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NEW JERSEY CENTER
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AP Physics 1

Simple Harmonic Motion

2017-07-20

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Period and Frequency

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SHM and Circular Motion

There is a deep connection between Simple Harmonic Motion (SHM) and Uniform Circular Motion (UCM).

Simple Harmonic Motion can be thought of as a one-dimensional projection of Uniform Circular Motion.

All the ideas we learned for UCM, can be applied to SHM...we don't have to reinvent them.

So, let's review circular motion first, and then extend what we know to SHM.

Click here to see how circular motion relates to simple harmonic motion.

Period

The time it takes for an object to complete one trip around a circular path is called its Period.

The symbol for Period is "T"

Periods are measured in units of time; we will usually use seconds (s).

Often we are given the time (t) it takes for an object to make a number of trips (n) around a circular path. In that case,

$$T = \frac{t}{n}$$

1 If it takes 50 seconds for an object to travel around a circle 5 times, what is the period of its motion?

- 2 s
- 5 s
- 10 s
- 25 s
- I need help

Answer

2 If an object is traveling in circular motion and its period is 7.0 s, how long will it take it to make 8 complete revolutions?

- A 18 s
- B 23 s
- C 41 s
- D 56 s
- E I need help

Answer

Frequency

The number of revolutions that an object completes in a given amount of time is called the frequency of its motion.

The symbol for frequency is "f"

Frequencies are measured in units of revolutions per unit time; we will usually use 1/seconds (s^{-1}). Another name for s^{-1} is Hertz (Hz). Frequency can also be measured in revolutions per minute (rpm), etc.

Often we are given the time (t) it takes for an object to make a number of revolutions (n). In that case,

$$f = \frac{n}{t}$$

3 An object travels around a circle 50 times in ten seconds, what is the frequency (in Hz) of its motion?

- 5 Hz
- 10 Hz
- 25 Hz
- 50 Hz
- I need help

Answer

4 If an object is traveling in circular motion with a frequency of 7.0 Hz, how many revolutions will it make in 20 s?

- A 14
- B 45
- C 105
- D 140
- E I need help

Answer

Period and Frequency

Since $T = \frac{t}{n}$ and $f = \frac{n}{t}$

then $T = \frac{1}{f}$ and $f = \frac{1}{T}$

5 An object has a period of 4.0s, what is the frequency of its motion (in Hertz)?

- A 0.13 Hz
- B 0.25 Hz
- C 0.63 Hz
- D 0.89 Hz
- E I need help

Answer

6 An object is revolving with a frequency of 8.0 Hz, what is its period (in seconds)?

- 0.070 s
- 0.090 s
- 0.13 s
- 0.18 s
- I need help

Answer

Velocity

Also, recall from Uniform Circular Motion....

$$v = \frac{2\pi r}{T}$$

and

$$v = 2\pi r f$$

7 An object is in circular motion. The radius of its motion is 2.0 m and its period is 5.0s. What is its velocity?

- A 2.0 m/s
- B 2.5 m/s
- C 5.0 m/s
- D 7.0 m/s
- E I need help

Answer

8 An object is in circular motion. The radius of its motion is 2.0 m and its frequency is 8.0 Hz. What is its velocity?

- A 16 m/s
- B 44 m/s
- C 88 m/s
- D 100 m/s
- E I need help

Answer

SHM and UCM

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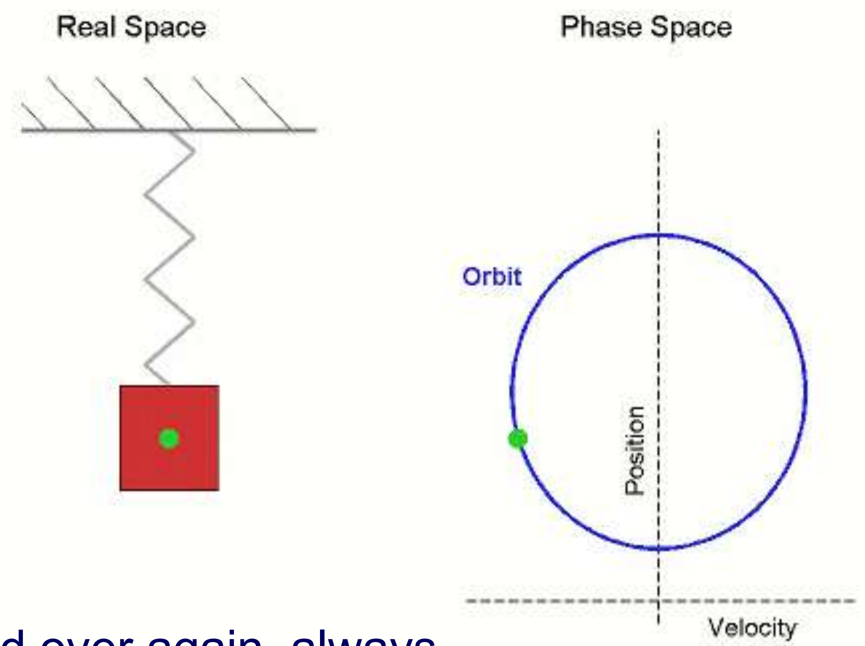
SHM and Circular Motion

In UCM, an object completes one circle, or cycle, in every T seconds. That means it returns to its starting position after T seconds.

In Simple Harmonic Motion, the object does not go in a circle, but it also returns to its starting position in T seconds.



Any motion that repeats over and over again, always returning to the same position is called " periodic".

Click here to see how simple harmonic motion relates to circular motion.

