☐ An external force must act on the charges to prevent them from getting closer together.
- ☐ Decreasing the distance between the two charges increases the electric potential energy of the system of charges.
- ☐ Substituting each charge with the same value, but opposite signs (positive to negative, negative to positive) will have no effect on the calculated electric potential energy.
- ☐ I need help

Electric Potential Energy of Multiple Charges

To get the total potential energy for multiple charges, you must first find the energy due to each pair of charges.

Then, add these energies together. Since energy is a scalar, there is no direction involved; there is a positive or negative sign associated with each energy pair.

For example, if there are three charges, the total potential energy is:

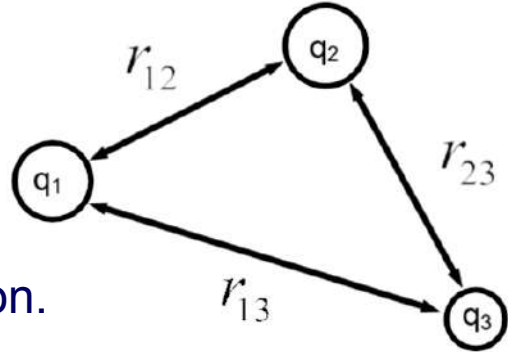
$$U_T = U_{12} + U_{23} + U_{13}$$

Where U_{xy} is the Potential Energy of charges x and y.

Electric Potential Energy of Multiple Charges

Compute the electric potential energy of the three charge configuration.

There are no values for the distances between the charges or for the charges. So, we're looking for an algebraic solution.



Quick notation review: r_{12} means the distance between charges q_1 and q_2 .

Here's the equation we're going to use:

$$U_T = U_{12} + U_{23} + U_{13}$$

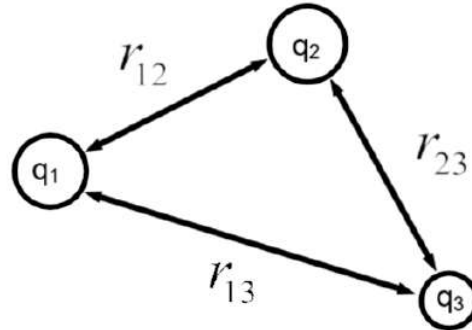
Try this within your groups before going to the next slide.

Electric Potential Energy of Multiple Charges

$$U_T = U_{12} + U_{23} + U_{13}$$

$$U_T = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

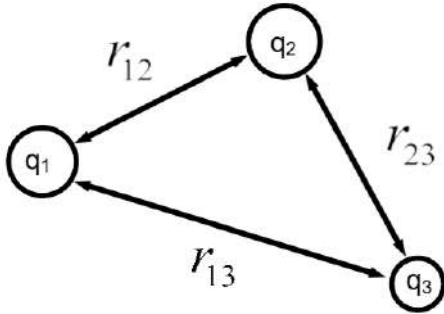
$$U_T = k \left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}} \right)$$



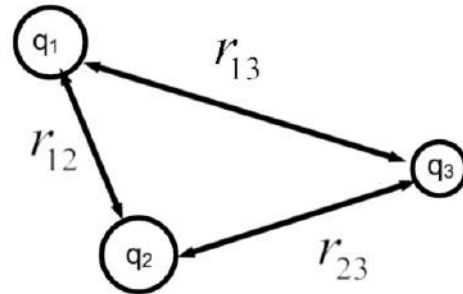
It's that easy! You should always solve physics problems algebraically first - it shows you understand the concept and it makes the math easier. This general equation can be used to solve any configuration of three charges - you just need to plug in the values.

The k was factored out so you only have to multiply by it one time. Saves calculations and reduces errors.

Electric Potential Energy of Multiple Charges

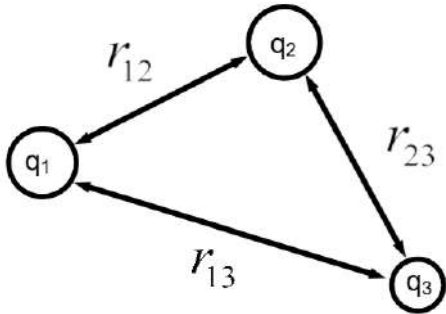


What if we kept all the q 's the same and the distances between each q_i the same, but flipped where q_2 was located. Would the potential energy of the system change?



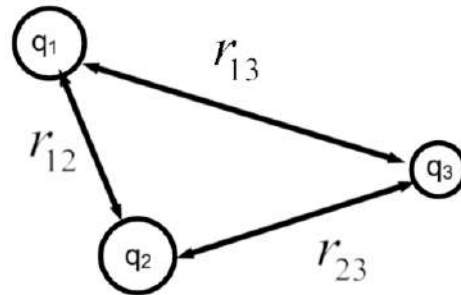
$$U_T = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}} \right)$$

Electric Potential Energy of Multiple Charges

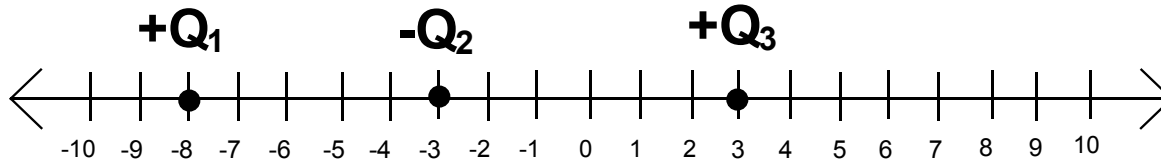


NO! Unlike Electric force and Field, Electric Potential Energy is a scalar. As long as the charges and the distances between them are the same, the Potential Energy stays the same.

$$U_T = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}} \right)$$



5 Compute the potential energy of the three charges in the following configuration:



A positive charge, $Q_1 = 5.00 \text{ mC}$ is located at $x_1 = -8.00 \text{ m}$, a negative charge $Q_2 = -4.50 \text{ mC}$ is located at $x_2 = -3.00 \text{ m}$, and a positive charge $Q_3 = 2.50 \text{ mC}$ is located at $x_3 = 3.00 \text{ m}$.

☐ $-5.74 \times 10^4 \text{ J}$

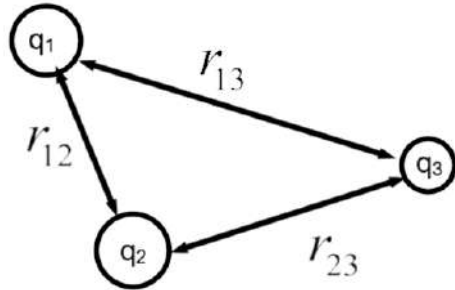
☐ $-4.72 \times 10^4 \text{ J}$

☐ $2.24 \times 10^4 \text{ J}$

☐ $6.91 \times 10^4 \text{ J}$

☐ I need help

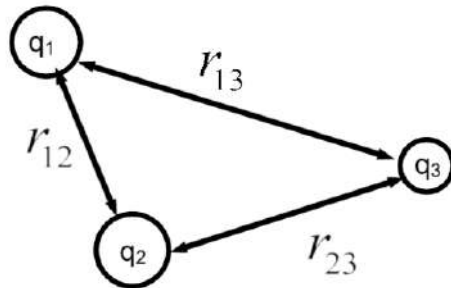
6 Compute the Electric Potential energy of the three charges in the following configuration.



$$\begin{aligned}q_1 &= 4.2 \mu\text{C} \\q_2 &= 3.6 \mu\text{C} \\q_3 &= -5.2 \mu\text{C} \\r_{12} &= 0.034 \text{ m} \\r_{13} &= 0.072 \text{ m} \\r_{23} &= 0.039 \text{ m}\end{aligned}$$

- ☐ -3.05 J
- ☐ -1.98 J
- ☐ 2.07 J
- ☐ 3.66 J
- ☐ I need help

- 7 Three positive charges are aligned as below. They are not equal to each other, and the distances between them are different. They are replaced with negative charges with the same magnitude as the positive charges, and the distances remain the same. Does the Electric Potential Energy increase, decrease, or stay the same? Justify your answer qualitatively, with no equations or calculations.



8 What factors are required to calculate the Electric Potential Energy of a system of charges? **Select two answers.**

- ☐ The direction of the Electric Field created by the charges.
- ☐ The spatial orientation of the system.
- ☐ Distance between each pair of charges.
- ☐ The sign of the charge (positive or negative).
- ☐ E I need help

Electric Potential (Voltage)

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Electric Potential or Voltage

Our study of electricity began with Coulomb's Law which calculated the electric force between two charges, Q and q .

By assuming q was a small positive charge, and dividing \mathbf{F} by q , the electric field \mathbf{E} due to the charge Q was defined.

$$\vec{F}_E = \frac{kQq}{r^2} \hat{r} \qquad \vec{E} = \frac{\vec{F}_E}{q} = \frac{kQ}{r^2} \hat{r}$$

The Electric Field is a property of the space surrounding the charge Q . More advanced physics courses call it a vector field.

Electric Potential or Voltage

The same process will be used to define the Electric Potential, or V , from the Electric Potential Energy, where V is a property of the space surrounding the charge Q :

$$U_E = \frac{kQq}{r} \qquad V = \frac{U_E}{q} = \frac{kQ}{r}$$

V is also called the voltage and is measured in volts. It is called a scalar field.

Electric Potential or Voltage

What we've done here is removed the system that was required to define Electric Potential Energy (needed two objects or a field and an object).

$$V = \frac{U_E}{q} = \frac{kQ}{r}$$

It tells you how much potential energy is in each charge - and if the charges are moving, how much work, per charge, they can do on another system.

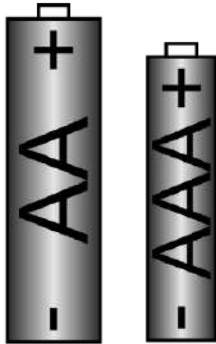
Electric Potential or Voltage

Voltage is the Electric Potential Energy per charge, which is expressed as Joules/Coulomb. Hence:

$$V = \frac{J}{C}$$

To make this more understandable, a Volt is visualized as a battery adding 1 Joule of Energy to every Coulomb of Charge that goes through the battery.

Electric Potential or Voltage



Batteries are a source of Electric Potential (Voltage), by using chemical reactions to separate the charges and put electrons in motion. Chemical energy is converted into Electrical Energy.

Despite the different size of these two batteries, they both have the same Voltage (1.5 V). That means that every electron that leaves each battery has the same Electric Potential - the same ability to do work.

The AA battery just has more chemicals for the reactions - so it will last longer than the AAA battery.

Electric Potential or Voltage

Another helpful equation can be found from $V = \frac{U_E}{q}$ by realizing that the work done on a positive charge by an external force will increase the potential energy of the charge so that:

$$W = U_E = qV$$

9 What is the Electric Potential (Voltage) 5.00 m away from a charge of $6.23 \times 10^{-6} \text{ C}$?

☐ $1.12 \times 10^4 \text{ V}$

☐ $1.84 \times 10^4 \text{ V}$

☐ $3.16 \times 10^4 \text{ V}$

☐ $5.59 \times 10^4 \text{ V}$

☐ I need help

10 What is the Electric Potential (Voltage) 7.50 m away from a charge of $-3.32 \times 10^{-6} \text{ C}$?

☒ $-7.19 \times 10^3 \text{ V}$

☐ $-5.54 \times 10^3 \text{ V}$

☐ $-3.98 \times 10^3 \text{ V}$

☐ $-2.24 \times 10^3 \text{ V}$

☐ I need help

Electric Potential of Multiple Charges

To get the total potential for multiple charges at a particular point in space, you must first find the potential due to each charge.

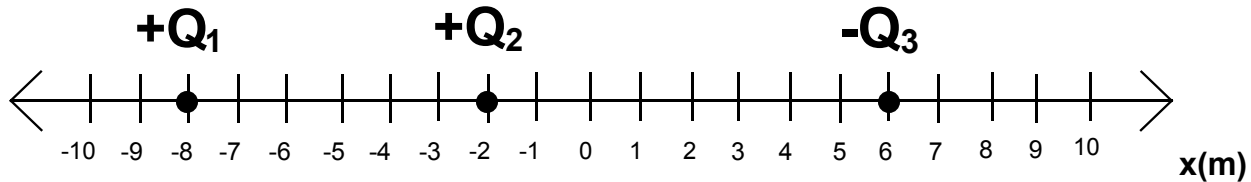
Then, you can add these potentials together. Since potential is a scalar, there is no direction involved - but, there is a positive or negative sign associated with each potential.

For example, if there are three charges, the total potential is:

$$V_{\text{net}} = V_1 + V_2 + V_3$$

Where V_{net} is the net potential and $V_1, V_2, V_3...$ is the electric potential of each individual charge at that point.

11 Compute the electric potential of three charges at the origin in the following configuration:

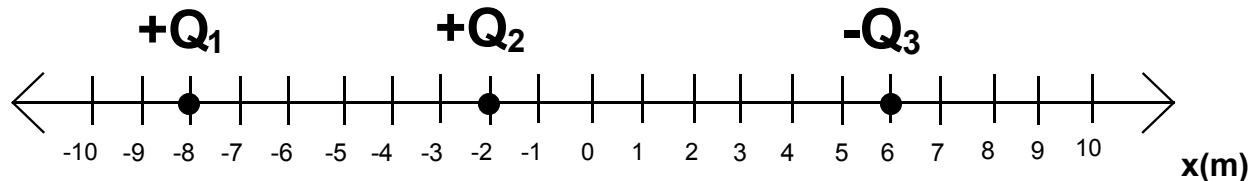


A positive charge, $Q_1 = 5.00$ nC is located at $x_1 = -8.00$ m, a positive charge $Q_2 = 3.00$ nC is located at $x_2 = -2.00$ m, and a negative charge $Q_3 = -9.00$ nC is located at $x_3 = 6.00$ m.

- ☐ A 3.48 V
- ☐ B 5.63 V
- ☐ C 7.11 V
- ☐ D 8.85 V
- ☐ I need help

Answer

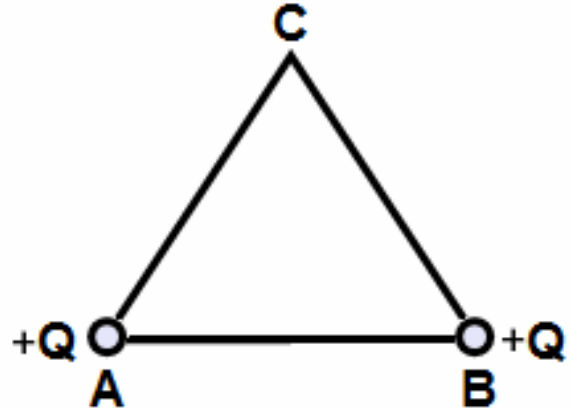
12 How much work must be done by an external force to bring a 1×10^{-6} C charge from infinity to the origin of the following configuration? The electric potential at the origin is 5.63 V.



A positive charge, $Q_1 = 5.00$ nC is located at $x_1 = -8.00$ m, a positive charge $Q_2 = 3.00$ nC is located at $x_2 = -2.00$ m, and a negative charge $Q_3 = -9.00$ nC is located at $x_3 = 6.00$ m.

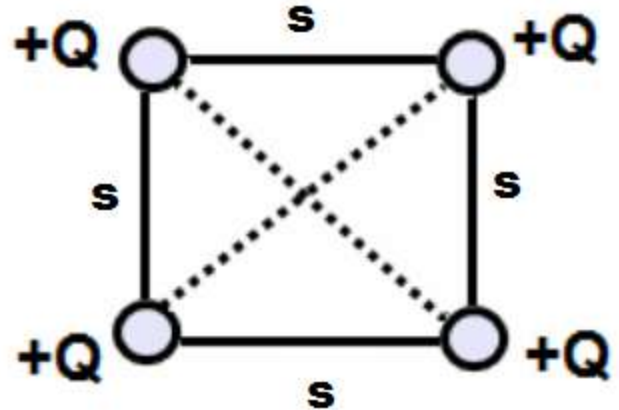
- ☐ A 5.63×10^{-6} J
- ☐ B 1.88×10^{-6} J
- ☐ C 7.92×10^{-5} J
- ☐ D 2.15×10^{-5} J
- ☐ E I need help

- 13 Two positive charges of magnitude $+Q$ are placed at the corners A and B of an equilateral triangle with a side r . Calculate the net electric potential at point C.



- ☐ A kQ/r
- ☐ B $2kQ/r$
- ☐ C $3kQ/r$
- ☐ D $9kQ/r$
- ☐ E I need help

14 Four charges of equal magnitude are arranged in the corners of a square. Calculate the net electric potential at the center of the square due to all charges.

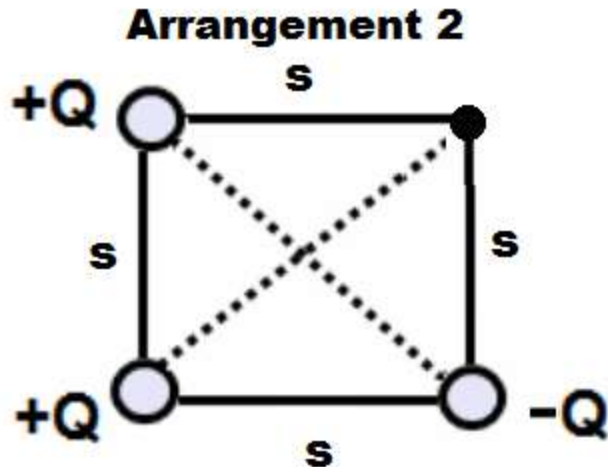
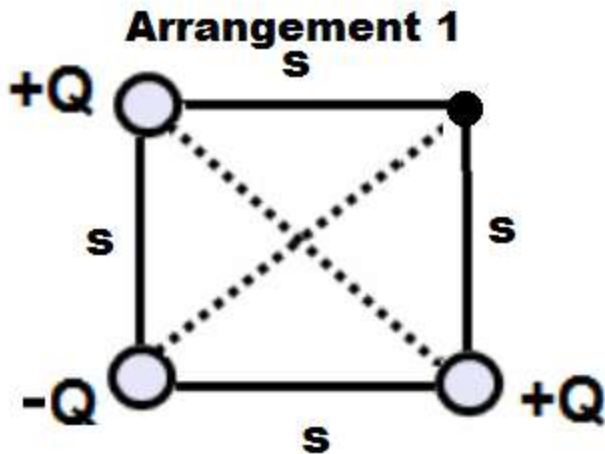


- ☐ A $\sqrt{2}kQ/s$
- ☐ B $2kQ/s$
- ☐ C $2\sqrt{2}kQ/s$
- ☐ D $4\sqrt{2}kQ/s$
- ☐ E I need help

Answer

- 15 Three charges of equal magnitude but different signs are placed in three corners of a square. In which of the arrangements is more work is required to move a positive test charge q from infinity to the empty corner of the square? First, make a prediction and discuss with your classmates. Second, calculate the work done on moving q for each arrangement with the given numerical values:

$$Q=9.00 \times 10^{-9} \text{ C}, q=1.00 \times 10^{-12} \text{ C}, s=1.00 \text{ m}$$



Electric Potential or Voltage

Like Electric Potential Energy, Voltage is NOT a vector, so multiple voltages can be added directly, taking into account the positive or negative sign.