9 It takes 4.0s for a system to complete one cycle of simple harmonic motion. What is the frequency of the system?

- A 0.25 Hz
- B 1.5 Hz
- C 2.9 Hz
- D 3.4 Hz
- E I need help

Answer

Spring Pendulum

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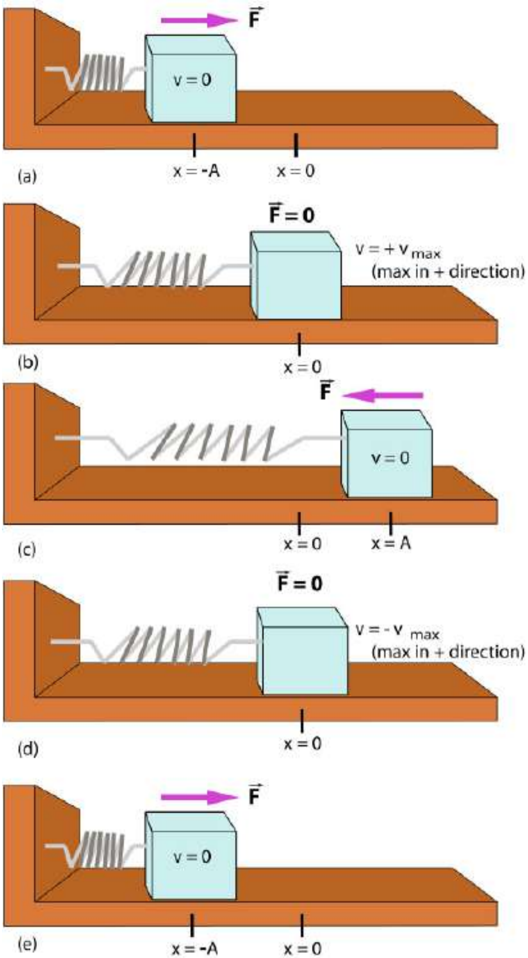


- Displacement is measured from the equilibrium point
- Amplitude is the maximum displacement *(equivalent to the radius, r, in UCM).*
- A cycle is a full to-and-fro motion *(the same as one trip around the circle in UCM)*
- Period is the time required to complete one cycle *(the same as period in UCM)*
- Frequency is the number of cycles completed per second *(the same as frequency in UCM)*

10 The period of a mass-spring system is 4.0s and the amplitude of its motion is 0.50m. How fardoes the mass travel in 4.0s?

- 2.0 m
- 5.0 m
- 8.0 m
- 11 m
- I need help

Answer

11 The period of a mass-spring system is 4.0 s and the amplitude of its motion is 0.50 m. How far does the mass travel in 6.0 s?

- A 1.0 m
- B 3.0 m
- C 6.0 m
- D 13 m
- E I need help

Answer

Simple Harmonic Motion

There is a point where the spring is neither stretched nor compressed; this is the equilibrium position.

We measure displacement from that point (x = 0 on the previous figure).

The force exerted by the spring depends on the displacement:

$$\vec{F} = -k\vec{x}$$

12 A spring whose spring constant is 20 N/m is stretched 0.20m from equilibrium; what is the magnitude of the force exerted by the spring?

- A 1 N
- B 2 N
- C 4 N
- D 10 N
- E I need help

Answer

13 A spring whose spring constant is 150 N/m exerts a force of 30 N on the mass in a mass-spring system. How far is the mass from equilibrium?

- A 0.2 m
- B 0.5 m
- C 0.8 m
- D 1.2 m
- E I need help

Answer

14 A spring exerts a force of 50 N on the mass in a mass-spring system when it is 2.0 m from equilibrium. What is the spring's spring constant?

- 13 N/m
- 19 N/m
- 22 N/m
- 25 N/m
- I need help

Answer

Simple Harmonic Motion

$$\vec{F} = -k\vec{x}$$

The minus sign indicates that it is a restoring force – it is directed to restore the mass to its equilibrium position.

k is the spring constant

The force is not constant, so the acceleration is not constant either

Simple Harmonic Motion

The maximum force exerted on the mass is when the spring is most stretched or compressed ($x = -A$ or $+A$):

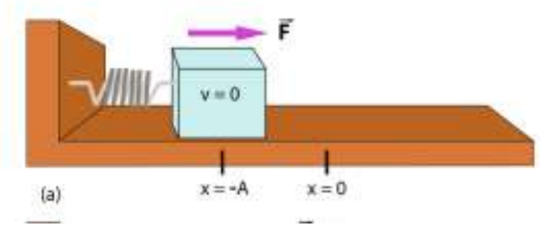
$$F = -kA \text{ (when } x = -A \text{ or } +A)$$

The minimum force exerted on the mass is when the spring is not stretched at all ($x = 0$)

$$F = 0 \text{ (when } x = 0)$$

Simple Harmonic Motion

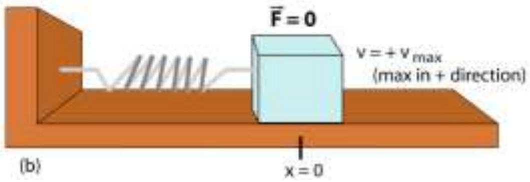
When the spring is all the way compressed:



- The displacement is at the negative amplitude.
- The force of the spring is in the positive direction.
- The acceleration is in the positive direction.
- The velocity is zero.

Simple Harmonic Motion

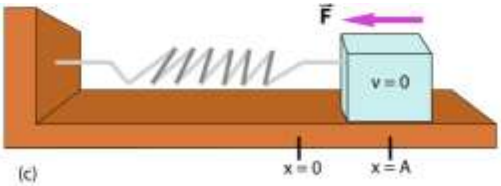
When the spring is at equilibrium and heading in the positive direction:



- The displacement is zero.
- The force of the spring is zero.
- The acceleration is zero.
- The velocity is positive and at a maximum.

Simple Harmonic Motion

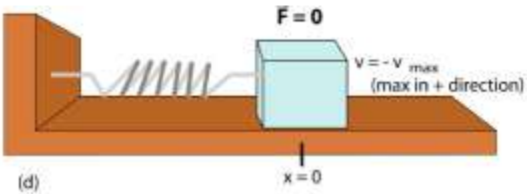
When the spring is all the way stretched:



- The displacement is at the positive amplitude.
- The force of the spring is in the negative direction.
- The acceleration is in the negative direction.
- The velocity is zero.