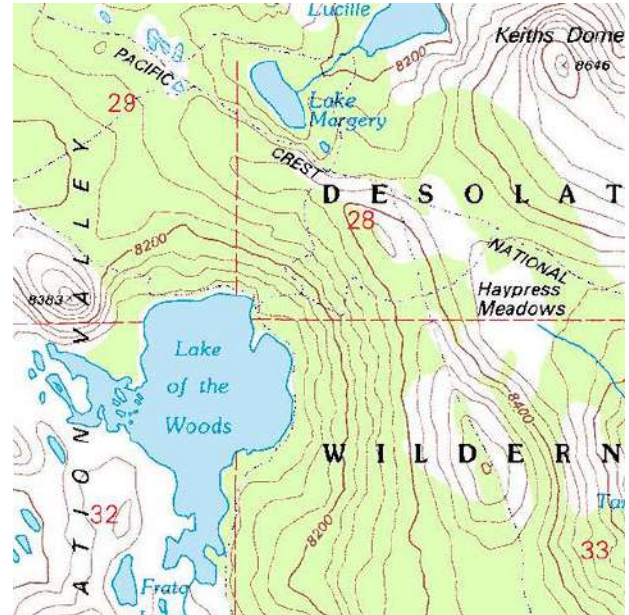
Like Gravitational Potential Energy, Voltage is not an absolute value - it is compared to a reference level. By assuming a reference level where $V=0$ (as we do when the distance from the charge generating the voltage is infinity), it is allowable to assign a specific value to V in calculations.

The next slide will continue the gravitational analogy to help understand this concept.

Topographic Maps

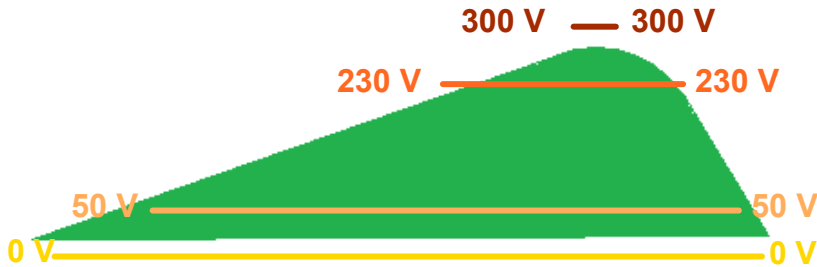
Each line represents the same height value. The area between lines represents the change between lines.

A big space between lines indicates a slow change in height. A little space between lines means there is a very quick change in height.

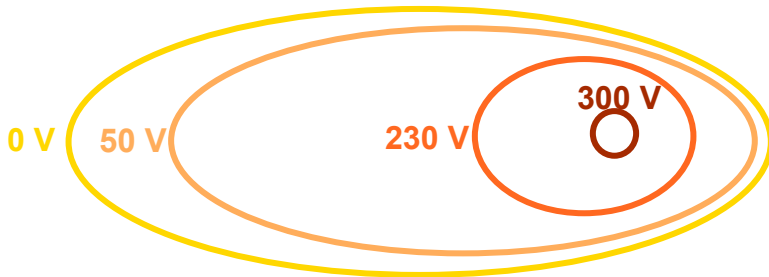


Where in this picture is the steepest incline?

Equipotential Lines



These "topography" lines are called "Equipotential Lines" when we use them to represent the Electric Potential - they represent lines where the Electric Potential is the same.



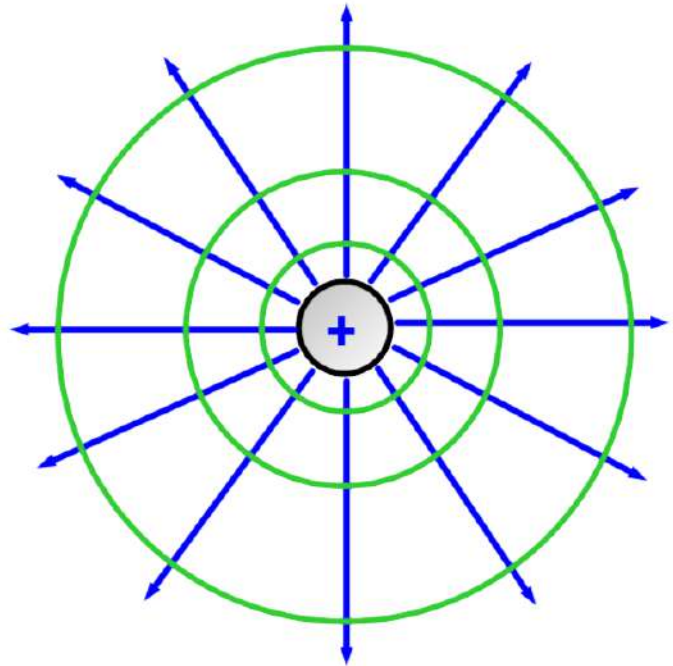
The closer the lines, the faster the change in voltage.... the bigger the change in Voltage, the larger the Electric Field.

Equipotential Lines

The direction of the Electric Field lines are always perpendicular to the Equipotential lines.

The Electric Field lines are farther apart when the Equipotential lines are farther apart.

The Electric Field points from high to low potential (just like a positive charge).



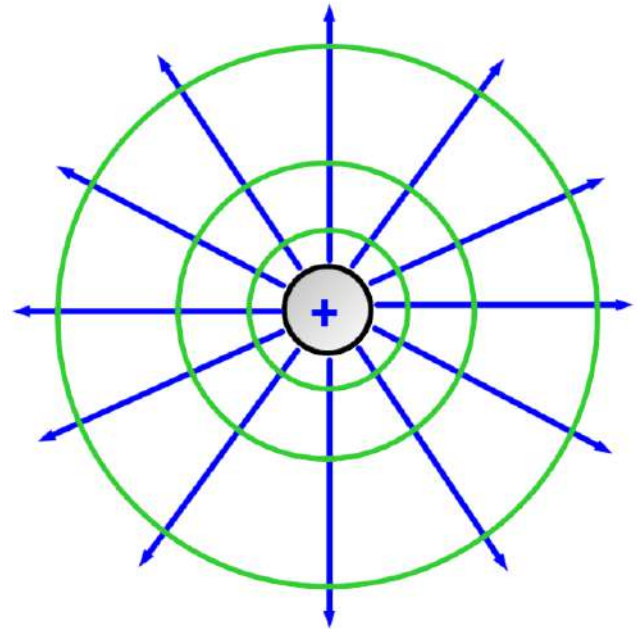
Why are the Electric Field lines always perpendicular to the Equipotential lines? Discuss this within your groups, then go to the next slide.

Equipotential Lines

Equipotential lines represent points in space where the Electric Potential (Voltage) is the same.

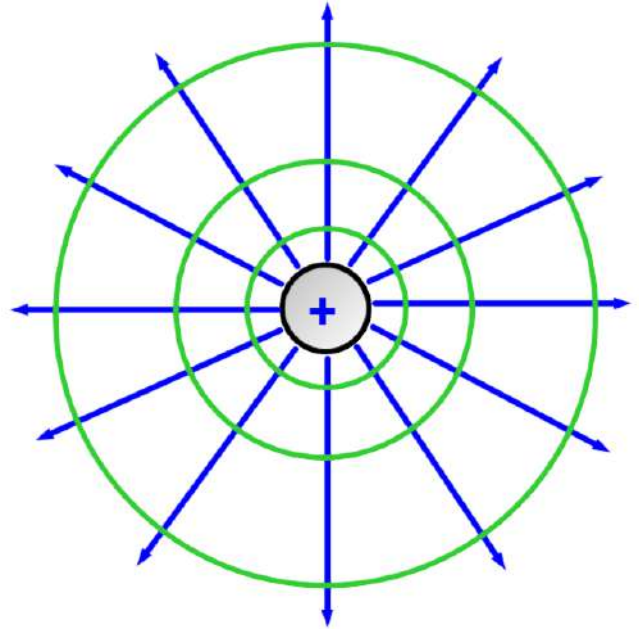
If the Voltage is the same, that means the Electric Field along the line must be zero (field lines point in the direction of decreasing potential).

The Electric field must have no vector component along the Equipotential line, hence the Electric field is perpendicular to the line at all points.



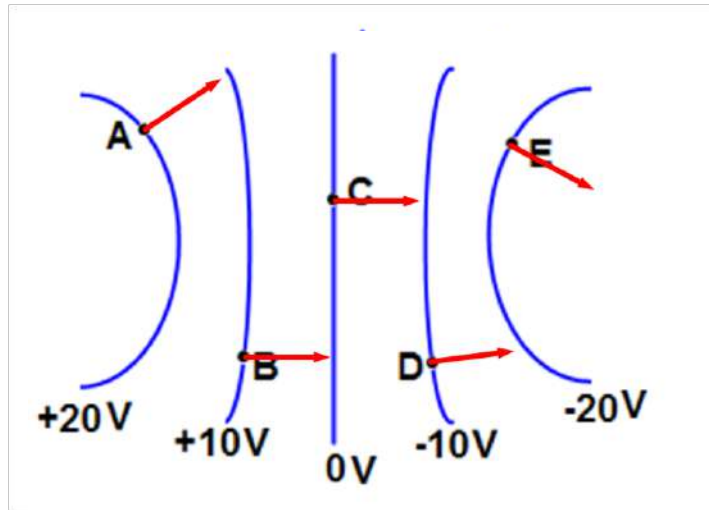
Equipotential Lines

For a positive charge like this one the equipotential lines are positive, and decrease to zero at infinity. A negative charge would be surrounded by negative equipotential lines, which would also go to zero at infinity.



More interesting equipotential lines (like the topographic lines on a map) are generated by more complex charge configurations.

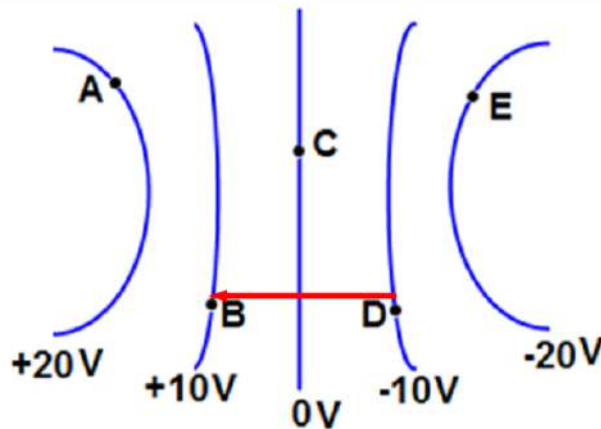
Equipotential Lines



This configuration is created by a positive charge to the left of the +20 V line and a negative charge to the right of the -20 V line.

Note the signs of the Equipotential lines, and the directions Electric Field vectors (in red) which are perpendicular to the Equipotential lines.

Work on a Charge

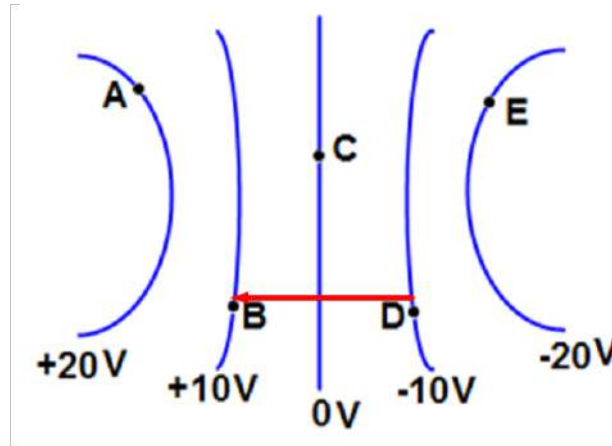


The work done by an external force to move a charge from infinity to a region of space with an Electric Potential of V has been shown to be:

$$W = qV$$

This assumed that the Electric Potential at infinity is 0 V. What if a charge is moved from point D to point B?

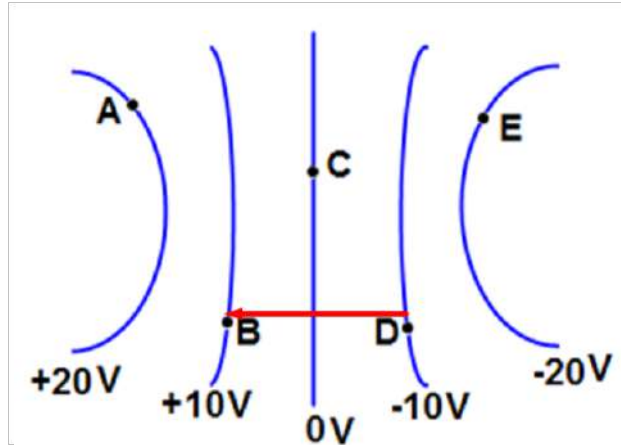
Work on a Charge



The voltage at both points has a non zero value. We need to use the change in Electric Potential to find the work done by an external force:

$$W = q\Delta V = q(V_f - V_0)$$

Work on a Charge

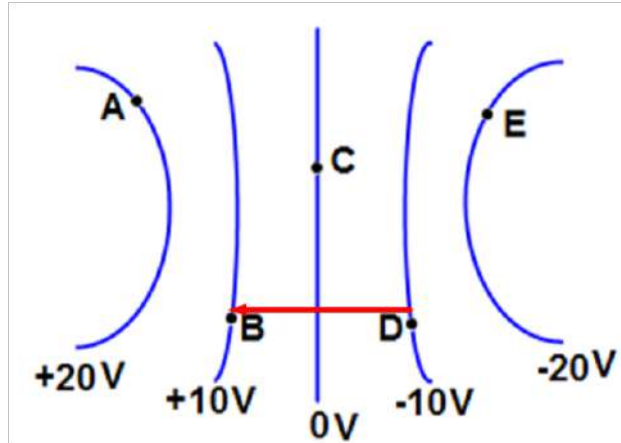


For a positive charge moving to a higher Electric Potential, we anticipate that positive work must be done by the external force, and that's what the equation shows:

$$W = q\Delta V = q(V_f - V_o)$$

$$W = q(10 - (-10))V = 20qV$$

Work on a Charge



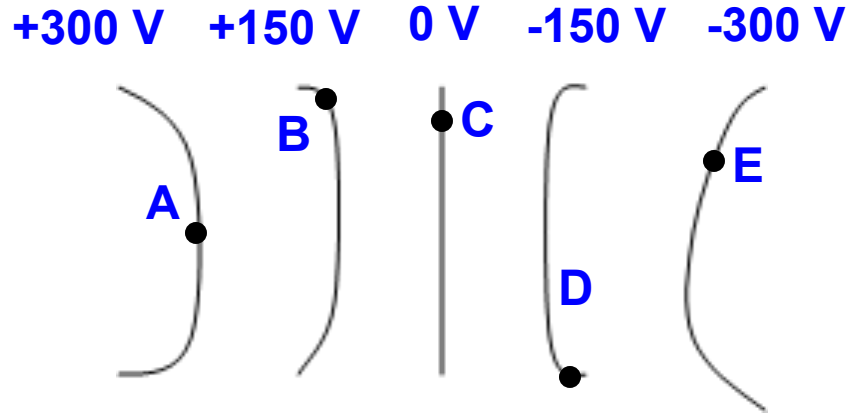
For a negative charge moving to a higher Electric Potential, we anticipate that negative work must be done by the external force, and that's what the equation shows:

$$W = q\Delta V = -q(V_f - V_o)$$

$$W = -q(10 - (-10))V = -20qV$$

16 At point A in the diagram, what is the direction of the Electric Field?

- ☐ A Up
- ☐ B Down
- ☐ C Left
- ☐ D Right
- ☐ E I need help



17 How much work is done by an external force on a $+10\text{ }\mu\text{C}$ charge that moves from point C to B?

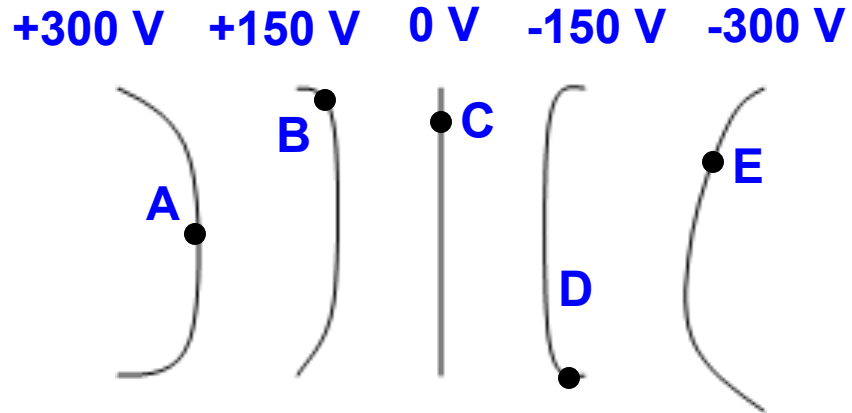
☐ $-2.2 \times 10^{-3}\text{ J}$

☐ $-1.5 \times 10^{-3}\text{ J}$

☐ $1.5 \times 10^{-3}\text{ J}$

☐ $3.7 \times 10^{-3}\text{ J}$

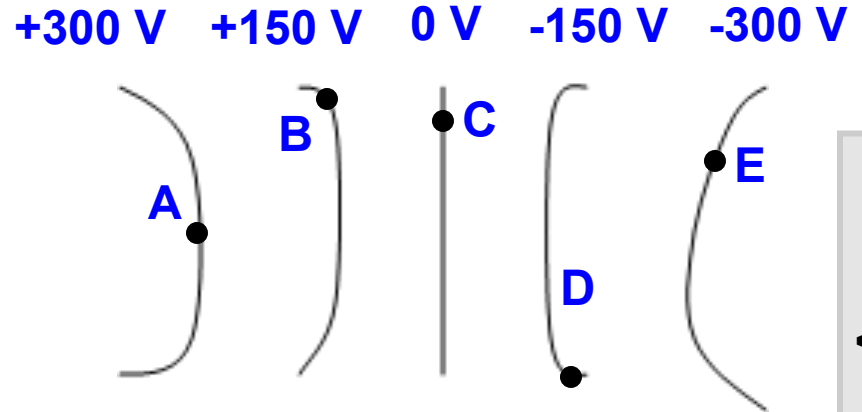
☐ I need help



Answer

18 How much work is done by an external force on a $-10\text{ }\mu\text{C}$ charge that moves from point C to B?

- ☐ A $-2.8 \times 10^{-3}\text{ J}$
- ☐ B $-1.5 \times 10^{-3}\text{ J}$
- ☐ C $2.0 \times 10^{-3}\text{ J}$
- ☐ D $3.1 \times 10^{-3}\text{ J}$
- ☐ E I need help

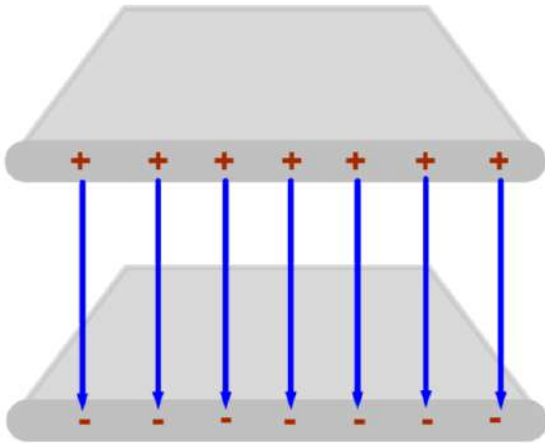


Answer

Electric Potential due to a Uniform Electric Field

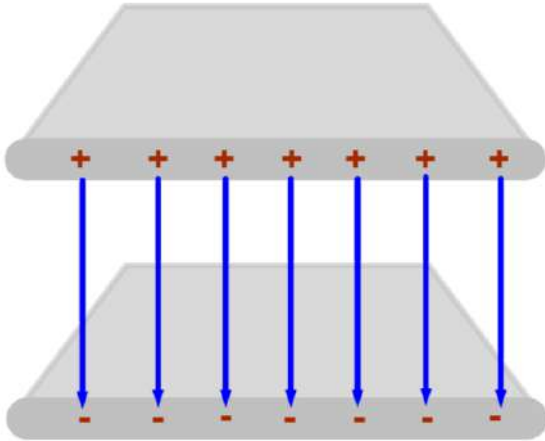
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Uniform Electric Field



Up until now, we've dealt with Electric Fields and Potentials due to individual charges. What is more interesting, and relates to practical applications, is when you have configurations of a massive amount of charges.