Simple Harmonic Motion

When the spring is at equilibrium and heading in the negative direction:

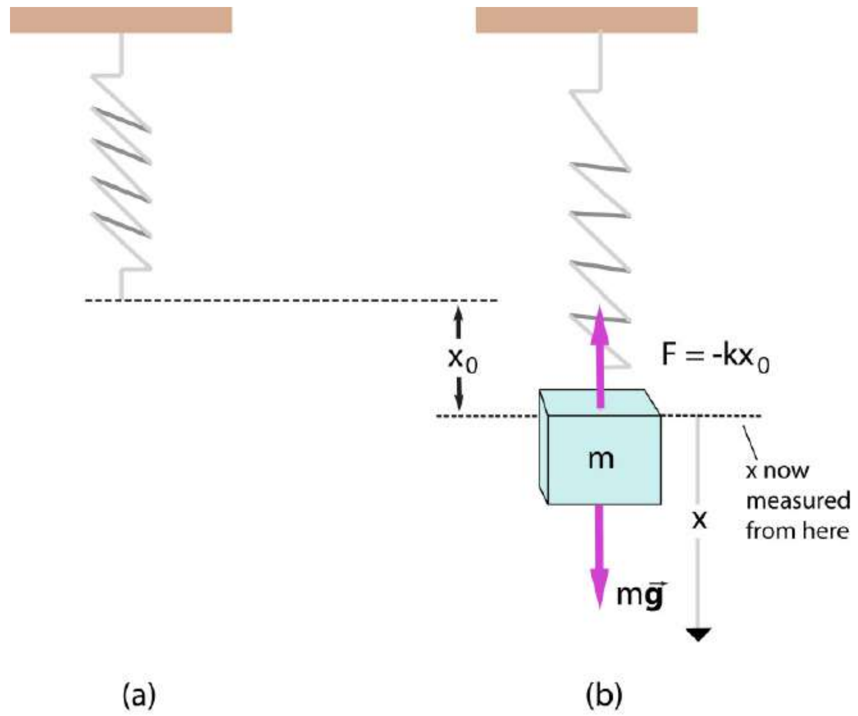


- The displacement is zero.
- The force of the spring is zero.
- The acceleration is zero.
- The velocity is negative and at a maximum.

Gravity does not affect the mass-spring system

If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

The effect of gravity is cancelled out by changing to this new equilibrium position.



15 At which location(s) is the magnitude of the force on the mass in a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

16 At which location(s) is the magnitude of the force on the mass in a mass-spring system a minimum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

Energy and Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.

Also, SHM requires that a system has two forms of energy and a method that allows the energy to go back and forth between those forms.

Energy in the Mass-Spring System

There are two types of energy in a mass-spring system.

The energy stored in the spring because it is stretched or compressed:

$$EPE = \frac{1}{2}kx^2$$

AND

The kinetic energy of the mass:

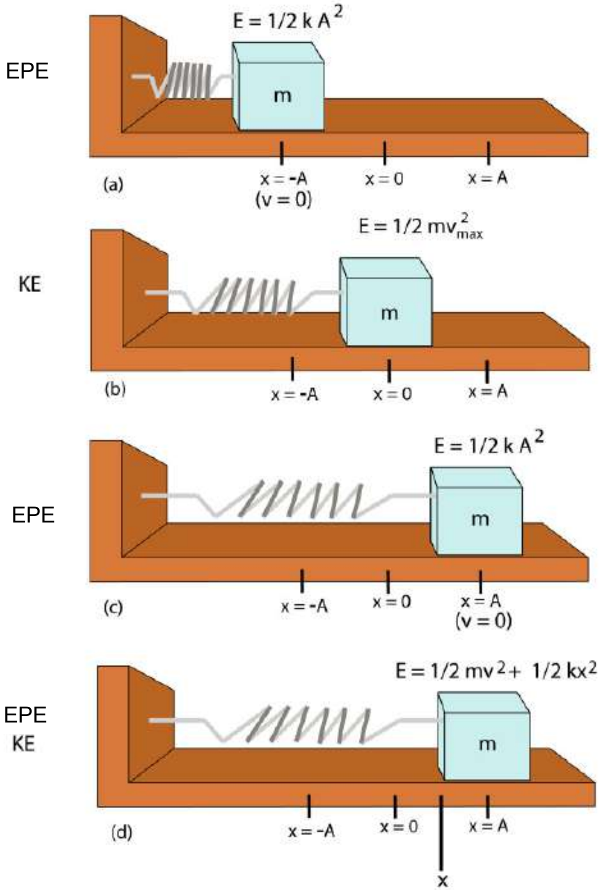
$$KE = \frac{1}{2}mv^2$$

Energy in the Mass-Spring System

At any moment, the total energy of the system is c omprised of those two forms.

$$E_{Total} = EPE + KE$$
$$E_{Total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

The total mechanical energy is constant.



When the mass is at the limits of its motion (x = A or x = -A), the energy is all potential:

$$E_{Total} = \frac{1}{2}kx^2$$

When the mass is at the equilibrium point (x=0) the spring is not stretched and all the energy is kinetic:

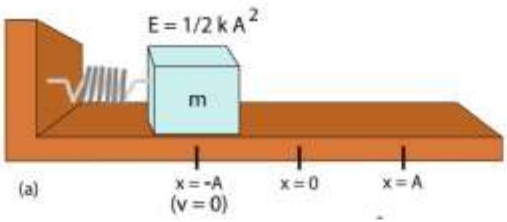
$$E_{Total} = \frac{1}{2}mv^2$$

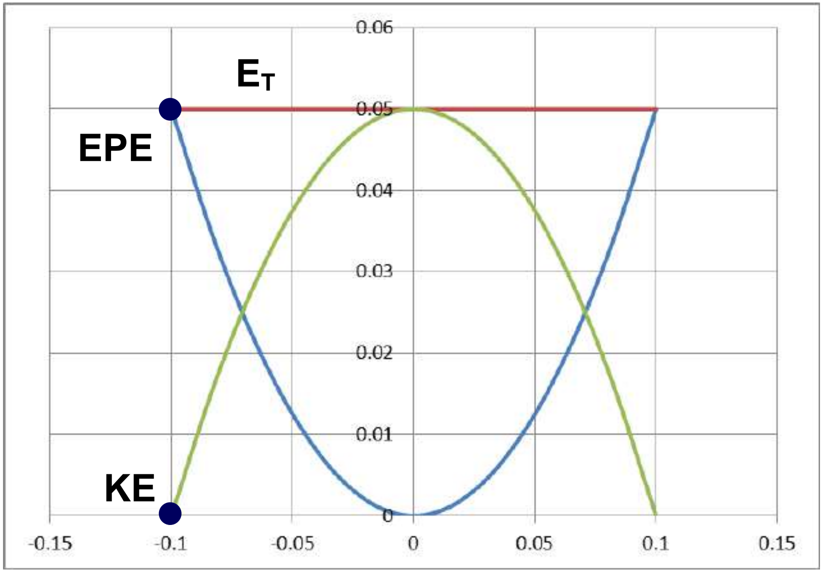
But the total energy is constant.

$$E_{Total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Energy in the Mass-Spring System

When the spring is all the way compressed....

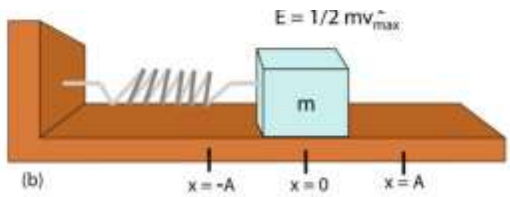


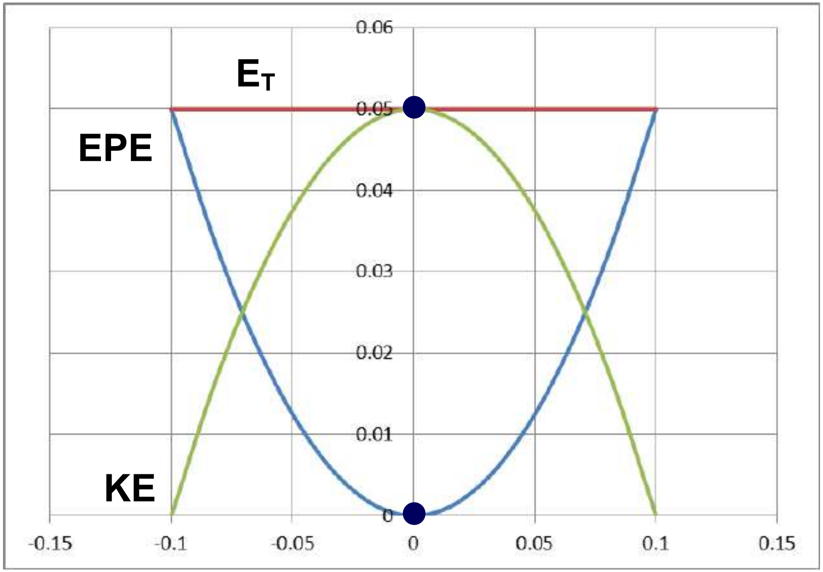


- EPE is at a maximum.
- KE is zero.
- Total energy is constant.

Energy in the Mass-Spring System

When the spring is passing through the equilibrium....

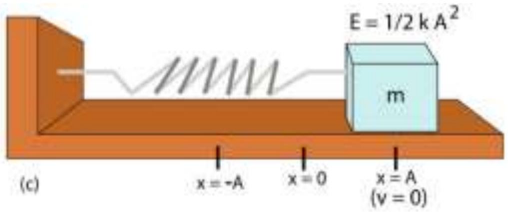


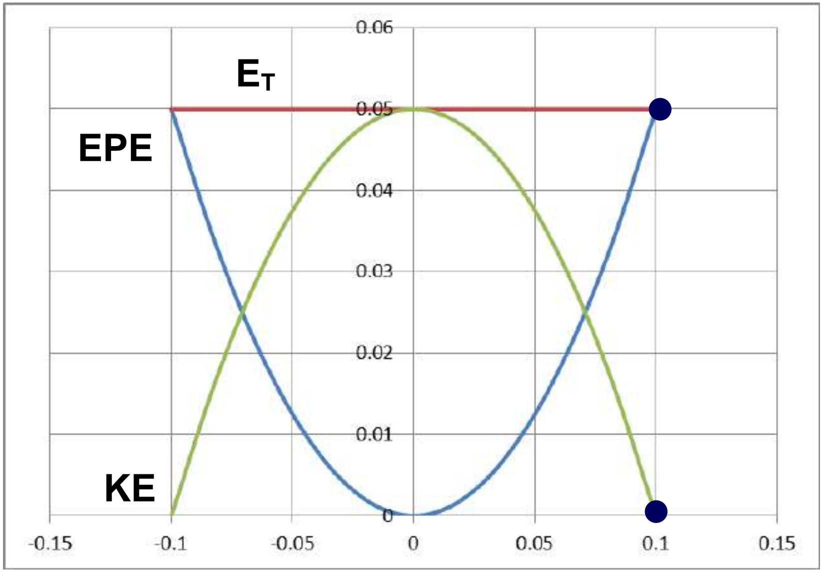


- EPE is zero.
- KE is at a maximum.
- Total energy is constant.

Energy in the Mass-Spring System

When the spring is all the way stretched....





- EPE is at a maximum.
- KE is zero.
- Total energy is constant.

17 At which location(s) is the kinetic energy of a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

18 At which location(s) is the spring potential energy (EPE) of a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

19 At which location(s) is the total energy of a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D All of the above
- E I need help

Answer

20 At which location(s) is the kinetic energy of a mass-spring system a minimum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

Problem Solving using Energy

Since the energy is constant, and the work done on the system is zero, you can always find the velocity of the mass at any location by using

$$E_0 = E_f$$

The most general equation becomes

$$\frac{1}{2}k{x_0}^2 + \frac{1}{2}m{v_0}^2 = \frac{1}{2}k{x_f}^2 + \frac{1}{2}m{v_f}^2$$

But usually this is simplified by being given the energy at some point where it is all EPE (x = A or -A) or when it is all KE (x = 0).

21 What is the total energy of a mass-spring system if the mass is 2.0 kg, the spring constant is 200 N/m and the amplitude of oscillation is 3.0 m?

- A 600 N/m
- B 700 N/m
- C 900 N/m
- D 1200 N/m
- E I need help

Answer

22 What is the maximum velocity of the mass in the mass-spring system from the previous slide: the mass is 2.0 kg, the spring constant is 200 N/m and the amplitude of oscillation is 3.0 m?