Uniform Electric Field

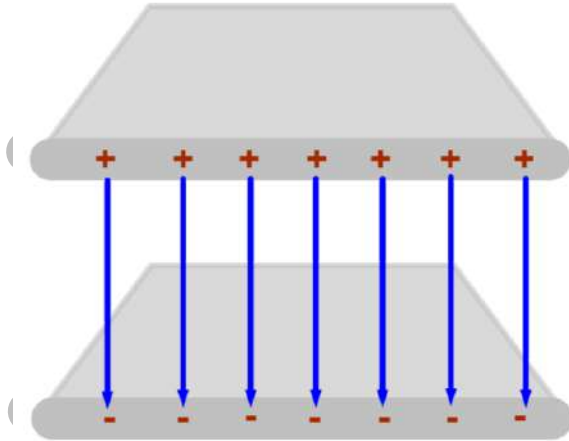


Let's begin by examining two infinite planes of charge that are separated by a small distance.

The planes have equal amounts of charge, with one plate being charged positively, and the other, negatively.

The diagram to the left represents two infinite planes (it's rather hard to draw infinity).

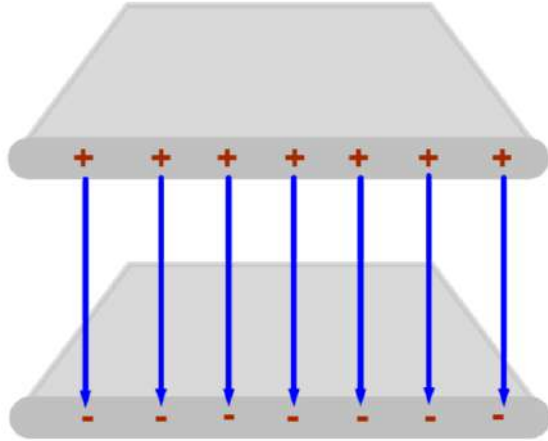
Uniform Electric Field



This is an idealized picture of a critical electric circuit element - the **Capacitor**. More on the Capacitor in the next chapter. Electric circuits will be explained in the next unit.

We are now going to work with Electric field, Electric Potential Energy and Electric Potential to explain the theory of the Capacitor.

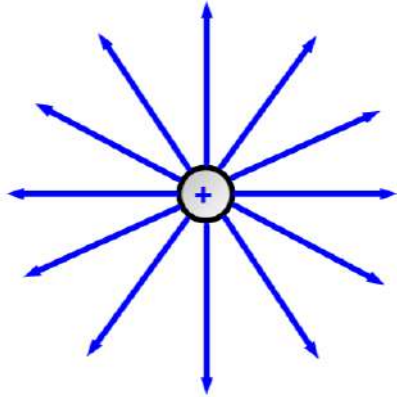
Uniform Electric Field



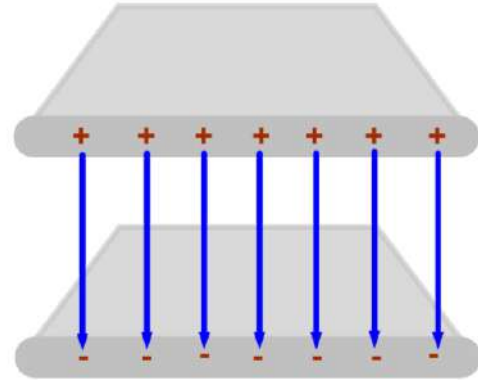
By applying Gauss's Law (a law that will be learned in AP C Physics), it is found that the strength of the Electric Field will be uniform between the planes - it will have the same magnitude and direction everywhere between the planes.

And, the Electric Field outside the two planes will equal zero.

Uniform Electric Field

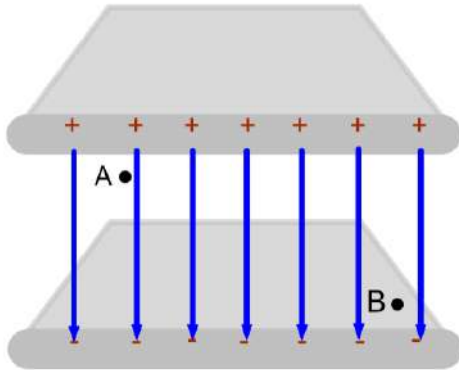


Point charges have a non-uniform field strength since the field weakens with distance.



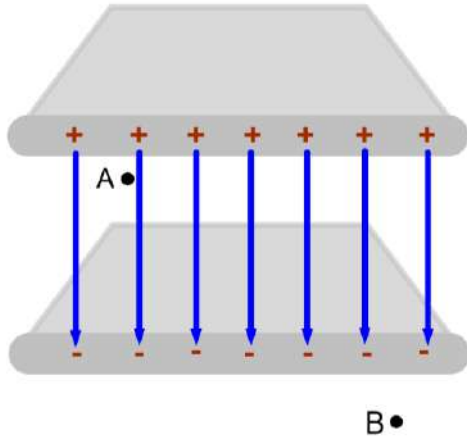
Only some equations we have learned will apply to uniform (uniform means constant magnitude and direction) electric fields.

- 19 If the strength of the Electric field at point A is 5,000 N/C, what is the strength of the Electric field at point B?



- ☐ A 0 N/C
- ☐ B 5000 N/C
- ☐ C 10000 N/C
- ☐ D 15000 N/C
- ☐ E I need help

20 If the strength of the Electric field at point A is 5,000 N/C, what is the strength of the Electric field at point B?

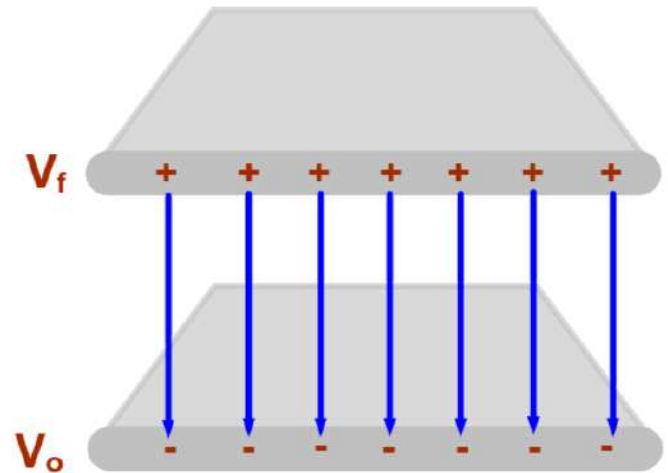


- ☐ 0 N/C
- ☐ 5000 N/C
- ☐ 10000 N/C
- ☐ 15000 N/C
- ☐ I need help

Uniform Electric Field & Potential Energy

For the parallel planes, the Electric Field is generated by the separation of charge - with the field lines originating on the positive charges and terminating on the negative charges.

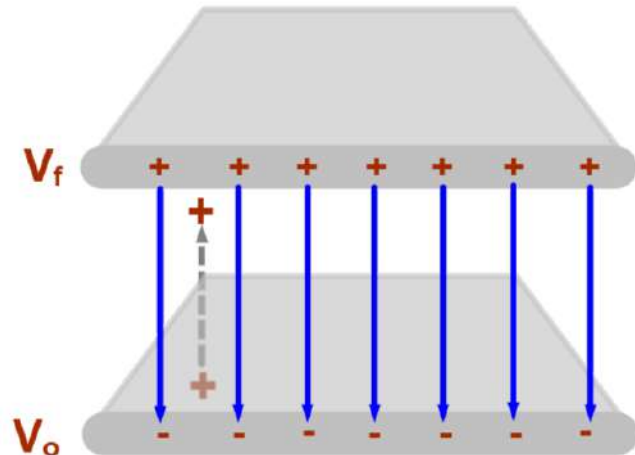
The difference in electric potential (voltage) is responsible for the electric field.



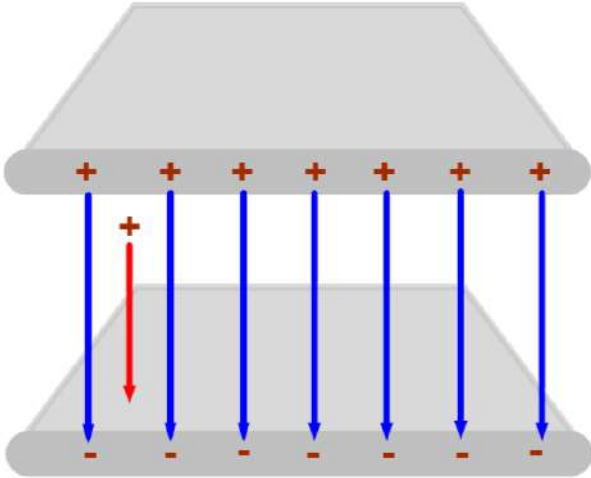
Uniform Electric Field & Potential Energy

Energy is being put into the parallel plane and charge system when a positive charge moves in the opposite direction of the electric field (or when a negative charge moves in the same direction of the electric field).

Positive work is being done by the external force, and since the positive charge is moving opposite the electric field - negative work is being done by the field.



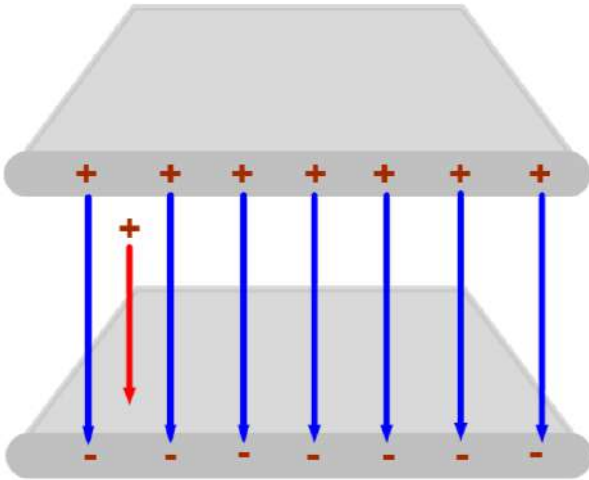
Uniform Electric Field & Potential Energy



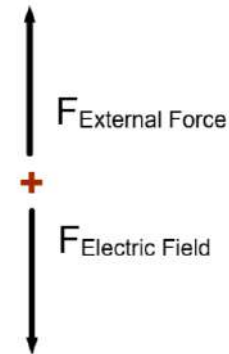
If the positive charge was just placed within the field, it will move from top to bottom. In this case, the Work done by the Electric Field is positive (the field is in the same direction as the charge's motion).

The potential energy of the system will decrease - this is directly analogous to the movement of a mass within a Gravitational Field.

Uniform Electric Field & Potential Energy

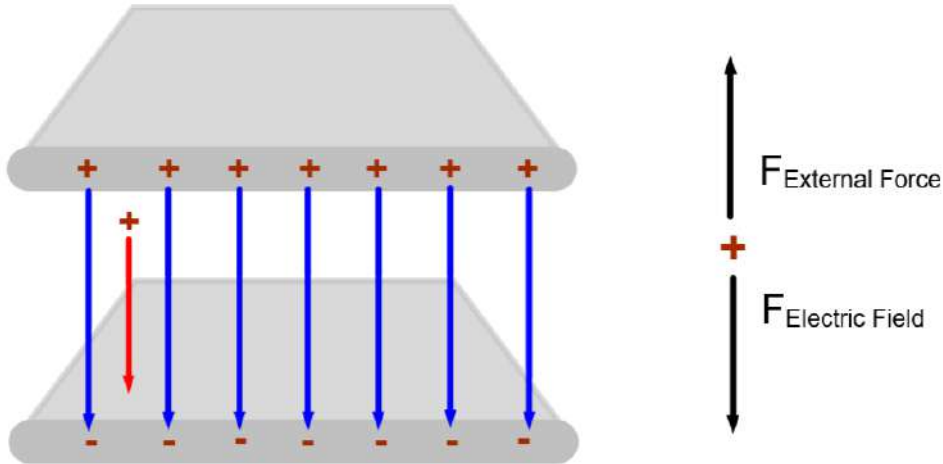


If there is no other force present, the charge will accelerate to the bottom by Newton's Second Law.



But, if we want the charge to move with a constant velocity, then an external force must act opposite to the Electric Field force. This external force is directed upwards. Since the charge is still moving down (but not accelerating), the Work done by the external force is negative.

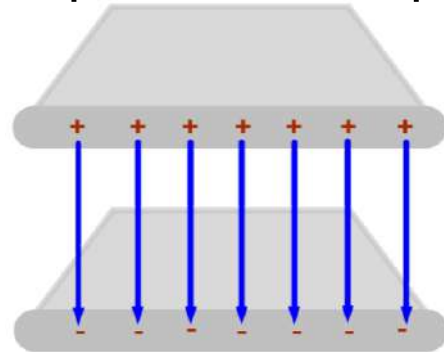
Uniform Electric Field & Potential Energy



The Work done by the external force is negative.
The Work done by the Electric Field is positive.
The Net force, and hence, the Net Work, is zero.
The Potential Energy of the system decreases.

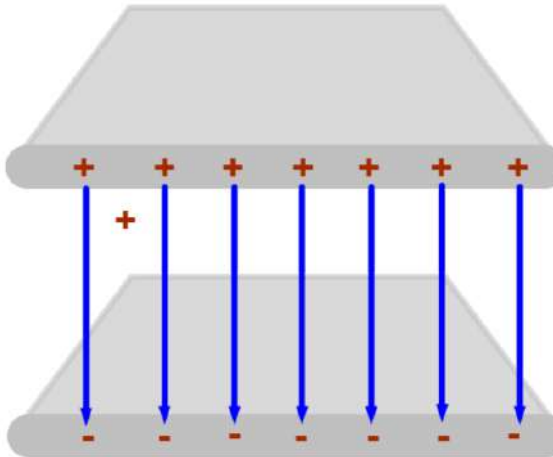
21 What generates the electric field between two parallel, oppositely charged plates?

- ☐ A The same voltage on both plates.
- ☐ B A difference in voltage between the top and bottom plates.
- ☐ C The charges moving between the plates.
- ☐ D Protons moving between the top and bottom plates.
- ☐ I need help



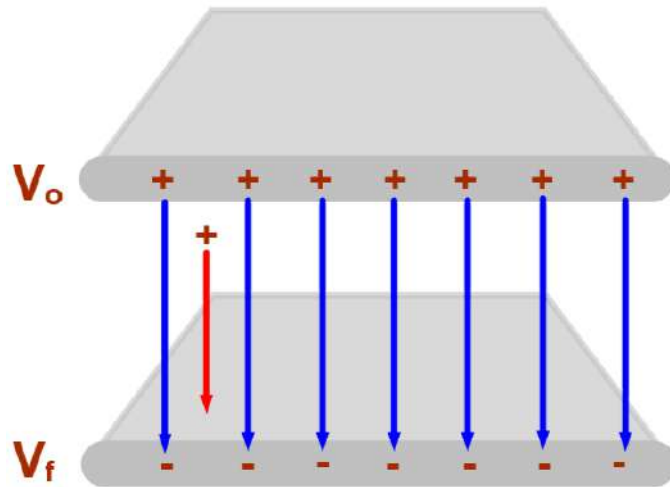
22 A positive charge is placed between two oppositely charged plates as shown below. Which way will the charge move?

- ☐ A Down
- ☐ B Up
- ☐ C To the left
- ☐ D To the right
- ☐ I need help



23 The positive charge moves from the top plate to the bottom plate, at a constant velocity, in the below configuration. What is the net work done on the charge?

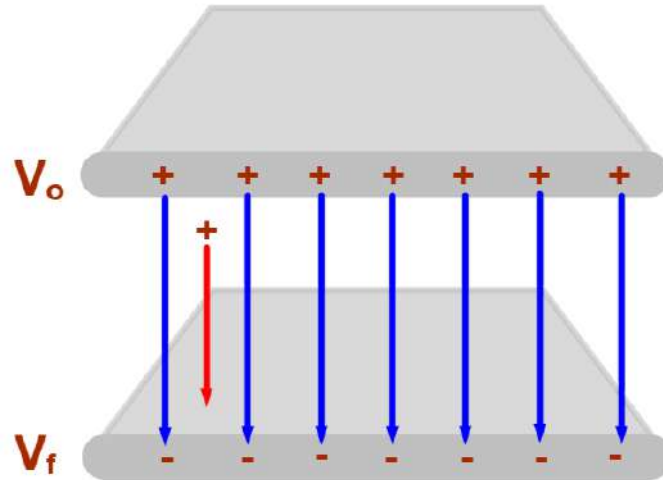
- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



Answer

24 The positive charge moves from the top plate to the bottom plate, at a constant velocity, in the below configuration. What is the work done by the Electric Field on the charge?