- A 22 m/s
- B 30 m/s
- C 35 m/s
- D 39 m/s
- E I need help

Answer

The Period and Frequency of a Mass-Spring System

We can use the period and frequency of a particle moving in a circle to find the period and frequency:

$$KE = EPE$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$mv^2 = kx^2$$

$$m\left(\frac{2\pi r}{T}\right)^2 = kx^2$$

$$T^2 = \frac{m(2\pi)^2 x^2}{kx^2} \qquad (r = x)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Demo

23 What is the period of a mass-spring system if the mass is 4.0 kg and the spring constant is 64 N/m?

- A 1.2 s
- B 1.6 s
- C 1.9 s
- D 2.0 s
- E I need help

Answer

24 What is the frequency of the mass-spring system from the previous slide; the mass is 4.0 kg and the spring constant is 64 N/m?

- A 0.64 Hz
- B 0.72 Hz
- C 0.91 Hz
- D 1.14 Hz
- E I need help

Answer

25 Multi Correct Question (Directions: For each of the following, two of the suggested answers will be correct. Select the best two choices to earn credit. No partial credit will be earned if only one correct choice is selected.)

A block is attached to a spring and oscillates on a horizontal frictionless surface. When the block is at the maximum displacement, another block is place on top of it without interrupting the oscillation. Which of the following are true?

- ☐ A The maximum kinetic energy is smaller.
- ☐ B The maximum kinetic energy is the same.
- ☐ C The period is greater.
- ☐ D The period is smaller.

☐ I need help

Answer

Simple Pendulum

Demo

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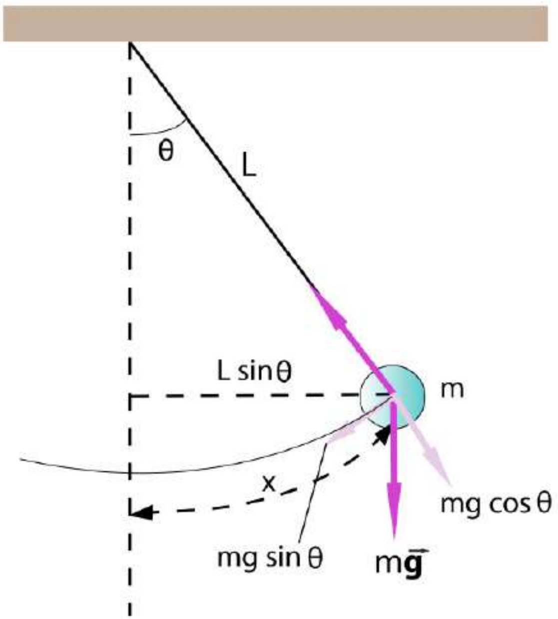
The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

*The Simple Pendulum



In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have:

$$F = -mg\sin\theta$$

which is proportional to sin θ and not to θ itself.

We don't really need to worry about this because for small angles (less than 15 degrees or so), $\sin\theta \approx \theta$ and $x = L\theta$. So we can replace $\sin\theta$ with x/L.

$$F \approx -\frac{mg}{L}x$$

The Simple Pendulum

$F \approx -\frac{mg}{L}x$ has the form of $F = -kx$ if $k = \frac{mg}{L}$

But we learned before that $T = 2\pi\sqrt{\frac{m}{k}}$

Substituting for k $T = 2\pi\sqrt{\frac{m}{\frac{mg}{L}}}$

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Notice the "m" canceled out, the mass doesn't matter.

The Simple Pendulum

So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.

26 What is the frequency of the pendulum of the previous slide (a length of 2.0 m near the surface of the earth)?

- A 0.090 Hz
- B 0.19 Hz
- C 0.27 Hz
- D 0.35 Hz
- E I need help

Answer

27 Which of the following factors affect the period of a pendulum?

- A the acceleration due to gravity
- B the length of the string
- C the mass of the pendulum bob
- D A & B
- E I need help

Answer

Energy in the Pendulum

The two types of energy in a pendulum are:

Gravitational Potential Energy

$$GPE = mgh$$

AND

The kinetic energy of the mass:

$$KE = \frac{1}{2}mv^2$$

Energy in the Pendulum

At any moment in time the total energy of the system is comprised of those two forms.

$$E_{total} = GPE + KE$$

$$E_{total} = mgh + \frac{1}{2}mv^2$$

The total mechanical energy is constant.

28 What is the total energy of a 1 kg pendulum if its height, at its maximum amplitude is 0.20 m above its height at equilibrium?

- A 2 J
- B 5 J
- C 9 J
- D 14 J
- E I need help

Answer

29 What is the maximum velocity of the pendulum's mass from the previous slide (its height at maximum amplitude is 0.20 m above its height at equilibrium)?

- A 0.5 m/s
- B 1.0 m/s
- C 1.2 m/s
- D 2.0 m/s
- E I need help

Answer

Sinusoidal Nature of SHM

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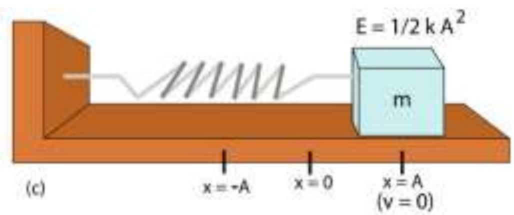
Position as a function of time

The position as a function of time for an object in simple harmonic motion can be derived from the equation:

$$x = A\cos\theta$$

Where A is the amplitude of oscillations.

Take note that it doesn't really matter if you are using sine or cosine since that only depends on when you start your clock. For our purposes lets assume that you are looking at the motion of a mass-spring system and that you start the clock when the mass is at the positive amplitude.