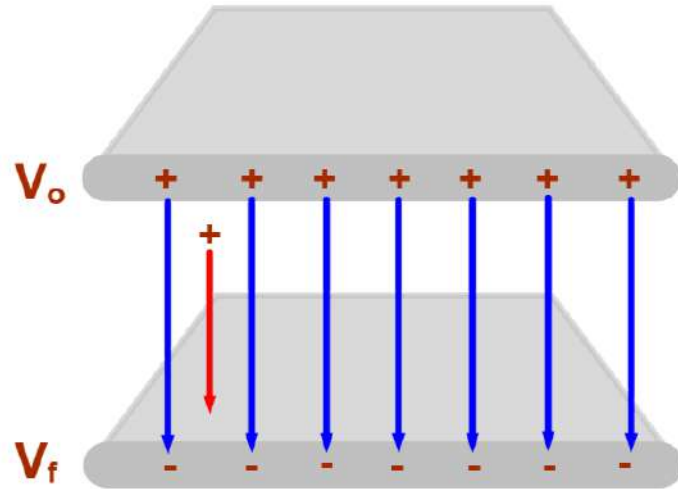
- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



Answer

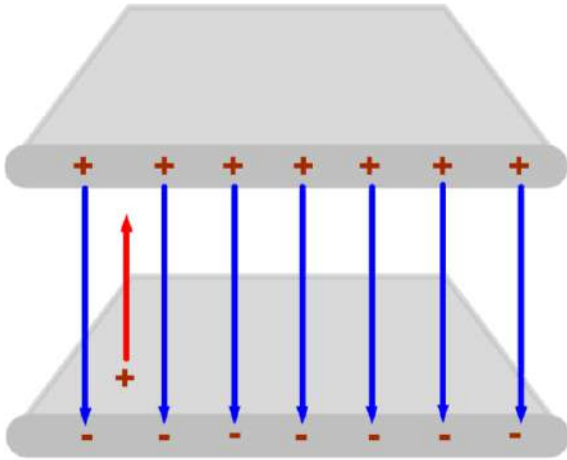
25 The positive charge moves from the top plate to the bottom plate, at a constant velocity, in the below configuration. What is the work done by the external force on the charge?

- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



Answer

Uniform Electric Field & Potential Energy



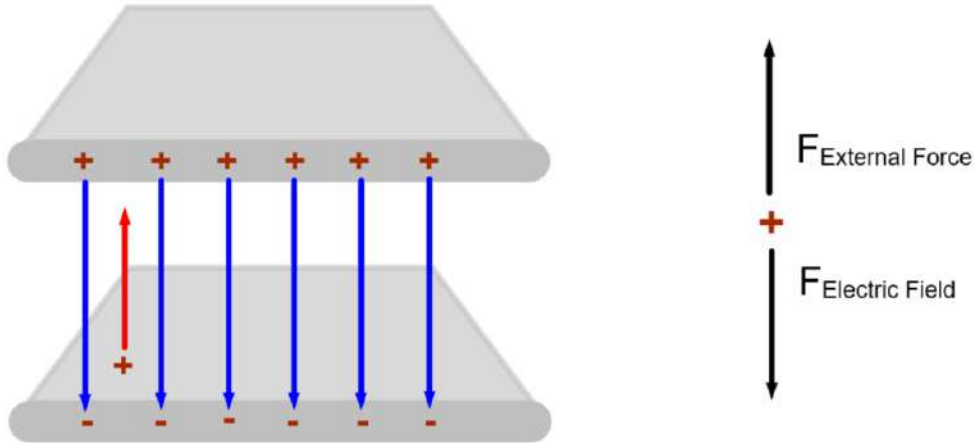
A positive charge moves from the bottom to the top plate.

An external force must act in the up direction to oppose the electric force which is directed down.

The Work done by the Electric field is negative as the field points in the opposite direction of the charge's motion.

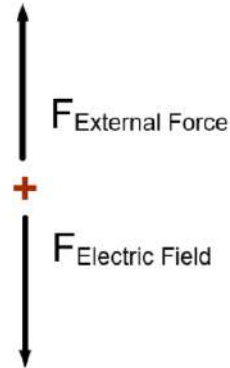
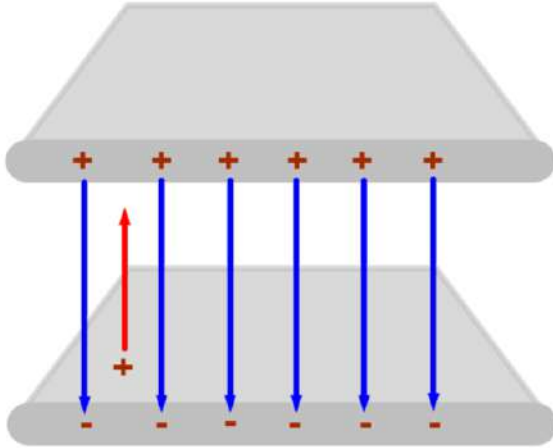
The potential energy of the system will increase - directly analogous to the movement of a mass within a gravitational field.

Uniform Electric Field & Potential Energy



If the charge moves with a constant velocity, then the external force is equal to the Electric Field force. Since the charge is moving up (but not accelerating), the Work done by the external force is positive.

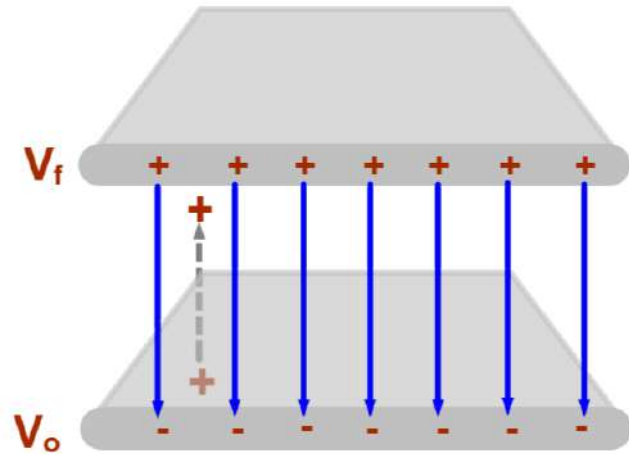
Uniform Electric Field & Potential Energy



The Work done by the external force is positive.
The Work done by the Electric Field is negative.
The Net force, and hence, the Net Work, is zero.
The Potential Energy of the system increases.

26 The positive charge moves from the bottom plate to the top plate, at a constant velocity in the below configuration. What happens to the Electric Potential Energy of the plane/charge system?

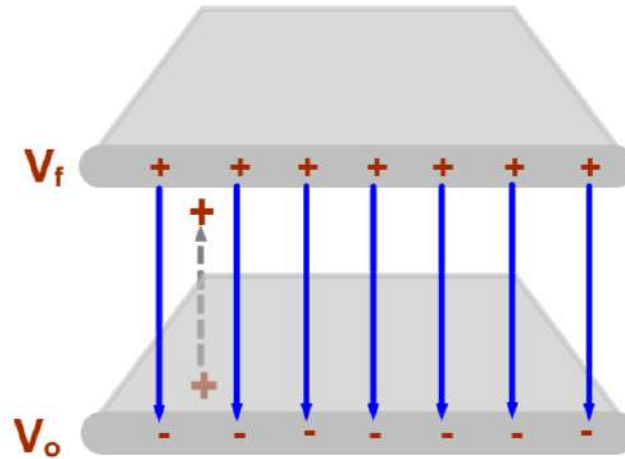
- ☐ A Increases
- ☐ B Remains constant
- ☐ C Decreases
- ☐ D Becomes zero
- ☐ E I need help



Answer

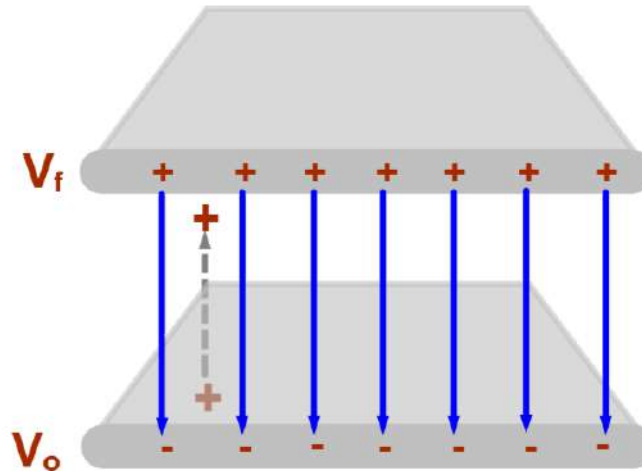
27 If a positive charge moves upwards in the following diagram with a constant velocity, what is the net work done on the charge?

- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



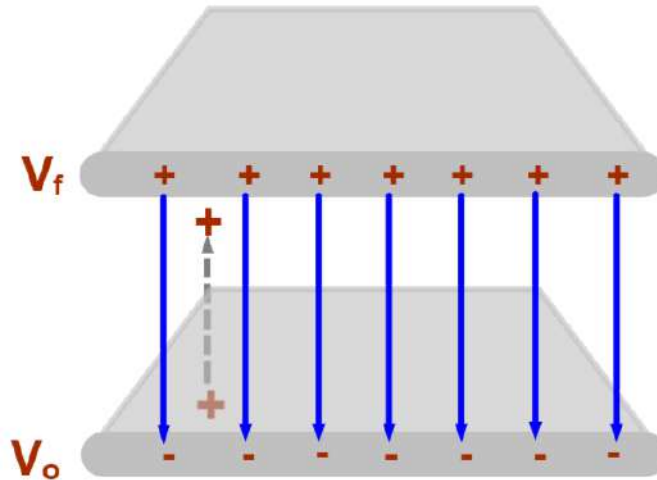
28 If a positive charge moves upwards in the following diagram with a constant velocity, what is the work done on the charge by the Electric Field?

- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



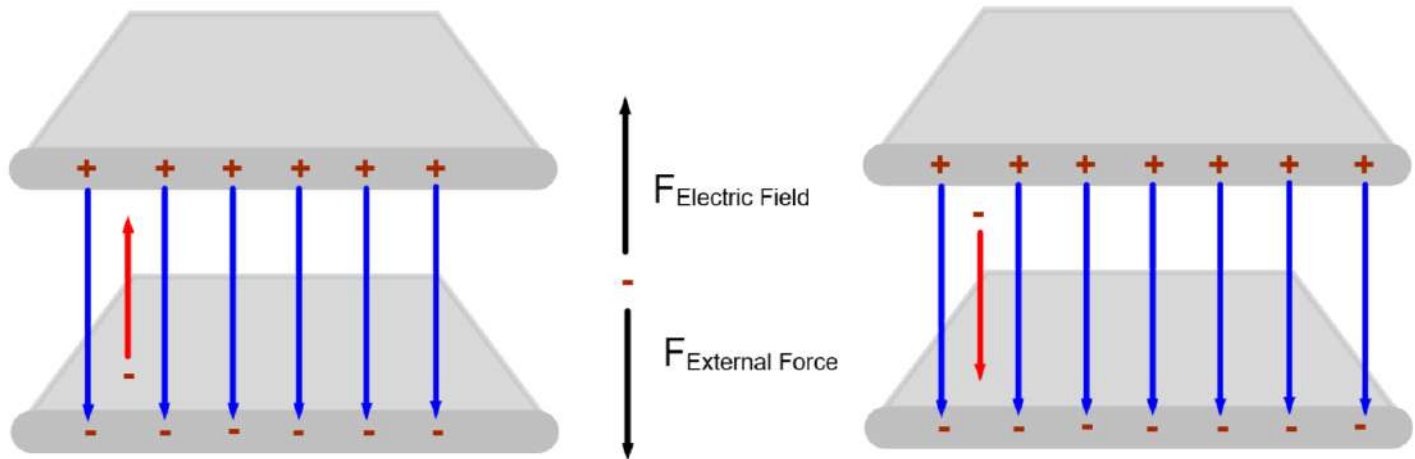
29 If a positive charge moves upwards in the following diagram with a constant velocity, what is the work done on the charge by the external force?

- ☐ A Negative
- ☐ B Zero
- ☐ C Positive
- ☐ D Infinite
- ☐ I need help



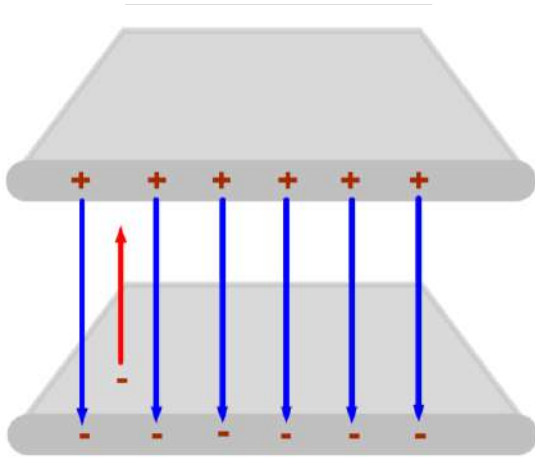
Answer

Uniform Electric Field & Potential Energy

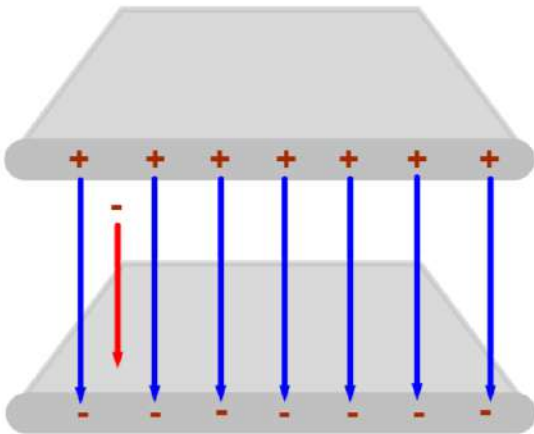
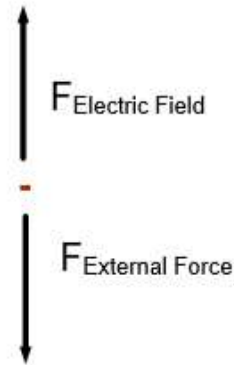


Similar logic works for a negative charge in the same Electric Field. But, the directions of the Electric Field force and the external force are reversed, which will change their signs, and the change in potential energy, as summarized on the next slide.

Uniform Electric Field & Potential Energy

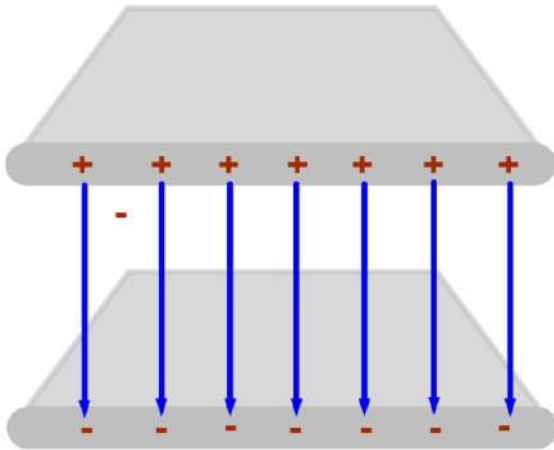


Work done by the external force is negative.
Work done by the Electric Field is positive.
Net force, and hence, the Net Work, is zero.
Potential Energy of the system decreases.



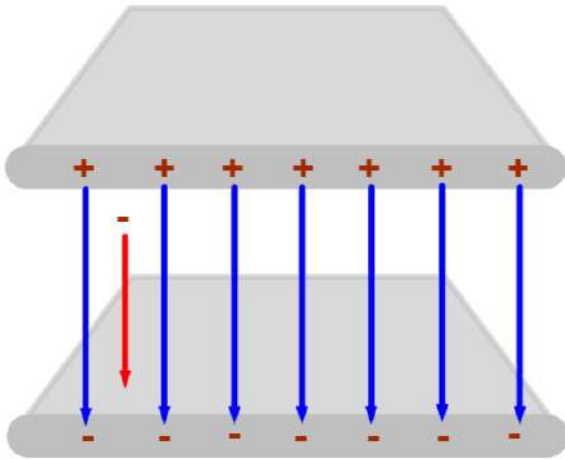
Work done by the external force is positive.
Work done by the Electric Field is negative.
Net force, and hence, the Net Work, is zero.
Potential Energy of the system increases.

- 30 A negative charge is placed between two oppositely charged plates as shown below. Which way will the charge move? What happens to the potential energy of the charge/plate system?



- ☐ A Down, increase
- ☐ B Down, decrease
- ☐ C Up, increase
- ☐ D Up, decrease
- ☐ E I need help

- 31 A negative charge is placed between two oppositely charged plates, and due to an external force moves down with a constant velocity, as shown below. What sign is the work done by the external force? What sign is the work done by the Electric field? What happens to the potential energy of the charge/plate system?



- ☐ A Positive, Negative, Decrease
- ☐ B Positive, Negative, Increase
- ☐ C Negative, Positive, Decrease
- ☐ D Negative, Positive, Increase
- ☐ E I need help

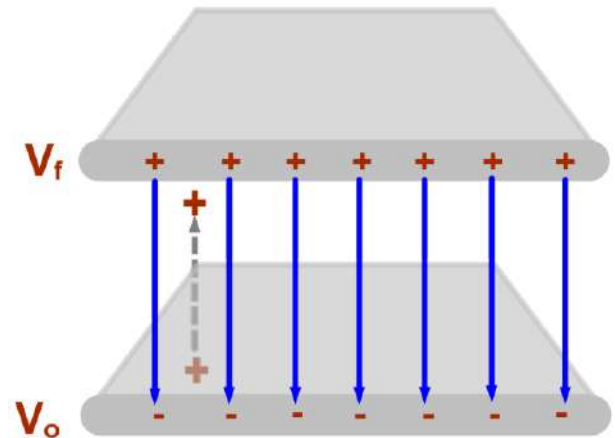
Uniform Electric Field & Voltage

Earlier, the Electric Potential (Voltage) at a point in space due to a charge Q was defined as the potential energy per unit charge in the electric field of Q .

$$V = \frac{U_E}{q} = \frac{kQ}{r}$$

We can't use this approach to calculate V within the infinite planes, because of the great amount of charges present.

V will not equal kQ/r .



Uniform Electric Field & Voltage

For the parallel plane configuration, a simple equation relating Voltage and the Electric Field can be derived by using the relationship of Work to Potential Energy.

The Potential Energy of a system with a conservative force is defined as $U = -W_{\text{force}}$.

Since the electric force is conservative, we will use the above relation.

Uniform Electric Field & Voltage

$$U_E = -W = -F\Delta x = -qE\Delta x$$

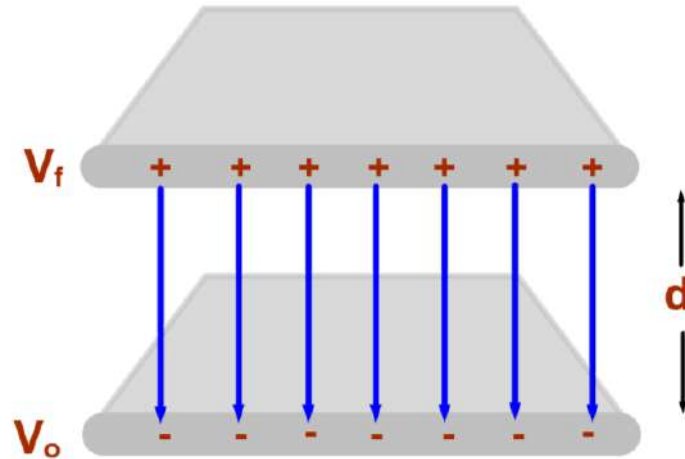
The definition of Work which is the force exerted on the charge, by the Electric Field, as the charge moves through Δx . The force on a charge is qE .

Divide both sides by q

$$\frac{U_E}{q} = -E\Delta x \quad \text{and recognize that} \quad \frac{U_E}{q} = \Delta V$$

$$\Delta V = -E\Delta x \quad \text{or} \quad E = -\frac{\Delta V}{\Delta x}$$

Uniform Electric Field & Voltage



The voltage difference ($\Delta V = V_f - V_o$) between the two planes, separated by a distance, d , is:

$$\Delta V = -E\Delta x$$

$$\Delta V = -Ed$$

Uniform Electric Field & Voltage

$$E = -\frac{\Delta V}{\Delta x}$$

Since the electric field points in the direction of the force on a hypothetical positive test charge, it must also point from higher to lower electric potential.

The negative sign just means that objects in a field feel a force directed from locations with greater potential energy to locations with lower potential energy.

This applies to all forms of potential energy.

Uniform Electric Field & Voltage

$$E = -\frac{\Delta V}{\Delta x}$$

A more intuitive way to understand the negative sign in the relationship is to consider that just like a mass falls down in a gravitational field, from higher gravitational potential energy to lower, a positive charge "falls down" from a higher electric potential (V) to a lower value.

Uniform Electric Field & Voltage

$$E = -\frac{\Delta V}{\Delta x}$$

The electric field can also be described in terms of Volts per meter (V/m) in addition to Newtons per Coulomb (N/C).

$$\frac{V}{m} = \frac{J/C}{m}$$

Since $V = J/C$.

$$\frac{J/C}{m} = \frac{N \cdot m/C}{m}$$

Since $J = N \cdot m$.

$$\frac{N \cdot m/C}{m} = \frac{N}{C}$$

The units are equivalent.

32 In order for a charged object to experience an electric force, there must be a:

- ☐ A large electric potential
- ☐ B small electric potential
- ☐ C the same electric potential everywhere
- ☐ D a difference in electric potential
- ☐ E I need help

33 What is the magnitude(in V/m) of the electric field between two metal plates 0.25 m apart if the potential difference between them is 100 V?

- ☐ 25 V/m
- ☐ 100 V/m
- ☐ 200 V/m
- ☐ 400 V/m
- ☐ I need help

34 An electric field of 3500 N/C is desired between two plates which are 0.0040 m apart; what is magnitude of the Voltage that should be applied?

☐ 14 V

☐ 36 V

☐ 81 V

☐ 102 V

☐ I need help

35 How much Work is done by a uniform 300 N/C Electric Field on a charge of 6.1 mC in accelerating it through a distance of 0.20 m?

☐ 0.18 J

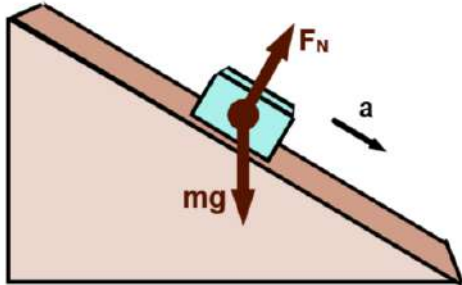
☐ 0.29 J

☐ 0.37 J

☐ 0.49 J

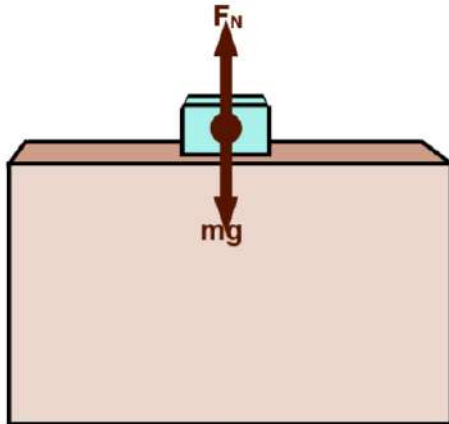
☐ I need help

Gravity Analogue



Consider a mass on an inclined plane. The slope of the plane determines the acceleration and the net force on the object.

The top mass is on an incline, hence it accelerates.

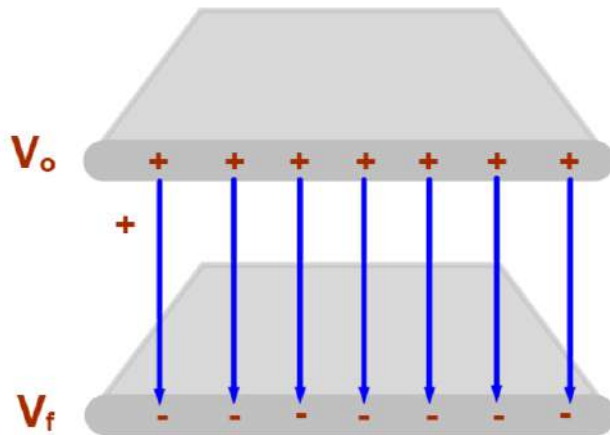


The second mass is on a level plane with a slope of 0, hence $F_{\text{net}} = 0$ and there is no acceleration.

Gravity Analogue

A similar relationship exists with uniform electric fields and voltage.


With the inclined plane, a difference in height was responsible for acceleration. Here, a difference in electric potential (voltage) is responsible for the acceleration. Let's see if we can relate these facts by placing a positive charge in the below field.



$$E = -\frac{\Delta V}{\Delta x}$$

Gravity Analogue

If we look at the energy of the mass on the inclined plane...

$$\begin{aligned} E_0 + W &= E_f & W = 0 & \text{assuming no external work on the block/ plane/earth system} \\ mg\Delta h &= \frac{1}{2}mv_f^2 \\ v_f^2 &= v_0^2 + 2a_m\Delta x \text{ where } v_0 = 0 \text{ then } v_f^2 = 2a_m\Delta x \\ mg\Delta h &= \frac{1}{2}m(2a_m\Delta x) \\ mg\Delta h &= ma_m\Delta x \\ g\Delta h &= a_m\Delta x \\ a_m &= g \frac{\Delta h}{\Delta x} \end{aligned}$$


Gravity Analogue

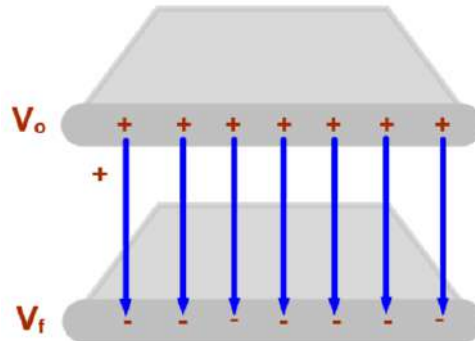
If we look at the energy of a positive charge in the Electric Field between the two planes...

$$E_0 + W = E_f \quad W = 0 \quad \text{assuming no external work on the charge/ plane system}$$

$$qV_0 = qV_f + \frac{1}{2}mv_f^2$$

$$qV_0 - qV_f = \frac{1}{2}mv_f^2$$

$$-q(V_f - V_0) = \frac{1}{2}mv_f^2$$



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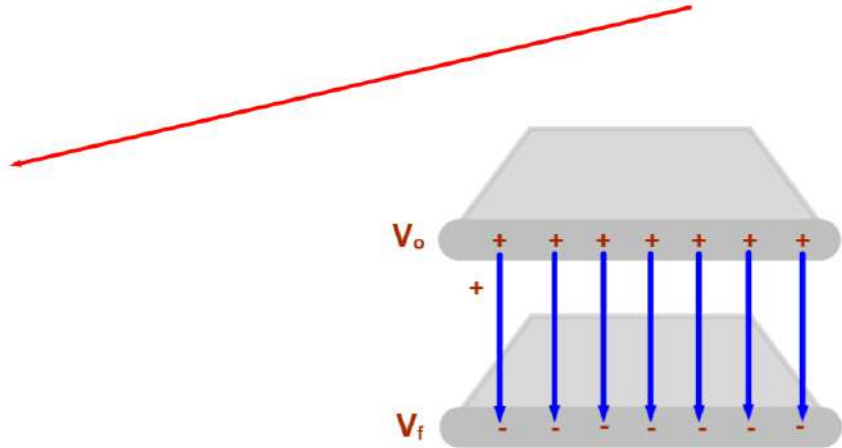
Gravity Analogue

$$-q(V_f - V_0) = \frac{1}{2}mv_f^2$$

$$-q\Delta V = \frac{1}{2}mv_f^2$$

$$-q\Delta V = \frac{1}{2}m(2a_q\Delta x)$$

$$v_f^2 = v_0^2 + 2a_q\Delta x \text{ where } v_0 = 0 \text{ then } v_f^2 = 2a_q\Delta x$$



Continued on the next page.....

Gravity Analogue

Continuing...

$$-q\Delta V = \frac{1}{2}m(2a_q\Delta x)$$

$$-q\Delta V = ma_q\Delta x$$

$$ma_q = F_E = qE$$

$$-q\Delta V = qE\Delta x$$

$$-\Delta V = E\Delta x$$

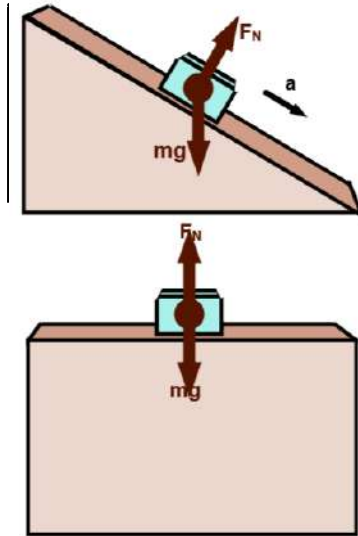
$$E = -\frac{\Delta V}{\Delta x}$$

The electric field is a measure of how the voltage changes over a displacement in space.

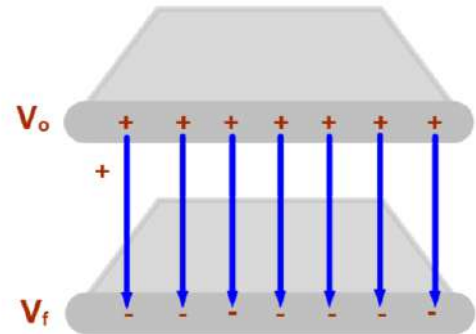
The negative sign tells us that the electric field vectors point in the direction of decreasing voltage. Or from positive to negative voltage.

Gravity Analogue

Let's look at what we've derived:



$$a_m = g \frac{\Delta h}{\Delta x}$$



$$E = -\frac{\Delta V}{\Delta x}$$

How can we compare these results to show how the gravitational field is analogous to the Electric Field?

Gravity Analogue

We want to compare the acceleration of a mass sliding down an incline to the acceleration of a charge moving in an Electric Field.

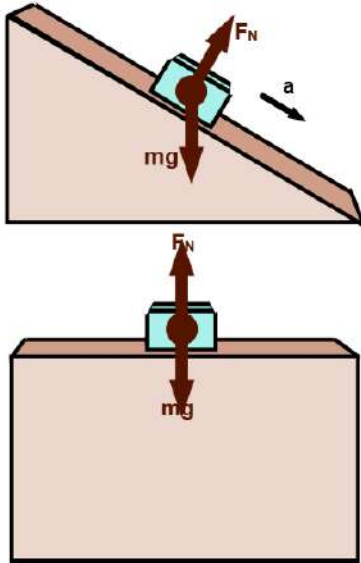
We have the acceleration of the mass: $a_m = g \frac{\Delta h}{\Delta x}$

Now find the acceleration of the charge:

$$F = qE = -q \frac{\Delta V}{\Delta x} = m_q a_q \quad \text{using Newton's Second Law}$$

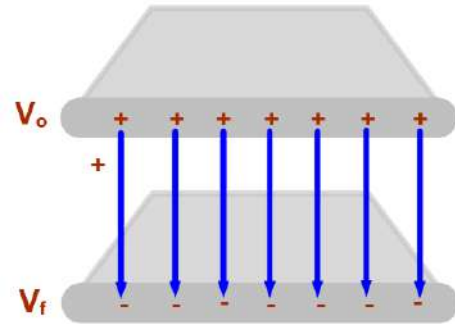
$$a_q = -\frac{q}{m_q} \frac{\Delta V}{\Delta x}$$

Gravity Analogue



$$a_m = g \frac{\Delta h}{\Delta x}$$

A greater g , or a greater angle of incline ($\Delta h/\Delta x$) results in a larger acceleration of the mass.



$$a_q = -\frac{q}{m_q} \frac{\Delta V}{\Delta x}$$

A greater q/m or a greater change in Voltage over distance results in a larger acceleration of the charge.

36 What affects the acceleration of an object down an incline? **Select two answers.**

- ☐ A The angle of the incline.
- ☐ B The magnetic field.
- ☐ C The electric field.
- ☐ D The gravitational field.
- ☐ I need help

37 Varying which of these parameters will guarantee an increase in the magnitude of the Electric Field?

- ☐ A Voltage
- ☐ B Distance between charged plates
- ☐ C Voltage change over time
- ☐ D Voltage change over distance
- ☐ I need help

38 What affects the acceleration of an object down an inclined plane? What impacts the acceleration of a charge in a uniform electric field?

Answer

Electric Force, Field, Potential Energy & Potential Summary

Use only with point charges.

Equations with the "k" are point charges only.

$$\vec{F}_E = \frac{kq_1q_2}{r^2}\hat{r}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$U_E = \frac{kq_1q_2}{r}$$

$$V = \frac{kq}{r}$$

As a handy mnemonic, note how moving left to right, q leaves. And moving from top to down, an r goes away!

Use in any situation.

$$\vec{F} = q\vec{E}$$

$$U_E = qV$$

$$E = -\frac{\Delta V}{\Delta x}$$

For point charges and uniform electric fields

$$U_E = -qEd$$

$$E = -\frac{\Delta V}{d}$$

Only for uniform electric fields

Capacitance and Capacitors

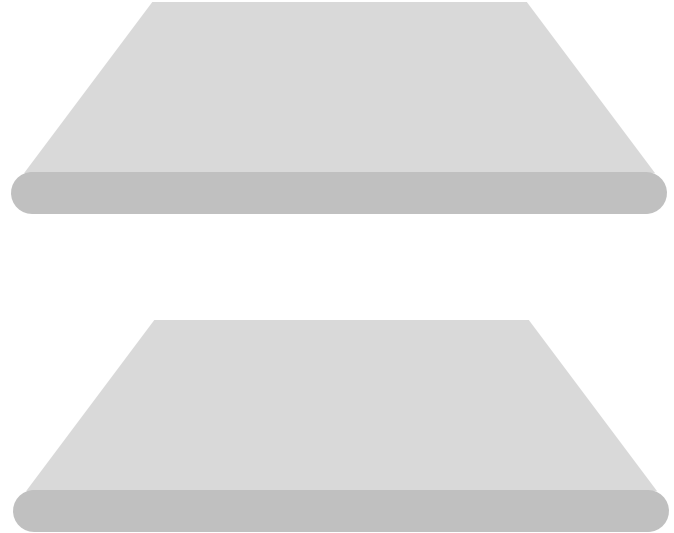
[**Return to
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Contents**](#)

Parallel Plate Capacitors

The simplest version of a capacitor is the

parallel plate capacitor,

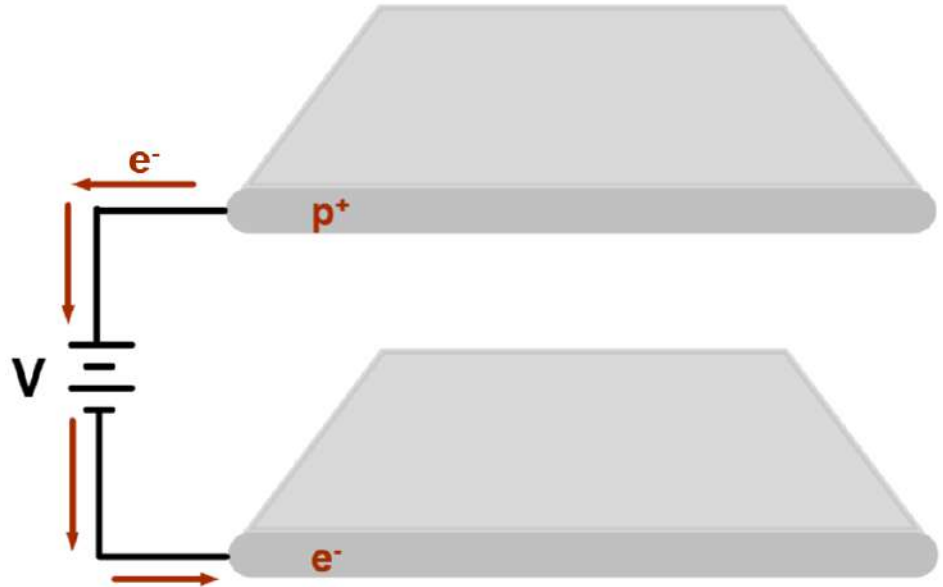
which consists of two metal plates that are parallel to one another and located a small distance apart.



Parallel Plate Capacitors

When a battery is connected to the plates, charge moves between them. Every electron that moves to the negative plate leaves a positive ion behind.

The plates have equal magnitudes of charges, but one is positive, the other negative.

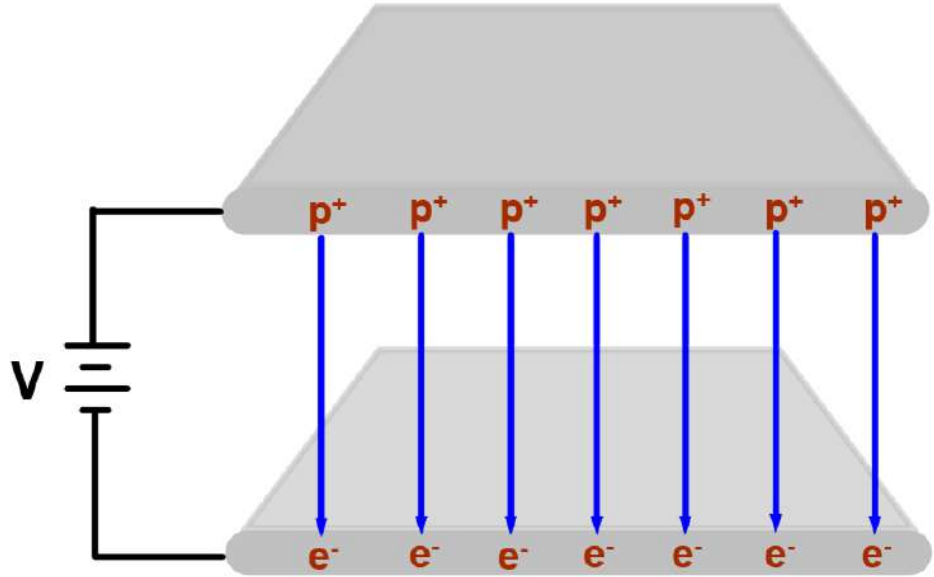


The electrons are attracted by the positive terminal of the battery and move through the wire and the battery to arrive on the other plate.

Parallel Plate Capacitors

Only unpaired protons and electrons are represented here.