Position as a function of time

Now we can derive the equation for position as a function of time.

$$x = A\cos\theta$$

Since $\omega = \dfrac{\theta}{t}$ we can replace θ with ωt.

$$x = A\cos(\omega t)$$

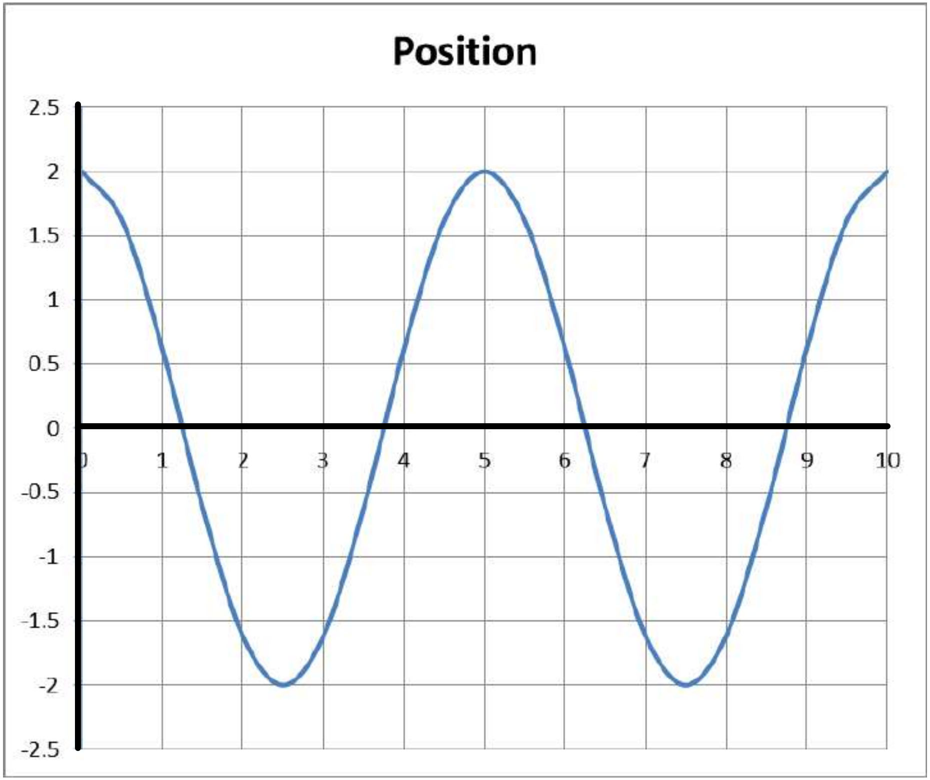
And we can also replace ω with $2\pi f$ or $2\pi/T$.

$$x = A\cos(2\pi f t) = A\cos\left(\frac{2\pi}{T}t\right)$$

Where A is amplitude, T is period, and t is time.

Position as a function of time

The graph of position vs. time for an object in simple harmonic motion with an amplitude of 2 m and a period of 5 s would look like this:



$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

Velocity as a function of time

We can also derive the equation for velocity as a function of time.

$$v = -v_0 \sin\theta$$

Since v=ωr can replace v_0 with ωA as well as θ with ωt.

$$v = -A\omega \sin(\omega t)$$

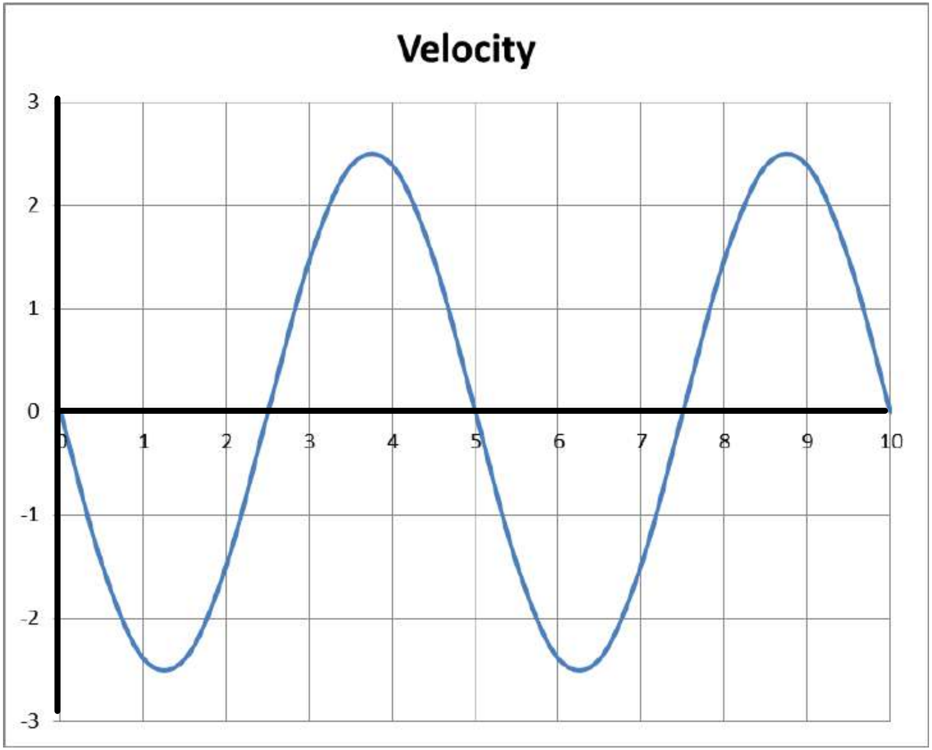
And again we can also replace ω with 2πf or 2π/T.

$$v = -A\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi}{T}t\right)$$

Where A is amplitude, T is period, and t is time.

Velocity as a function of time

The graph of velocity vs. time for an object in simple harmonic motion with an amplitude of 2 m and a period of 5 s would look like this:



$$v = -A\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi}{T}t\right)$$

Acceleration as a function of time

We can also derive the equation for acceleration as a function of time.

$$a = -a_0 \cos\theta$$

Since $a = r\omega^2$, we can replace a_0 with $A\omega^2$ as well as θ with ωt.