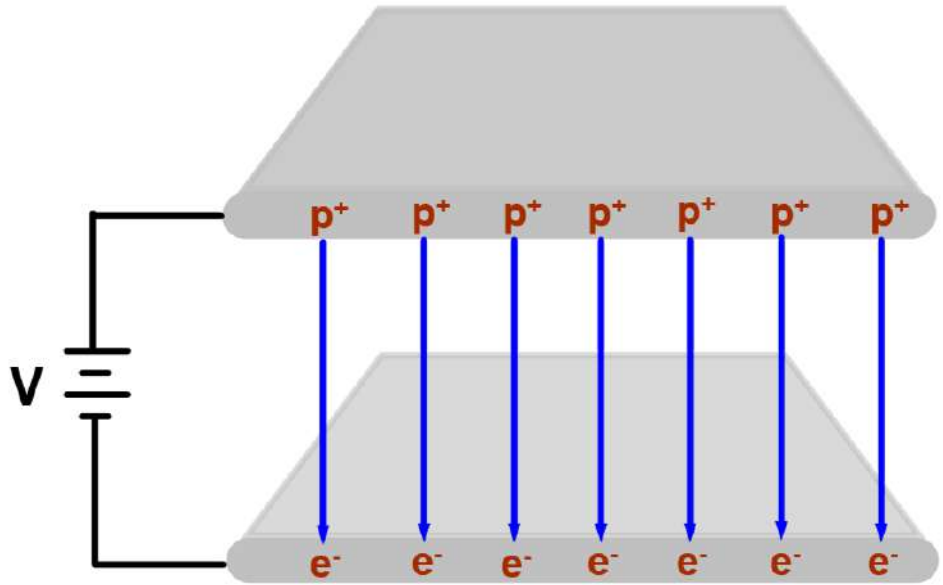
Most of the atoms are neutral since they have equal numbers of protons and electrons.



Parallel Plate Capacitors

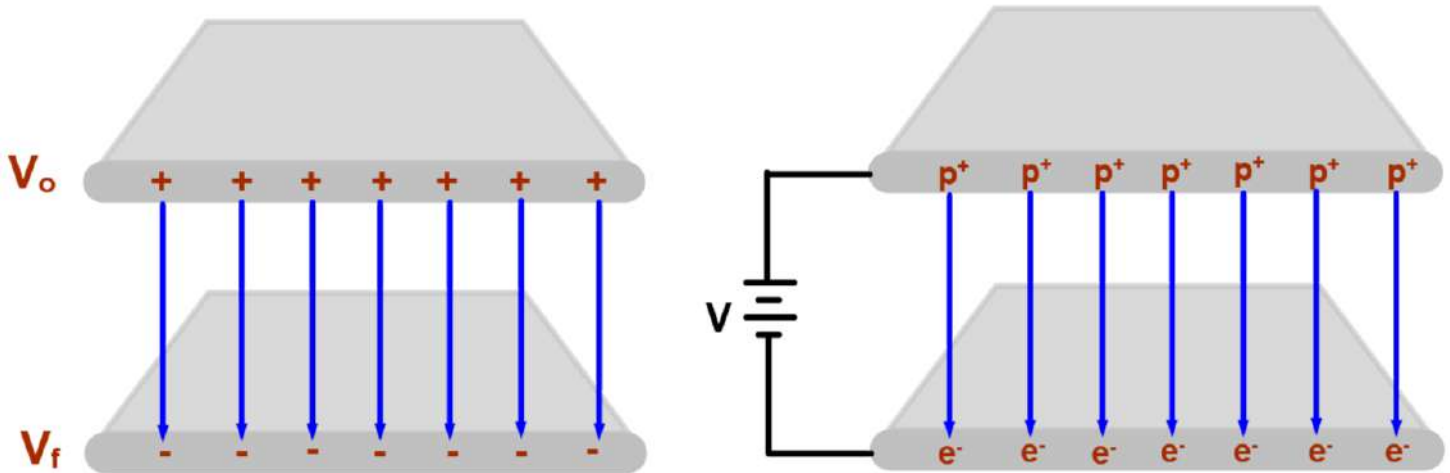
Drawing the Electric Field from the positive to negative charges indicates that the Electric Field is uniform everywhere in a capacitor's gap.

Also, there is no field outside the gap.



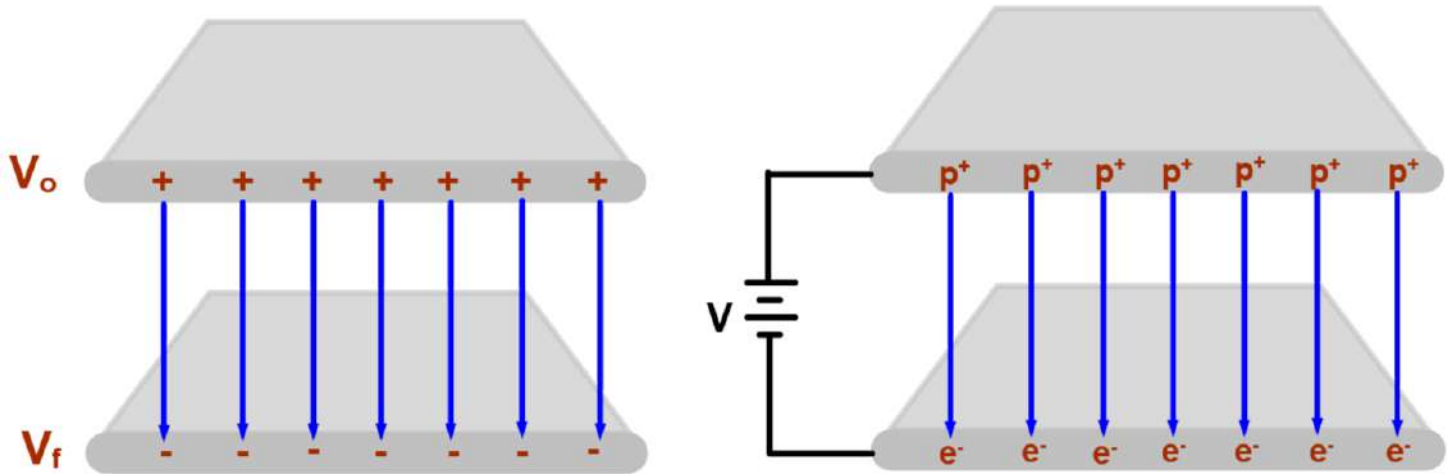
How does this relate to the work done in the previous chapter on parallel infinite planes of charge?

Parallel Plate Capacitors



The planes were assumed to be of infinite size when we made the assumption that the Electric Field was uniform and we placed equal amounts of positive and negative charge on each plane. We stated that there was no Electric Field outside the planes.

Parallel Plate Capacitors



The parallel plates of the capacitor have a finite size, but they are designed so that the plate size is very much greater than the distance between them - so the capacitor properties are very close to the infinite plane case.

The charge separation for the plates is caused by the battery.

39 When a voltage source (battery) is hooked up between two conducting plates, one plate becomes positive and the other negative. How is this accomplished?

- ☐ A Electrons move through the air gap between the plates.
- ☐ B Electrons move through the connecting wire and battery.
- ☐ C Protons move through the air gap between the plates.
- ☐ D Protons move through the connecting wire and battery.
- ☐ I need help

40 When the plates are charged, compare the amount of positive charge on one plate to the negative charge on the other.

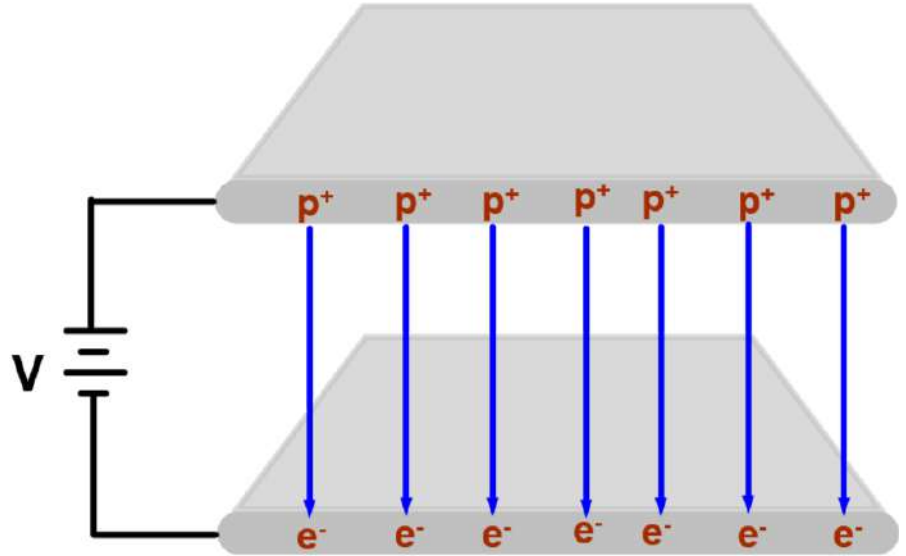
- ☐ A There is more positive charge than negative charge.
- ☐ B There is more negative charge than positive charge.
- ☐ C The amount of negative charge on one plate equals the amount of positive charge on the other.
- ☐ D There is no charge on either plate; both are neutral because of the Conservation of Charge.
- ☐ I need help

Parallel Plate Capacitors

ANY capacitor can store a certain amount of charge for a given voltage. That is called its capacitance, C.

$$C = \frac{Q}{V}$$

This is a definition and is true for all capacitors, not just parallel plate capacitors.



Parallel Plate Capacitors

$$C = \frac{Q}{V}$$

The unit of capacitance is the farad (F). A farad is a Coulomb per Volt (can you see that from the definition above?).

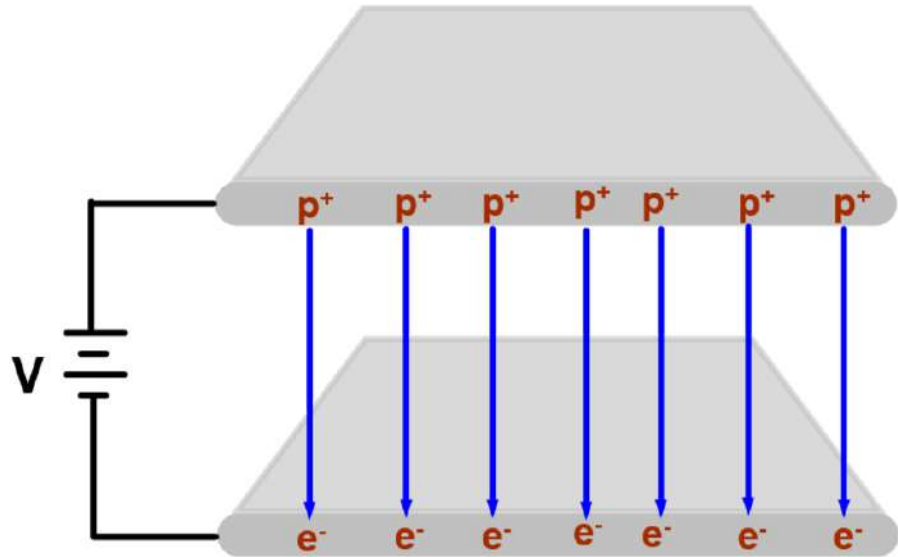
A farad is huge; so capacitance is given as

picofarad ($1\text{pf} = 10^{-12}\text{ F}$)

nanofarad ($1\text{nf} = 10^{-9}\text{ F}$)

microfarad ($1\mu\text{f} = 10^{-6}\text{ F}$)

millifarads ($1\text{mf} = 10^{-3}\text{ F}$)



41 What is the capacitance of a fully charged capacitor that has a charge of $25 \mu\text{C}$ and a potential difference of 50 V ?

☐ A $0.5 \mu\text{F}$

☐ B $2 \mu\text{F}$

☐ C $0.4 \mu\text{F}$

☐ D $0.8 \mu\text{F}$

☐ I need help

42 How much charge is on the positive plate of a capacitor with an applied Voltage of 9.0 V and a capacitance of $1.2 \mu\text{F}$?

☐ A $0.13 \mu\text{C}$

☐ B $0.26 \mu\text{C}$

☐ C $7.5 \mu\text{C}$

☐ D $11 \mu\text{C}$

☐ I need help

43 What is the applied voltage that results in $14\ \mu\text{C}$ of charge being placed on a capacitor with a capacitance of $9.6\ \mu\text{F}$?

☐ A $0.75\ \text{V}$

☐ B $1.5\ \text{V}$

☐ C $7.0\ \text{V}$

☐ D $13\ \text{V}$

☐ I need help

Answer

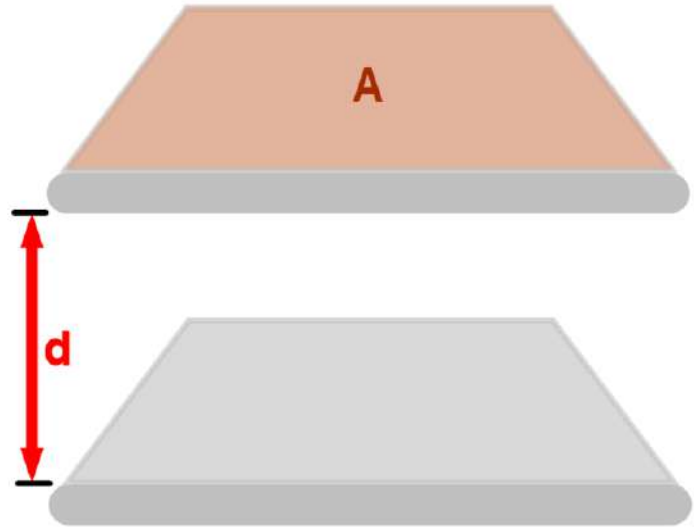
44 A fully charged $50\ \mu\text{F}$ capacitor has a potential difference of $100\ \text{V}$ across its plates. How much charge is stored on the positive plate of the capacitor?

- ☐ A $6\ \text{mC}$
- ☐ B $4\ \text{mC}$
- ☐ C $5\ \text{mC}$
- ☐ D $9\ \text{mC}$
- ☐ E I need help

Parallel Plate Capacitors

The Area of the capacitor is the surface area of **one plate**, and is represented by the letter A .

The distance between the plates is represented by the letter d .



Remember, both plates have the same surface area, A .

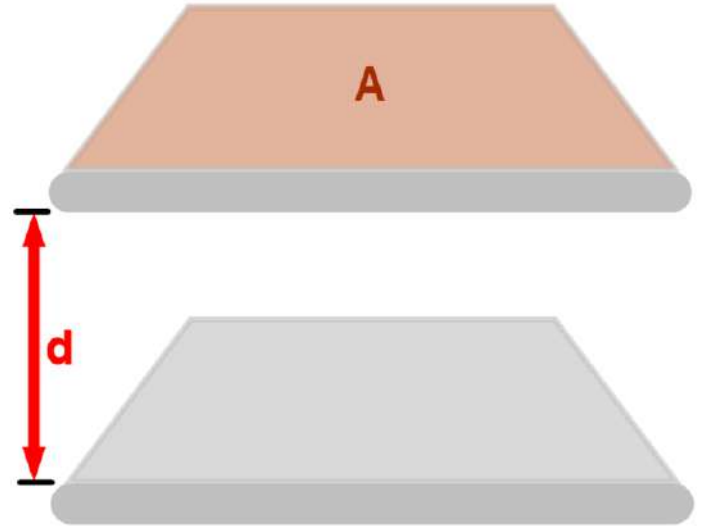
Parallel Plate Capacitors

For Parallel Plate Capacitors, the capacity to store charge increases with the area of the plates and decreases as the plates get farther apart.

$$C \propto A \quad C \propto \frac{1}{d}$$

Combining these two relationships:

$$C \propto \frac{A}{d}$$



Parallel Plate Capacitors

Once you have an equation that involves a proportion sign, you can replace it by an equal sign and a constant - the constant of proportionality.

For capacitors, it has been theoretically predicted and experimentally measured, and verified, that the constant of proportionality for this equation is the:

Permittivity of Free Space

symbolized by ϵ_0

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

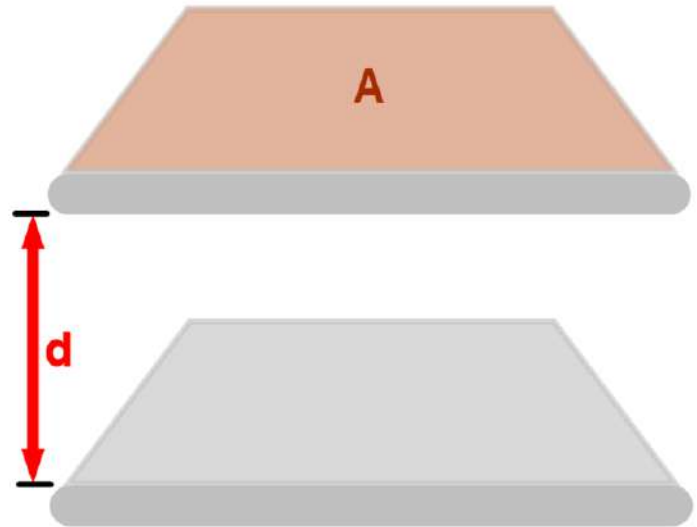
Parallel Plate Capacitors

So for Parallel Plate Capacitors:

$$C \propto \frac{A}{d} \text{ becomes } C = \frac{\epsilon_0 A}{d}$$

The larger the Area, A,
the higher the capacitance.

The closer together the
plates get, the higher the
capacitance.



45 A parallel plate capacitor has a capacitance C_0 . If the area on each plate is doubled and the distance between the plates is cut in half, what will be the new capacitance?

- ☐ A $C_0/4$
- ☐ B $C_0/2$
- ☐ C $4C_0$
- ☐ D $2C_0$
- ☐ E I need help

46 What is the capacitance of a capacitor that has a plate area of $1.2 \times 10^{-3} \text{ m}^2$ and a distance between the plates of $8 \times 10^{-3} \text{ m}$?

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

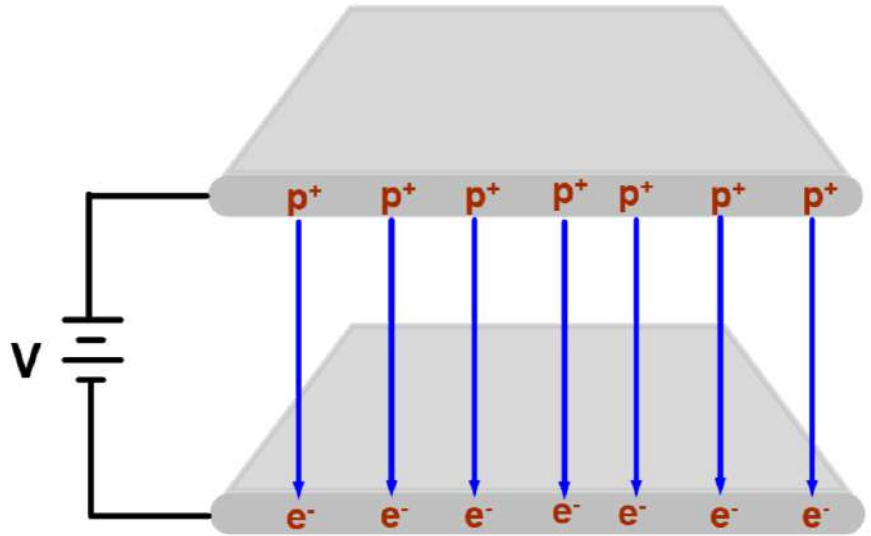
- ☐ A 1.3 pF
- ☐ B 1.5 pF
- ☐ C 6.7 pF
- ☐ D 7.5 pF
- ☐ I need help

- 47 A fellow student tells you that she has a 2.0 pF capacitor, and its dimensions are $A = 3.6 \times 10^{-4} \text{ m}^2$ and the distance between the plates is $4.1 \times 10^{-3} \text{ m}$. Does that sound right? Justify your answer.

Parallel Plate Capacitors

Imagine you have a fully charged capacitor.

If you disconnect the battery and change either the area or distance between the plates, what do you know about the charge on the capacitor?



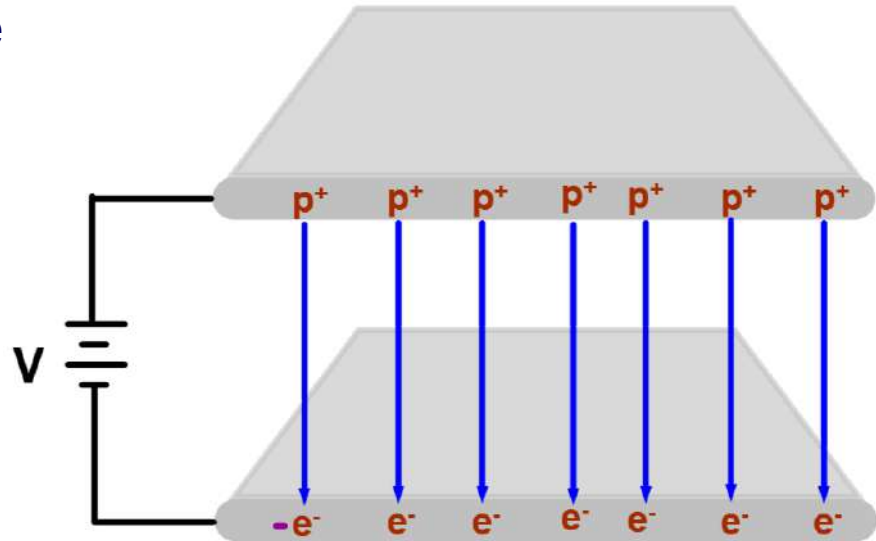
Think about the Conservation of Charge and how charges flow before going to the next slide for the answer.

Parallel Plate Capacitors

The charge remains the same
there is no where for it to go.

Charge needs to flow
through a conductor, and
once the battery is
disconnected, there is no
conduction path, so no
charges flow.

The Conservation of Charge - the
charge you start with is equal to the
final charge.

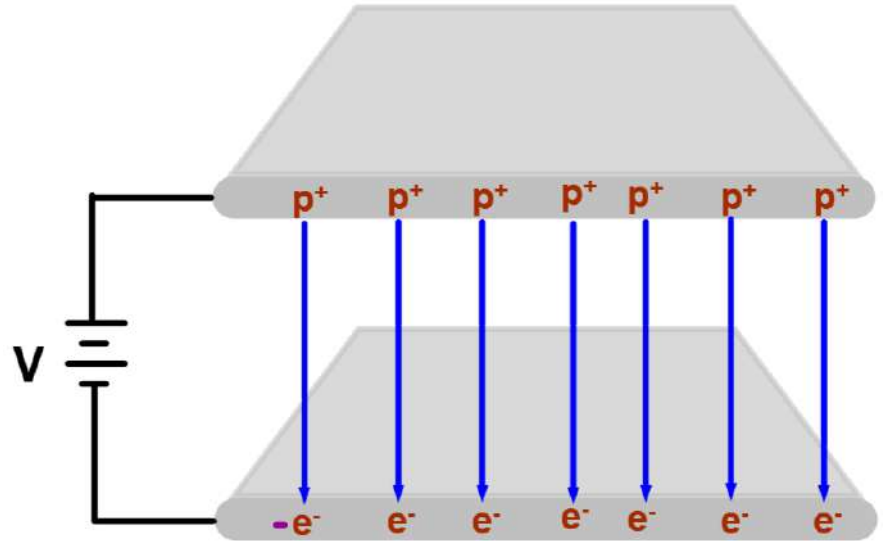


*That's why you should never handle a
capacitor if you don't know where it has
been - it might be holding a charge that
would go through your body to ground.
That can be very dangerous.*

Parallel Plate Capacitors

Imagine you have a fully charged capacitor.

If you keep the battery connected and change either the area or distance between the plates, what do you know about the voltage across the plates?



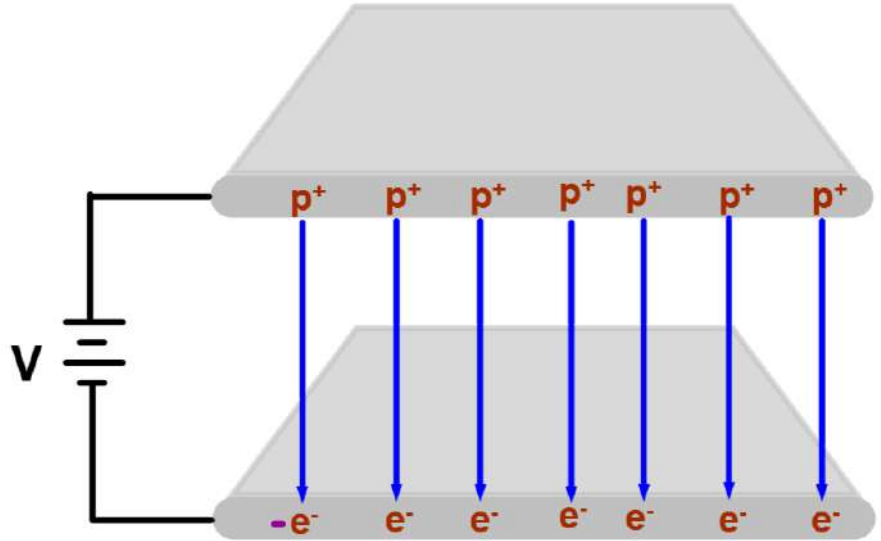
Before going to the next slide for the answer, think of the Conservation of Energy.

Parallel Plate Capacitors

The voltage remains the same.

Voltage is the amount of energy delivered to each charge as it passes through the battery.

Energy is conserved, regardless of the capacitor's physical configuration. The energy stays the same, hence the Voltage stays the same.



48 A parallel plate capacitor is charged by connection to a battery and the battery is disconnected. What will happen to the charge on the capacitor and the voltage across it if the area of the plates decreases and the distance between them increases?

- ☐ A Both increase.
- ☐ B Both decrease.
- ☐ C The charge remains the same and voltage increases.
- ☐ D The charge remains the same and voltage decreases.
- ☐ E I need help

49 A parallel plate capacitor is charged by connection to a battery and remains connected. What will happen to the charge on the capacitor and the voltage across it, if the area of the plates increases and the distance between them decreases?

- ☐ A Both increase.
- ☐ B Both decrease.
- ☐ C The voltage remains the same and the charge increases.
- ☐ D The voltage remains the same and the charge decreases.
- ☐ I need help

50 When a charged capacitor is left connected to a battery, and its physical dimensions change, what law explains why the voltage across the capacitor remains the same? If the capacitor is fully charged, and then disconnected from the battery, what law explains why the charge on the plates remains the same? **Select two answers.**

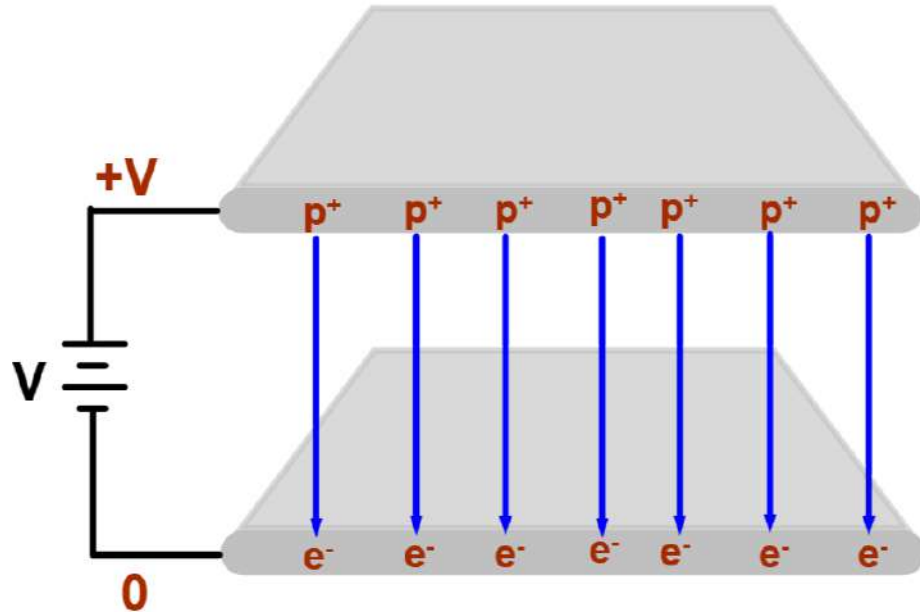
- ☐ A Conservation of Mass
- ☐ B Conservation of Energy
- ☐ C Conservation of Charge
- ☐ D Newton's Third Law
- ☐ I need help

Parallel Plate Capacitor

Electric Field and Potential

After being fully charged, there is a potential difference, ΔV , between the two plates. ΔV is equal to the voltage supplied by the battery.

There is a uniform electric field, \mathbf{E} , between the plates.



We learned earlier that with a uniform Electric Field, $\Delta V = -E\Delta x$; this is true in the case of the parallel plate capacitor.

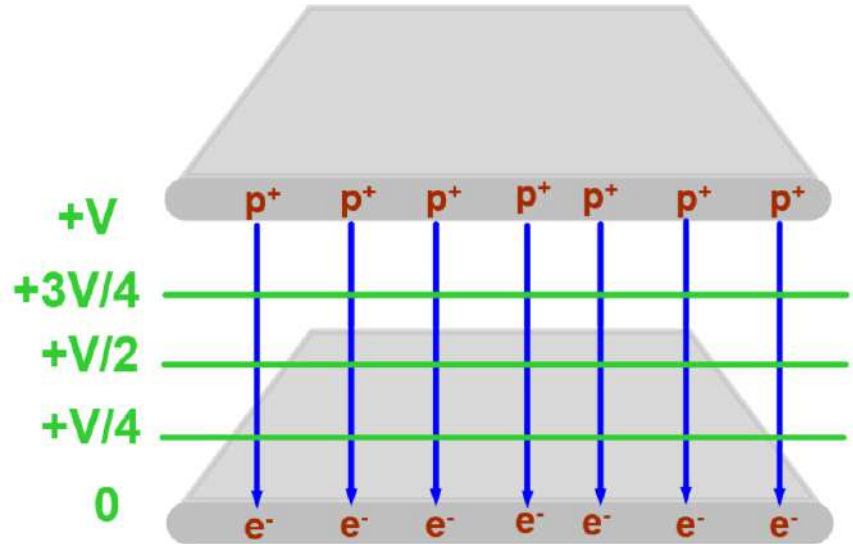
Parallel Plate Capacitor

Electric Field and Potential

The Electric Field is constant everywhere in the gap between the plates.

The Electric Potential (Voltage) declines uniformly from $+V$ to 0 within the gap.

The equipotential lines (in green) are always perpendicular to the Electric Field.



$$\Delta V = -E\Delta x$$

Parallel Plate Capacitor

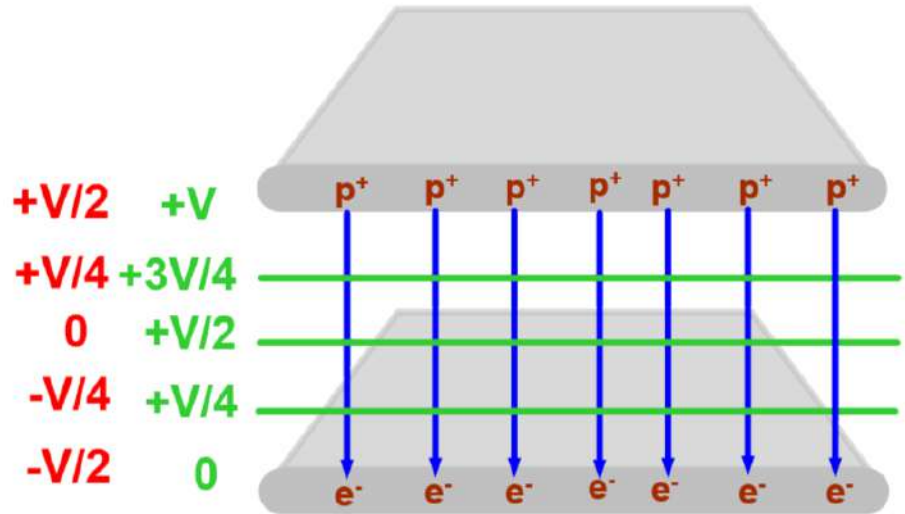
Electric Field and Potential

Why is the negatively charged plate showing a voltage of 0 V? Shouldn't it be negative?

It's somewhat arbitrary.
The important fact is that $\Delta V = V$.

We could've labeled the voltages as shown in red.

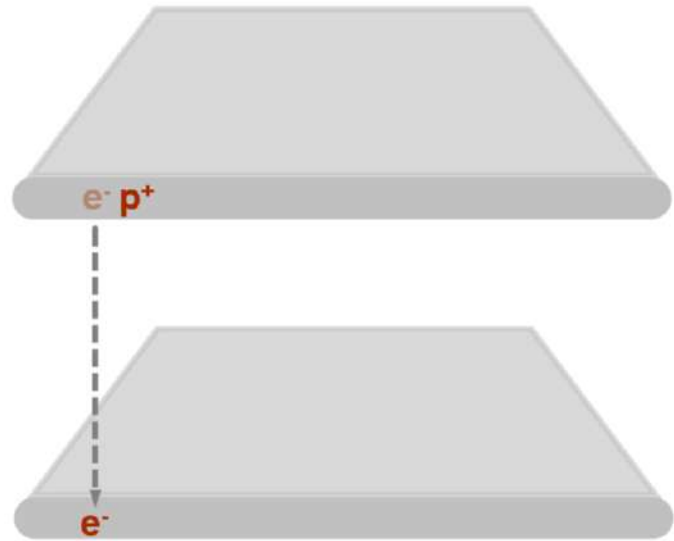
But people like positive numbers better than negative numbers.....



Energy stored in a Capacitor

The energy stored in ANY capacitor is given by formulas most easily derived from the parallel plate capacitor.

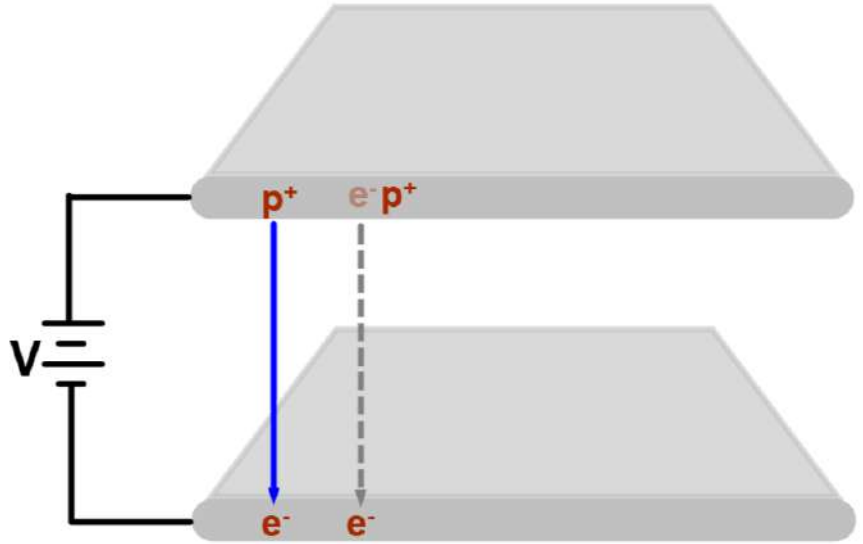
Consider how much work it would take to move a single electron between two initially uncharged plates.



Energy stored in a Capacitor

That takes ZERO work since there is no difference in voltage.

However, to move a second electron to the negative plate requires work to overcome the repulsion from the first electron...and to overcome the attraction of the positive plate.

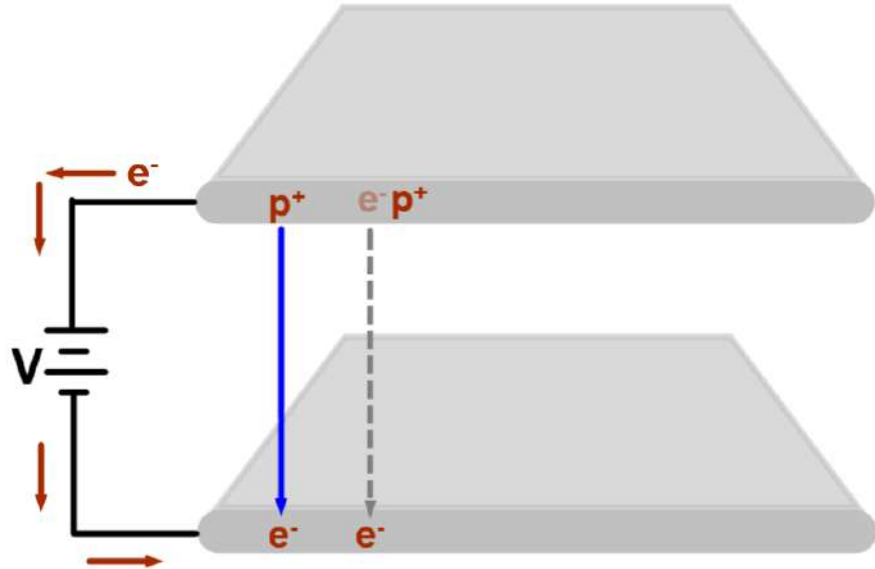


The work is provided by the potential difference within the battery connecting the two plates.

Energy stored in a Capacitor

The electrons are attracted by the positive terminal of the battery and move through the wire and the battery to arrive on the other plate.

Since the Electric force is conservative, the work done on this longer path is equal to the work done if the electron just went straight between the plates, and we'll use that as a model.



The electrons can't move directly between the plates, because the air between them is a very good insulator.

51 Does it take work to move an electron between two surfaces at the same electric potential? Explain.

Answer

52 An electron moves in a uniform electric field. Over which path is the most work done by the Electric field?

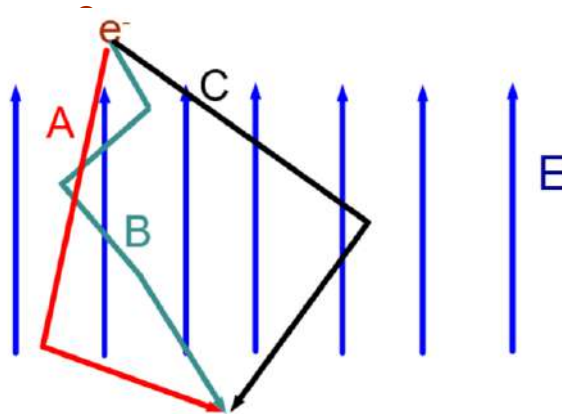
☐ A A

☐ B B

☐ C C

☐ D The work is the same for all paths shown.