$$a = -A\omega^2 \cos(\omega t)$$

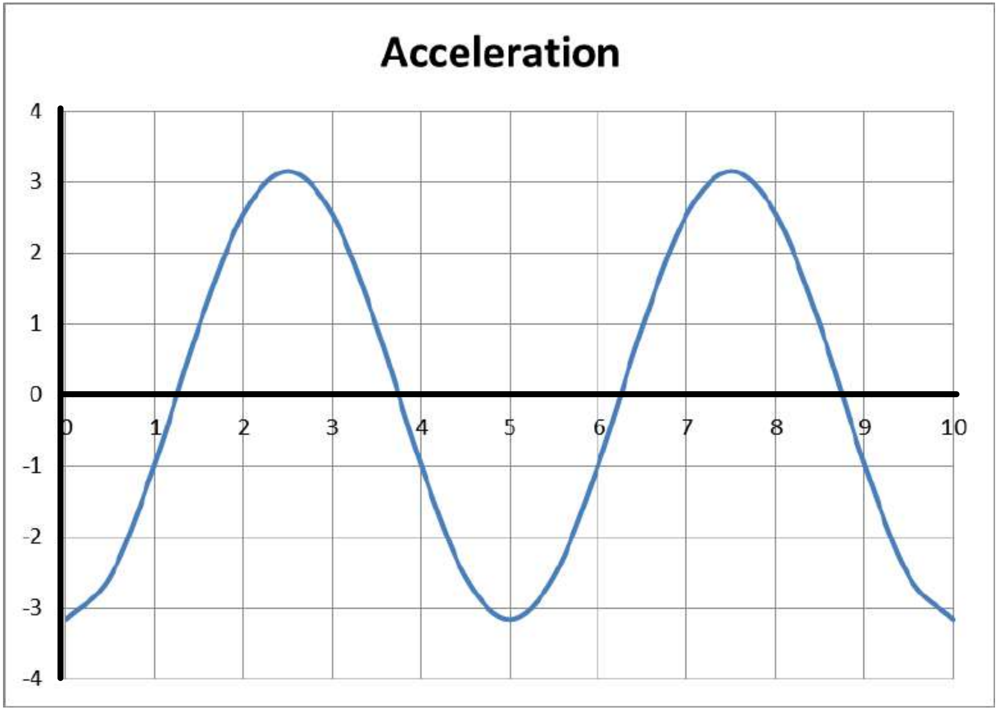
And again we can also replace ω with $2\pi f$ or $2\pi/T$.

$$a = -A\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi}{T}t\right)$$

Where A is amplitude, T is period, and t is time.

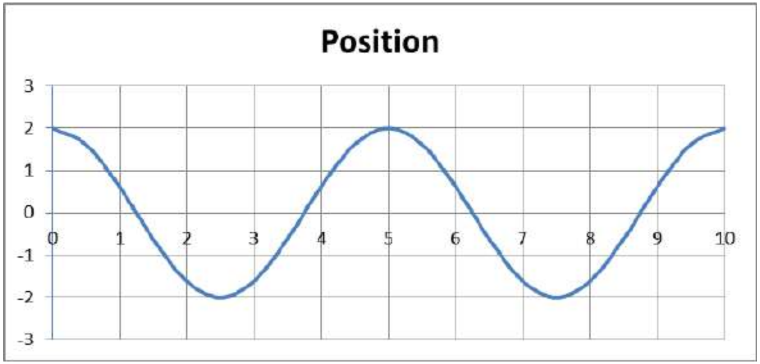
Acceleration as a function of time

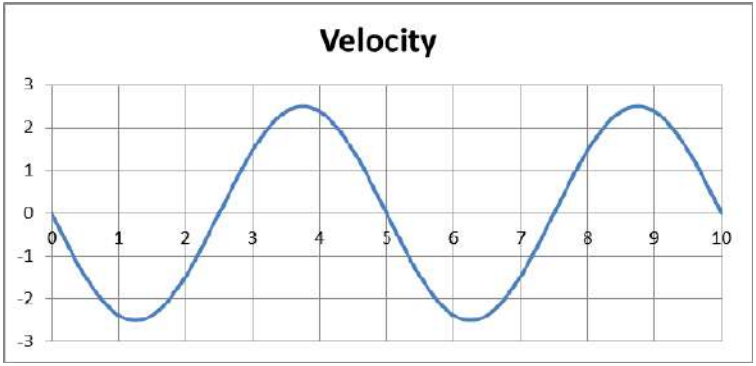
The graph of acceleration vs. time for an object in simple harmonic motion with an amplitude of 2 m and a period of 5 s would look like this:

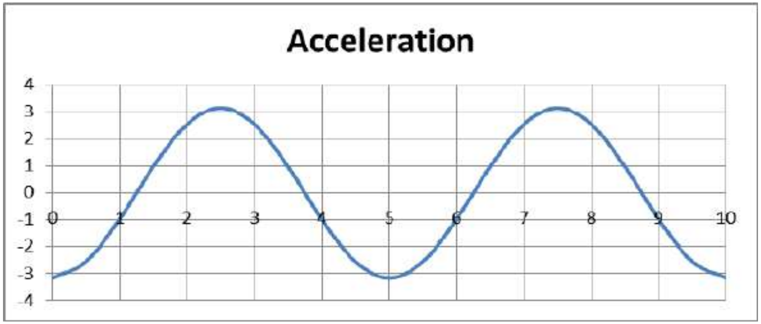


$$a = -A\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi}{T}t\right)$$

The Sinusoidal Nature of SHM







Now you can see all of the graphs together.

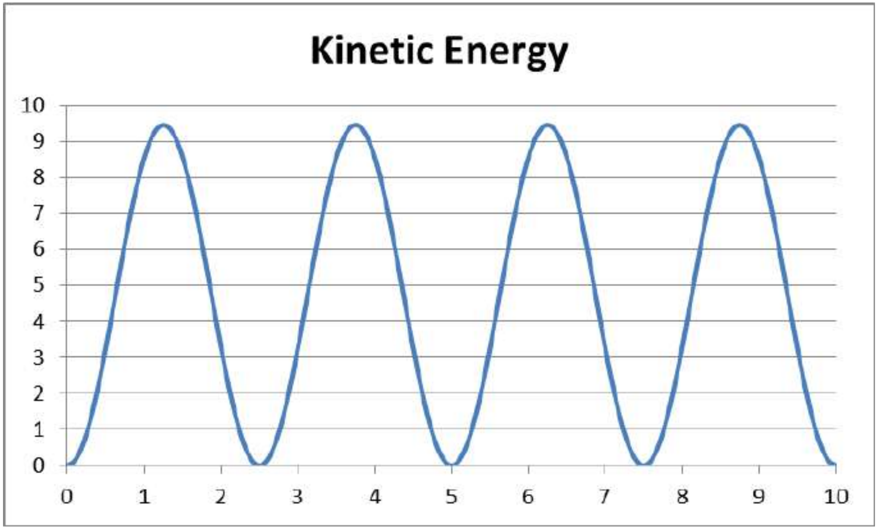
Take note that when the position is at the positive amplitude, the acceleration is negative and the velocity is zero.