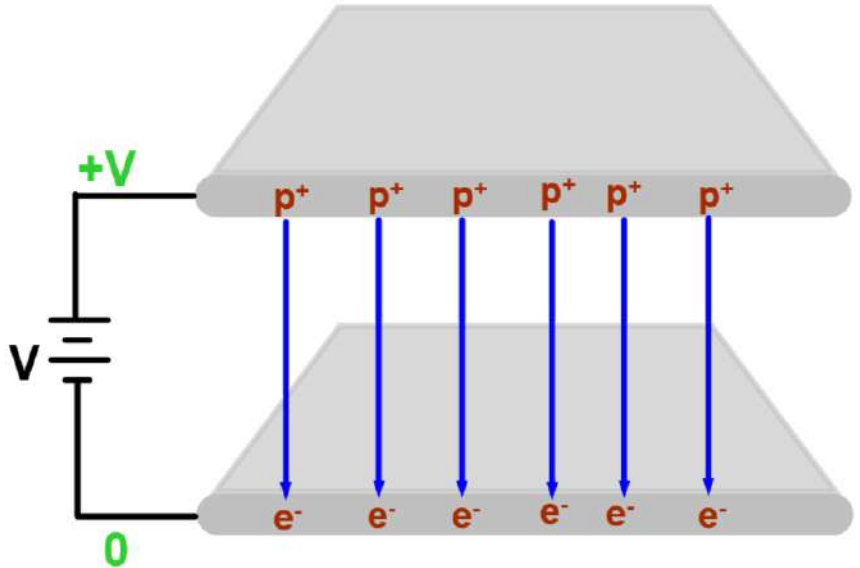
☐ I need help



Energy stored in a Capacitor

To move the last electron from the positive plate to the negative plate requires carrying it through a voltage difference of ΔV .

The work required to do that is $q\Delta V$...



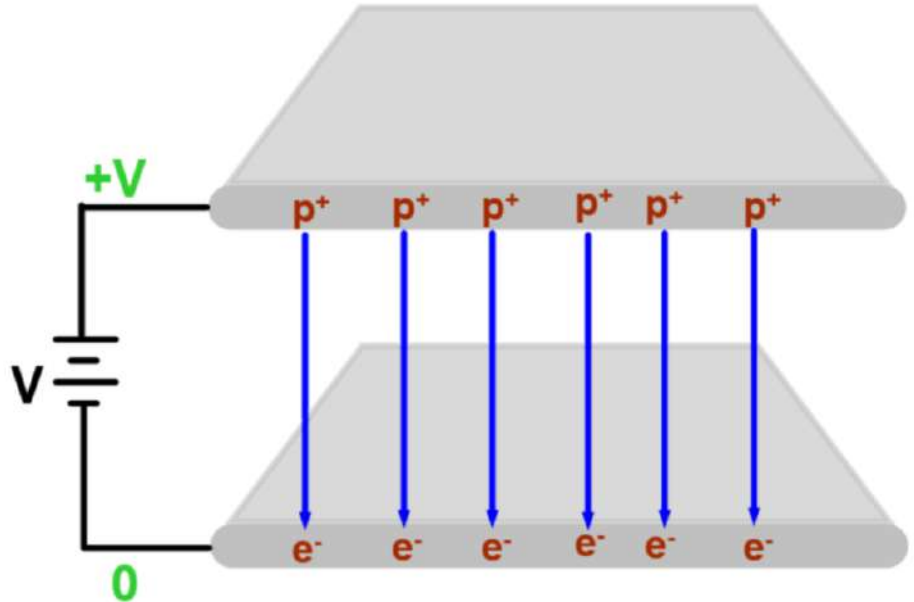
Since the charge of a electron is $-e$, and the difference in potential for the last electron is V , the work to move that electron is equal to $-e(0-V) = eV$.

Energy stored in a Capacitor

The work to move the first electron is zero.

The work to move the last electron is eV .

Then the AVERAGE work for all electrons is $eV/2$.



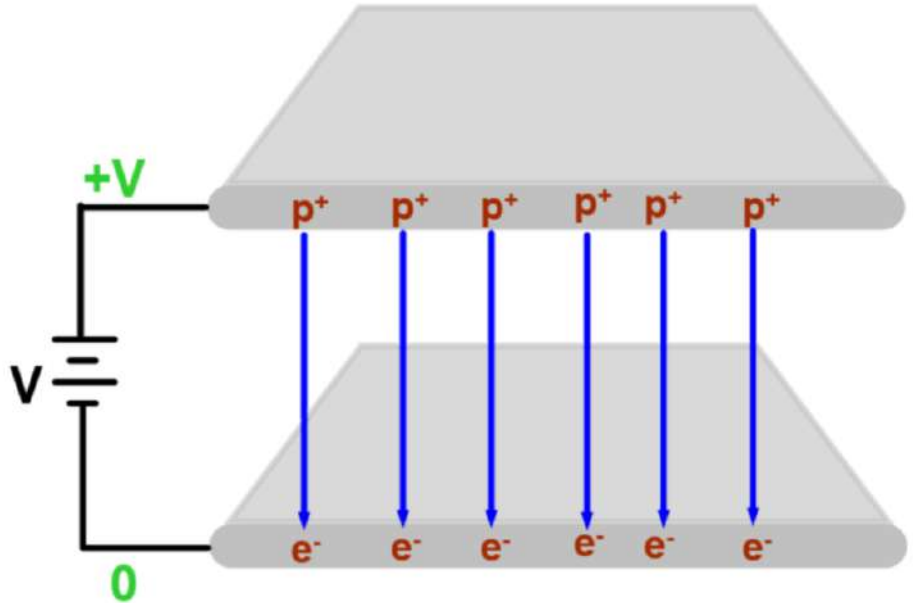
The reason we can use this average value is that the Voltage decreases linearly from the top to the bottom plate.

Energy stored in a Capacitor

Then, the work needed to move a total charge Q from one plate to the other is given by

$$W = QV/2$$

That energy is stored in the electric field within the capacitor.



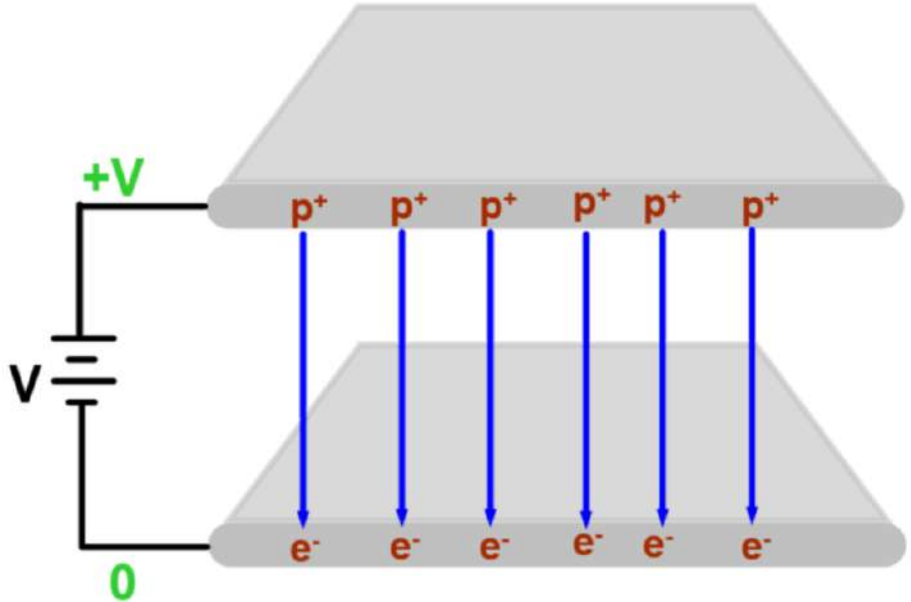
This is one of the functions of a capacitor - it stores energy to be used at a later time.

Energy stored in a Capacitor

The energy stored in a capacitor is given by:

$$U_c = \frac{QV}{2}$$

Where Q is the charge on one plate, V is the voltage difference between the plates, and U_c is the stored potential energy.



Note that the top plate is charged to $+Q$ and the bottom plate is charged to $-Q$. Thus charge is conserved as the original plates were neutral.

Energy stored in a Capacitor

Using our equation for capacitance ($C=Q/V$) and our equation for electric potential energy in a capacitor, we can derive two more expressions for the potential energy.

$$\begin{array}{ccccc} C = \frac{Q}{V} & \xrightarrow{\text{solve for } V} & V = \frac{Q}{C} & \xrightarrow{\text{solve for } Q} & Q = CV \\ & & \downarrow \text{substitute} & & \downarrow \text{substitute} \\ U_C = \frac{QV}{2} & & U_C = \frac{QV}{2} & & U_C = \frac{QV}{2} \\ & & U_C = \frac{Q}{2} \frac{Q}{C} & & U_C = \frac{(CV)V}{2} \\ & \cdots & U_C = \frac{Q^2}{2C} & \cdots & U_C = \frac{1}{2} CV^2 \end{array}$$

53 How much energy is stored in a fully charged capacitor with 15 nC of charge on one plate and 20.0 V across its plates?

- ☐ A 0.10 μJ
- ☐ B 0.15 μJ
- ☐ C 0.30 μJ
- ☐ D 0.60 μJ
- ☐ E I need help

54 How much energy is stored in a fully charged 3.0 mF parallel plate capacitor with 2.0 V across its plates?

- ☐ A 6.0 mJ
- ☐ B 3.0 mJ
- ☐ C 5.0 mJ
- ☐ D 12 mJ
- ☐ E I need help

55 How much energy is stored in a fully charged 12.0 pF capacitor that has 9.00 μC of charge?

- ☐ A 2.25 J
- ☐ B 3.38 J
- ☐ C 4.20 J
- ☐ D 5.80 J
- ☐ E I need help

56 A parallel plate capacitor is connected to a battery. The capacitor becomes fully charged and stays connected to the battery. What will happen to the energy held in the capacitor if the area of the plates increases?

- ☐ A Remains the same
- ☐ B Increase
- ☐ C Decrease
- ☐ D Zero
- ☐ E I need help

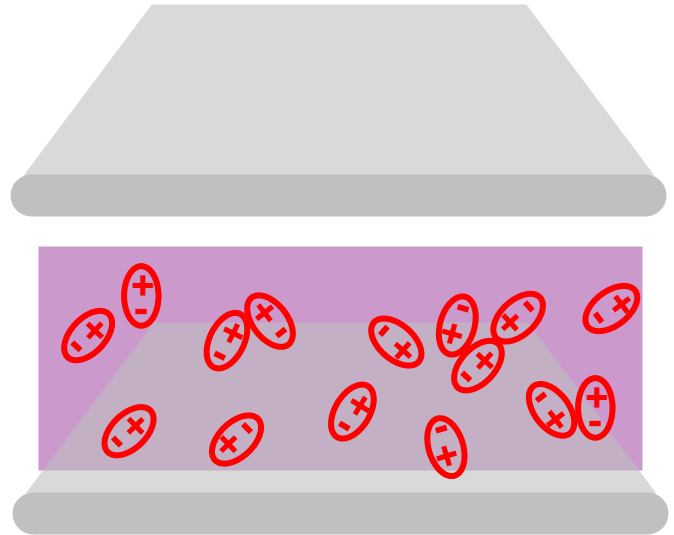
57 A parallel plate capacitor is connected to a battery. The capacitor becomes fully charged and is disconnected from the battery. What will happen to the energy stored in the capacitor if the plates are separated to a greater distance?

- ☐ A Remains the same
- ☐ B Increases
- ☐ C Decreases
- ☐ D Zero
- ☐ E I need help

Dielectric impact on Capacitance

Capacitance can be increased by inserting a **dielectric** (an insulator) into the gap. The atoms in a dielectric are slightly polarized (the electrons are not uniformly distributed, resulting in a separation of charge).

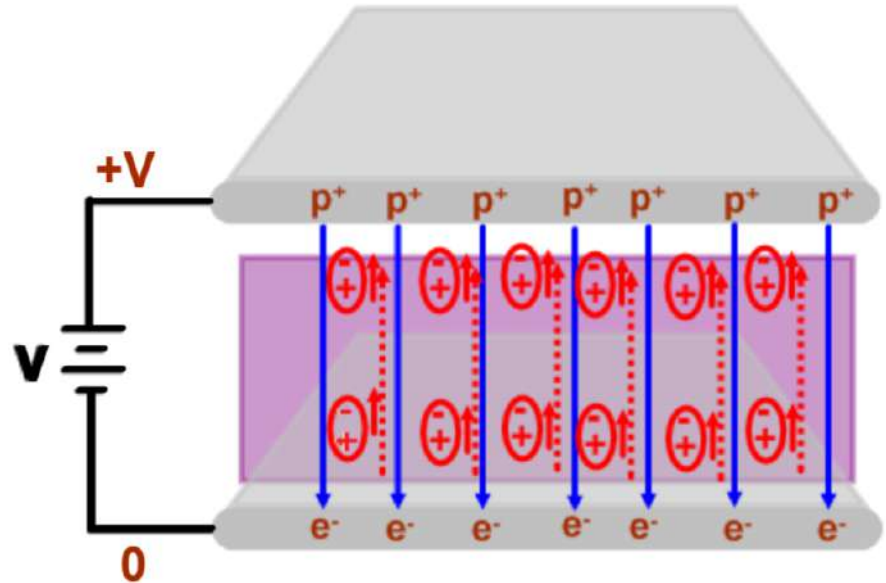
Before the plates are charged, the group of the dielectric's atoms are unpolarized, and each atom points in a random direction.



Dielectric impact on Capacitance

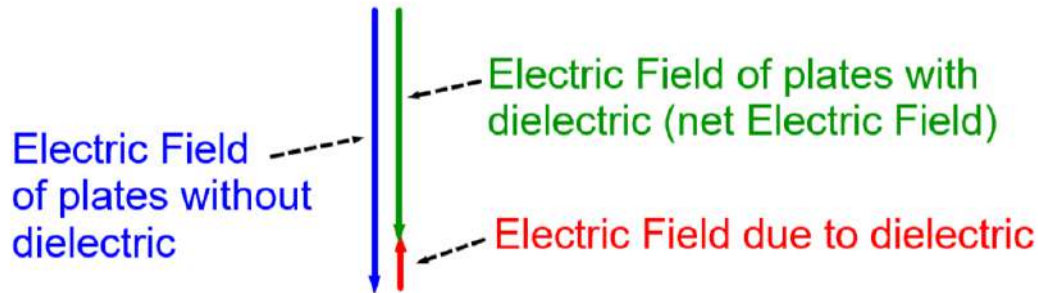
When the plates are charged, the atoms line up with their negative ends closer to the positive plate - the dielectric material is polarized.

An internal Electric field is now generated by the dielectric and it opposes the applied Electric field from the battery.



The strength of this internal field depends on the material of the dielectric.

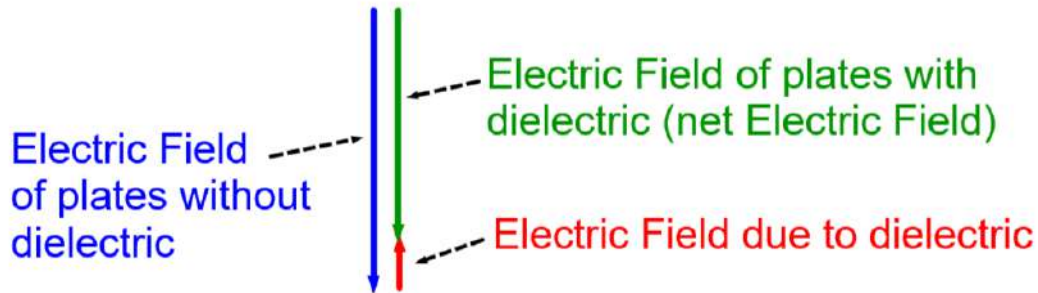
Dielectric impact on Capacitance



This reduces the net Electric field for a given applied voltage.

This is helpful, because if the Electric field between the plates gets too large, it will cause a "dielectric breakdown" in that space.

Dielectric impact on Capacitance



This will enable charge to flow directly between the two plates - a similar effect to lightning, or the sparks you get when you touch a door knob after rubbing your feet on carpet!

This is not good. The capacitor is then discharged and will not store energy - it could also harm the circuit or a person touching it.

Dielectric impact on Capacitance

The dielectric material was originally neutral. Once it became polarized, more charge was now present (the charge on the plates plus the charge induced in the dielectric). Thus the total charge in the capacitor increased.

The voltage stayed the same (it is still connected to the same battery).

Since $C = Q/V$, the capacitance of the configuration increases.

An engineering benefit to the dielectric is that it insulates the plates from each other, and thus the distance between them may be made very small, and charges will not flow between the plates (think sparks), but will flow through the battery.

Dielectric impact on Capacitance

Every material has a dielectric constant, κ (kappa). A sample table of values is shown to the right.

For a vacuum, $\kappa = 1$;
 κ_{air} is about 1. If a dielectric is present, then:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

Material	Dielectric constant (κ)
Acrylic	2.7
Air	1.0
Asbestos	4.8
Bakelite	3.5
Paper	3.0
Silicon	11

The larger the value of κ , the larger the capacitance (C).

58 What is the capacitance of a capacitor that has a plate area of $1.2 \times 10^{-3} \text{ m}^2$ and a distance between the plates of $8.0 \times 10^{-3} \text{ m}$? The capacitor has a dielectric made of acrylic between the two plates ($\kappa = 2.7$).

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

- ☐ A 3.6 pF
- ☐ B 4.1 pF
- ☐ C 18 pF
- ☐ D 20 pF
- ☐ E I need help

59 A capacitor of capacitance 4.6 pF has a plate area of $1.2 \times 10^{-3} \text{ m}^2$ and a distance between the plates of $8.0 \times 10^{-3} \text{ m}$, with a dielectric between the plates. Using the below table, determine which dielectric is present .

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

- ☐ A Air
- ☐ B Bakelite
- ☐ C Paper
- ☐ D Silicon
- ☐ E I need help

Material	Dielectric constant (κ)
Acrylic	2.7
Air	1.0
Asbestos	4.8
Bakelite	3.5
Paper	3.0
Silicon	11

60 Why is silicon a better material to use as a dielectric in a capacitor than asbestos? You may use the chart to guide your answer.

Material	Dielectric constant (κ)
Acrylic	2.7
Air	1.0
Asbestos	4.8
Bakelite	3.5
Paper	3.0
Silicon	11

Capacitor Summary

These two equations are true for all capacitors.

$$C = \frac{Q}{V}$$

$$U_c = \frac{QV}{2} = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

The following equation is true for Parallel Plate capacitors as it depends on the specific geometry of parallel planes. Unless indicated otherwise, $\kappa = 1$.

$$C = \frac{\kappa\epsilon_0 A}{d}$$

Combining these equations can solve many problems related to the voltage, charge, Electric field and capacitance of a capacitor.