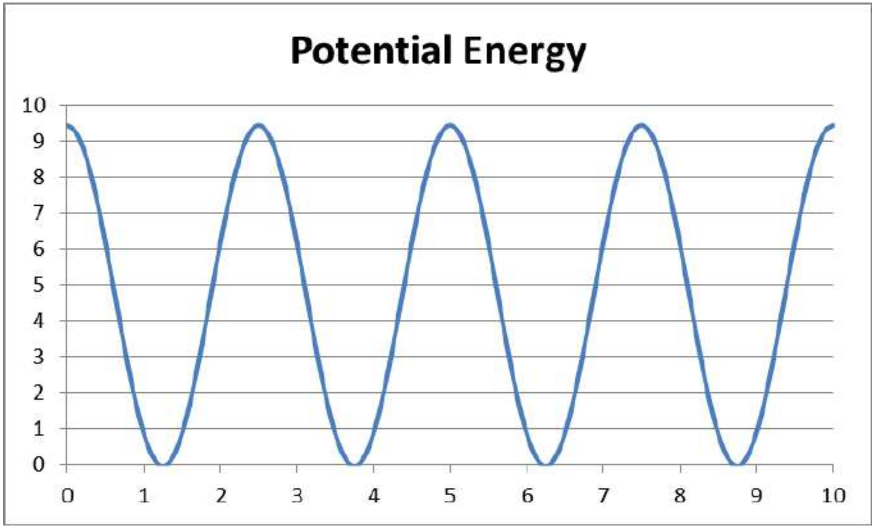
Or when the velocity is at a maximum both the position and acceleration are zero.

http://www.youtube.com/watch?v=eeYRkW8V7Vg&feature=PlayList&p=3AB590B4A4D71006&index=0

Energy as a function of time

The graphs of Kinetic Energy and Potential Energy vs. time for an object in simple harmonic motion with an amplitude of 2 m and a period of 5 s would look like this:

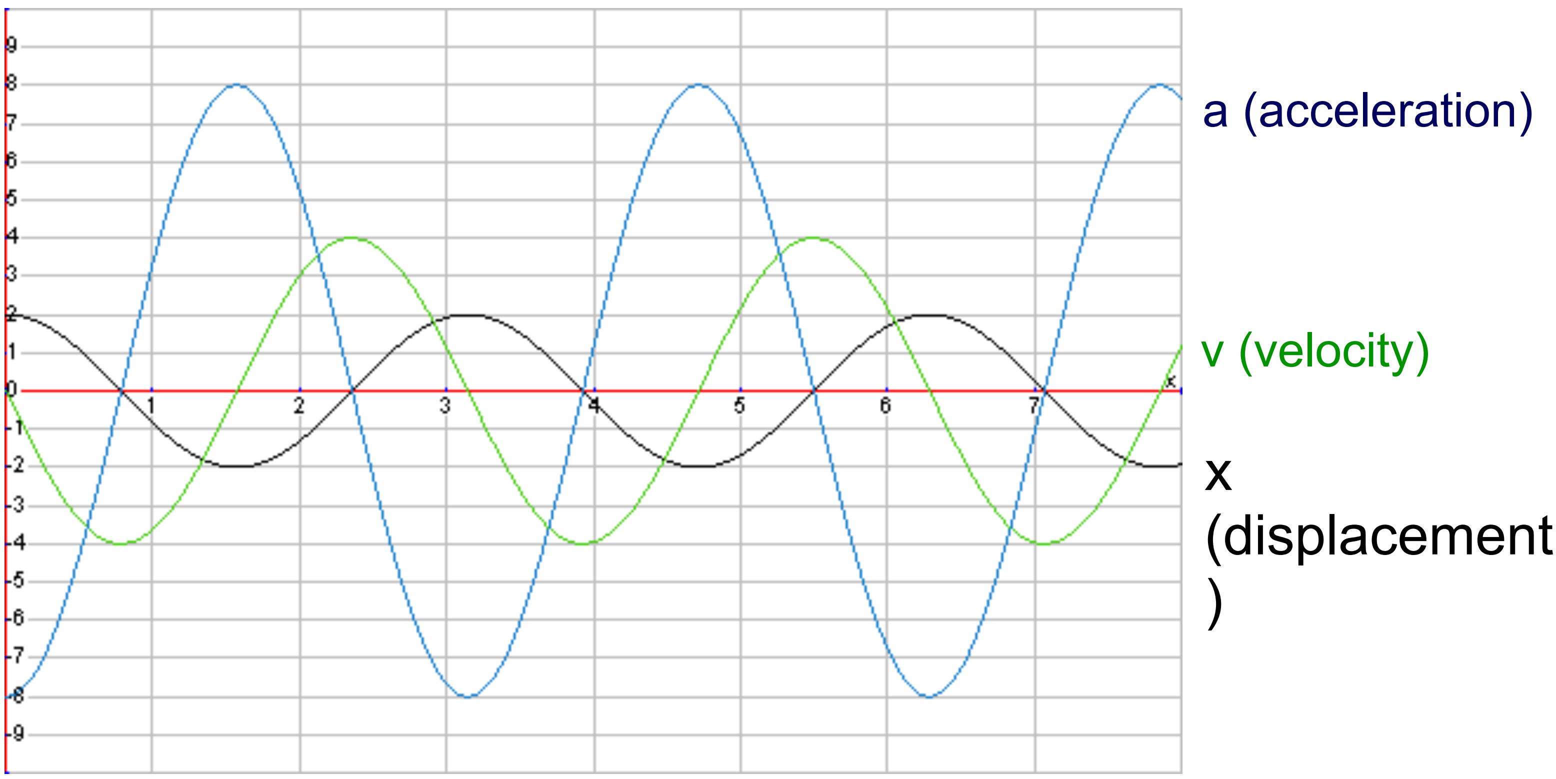




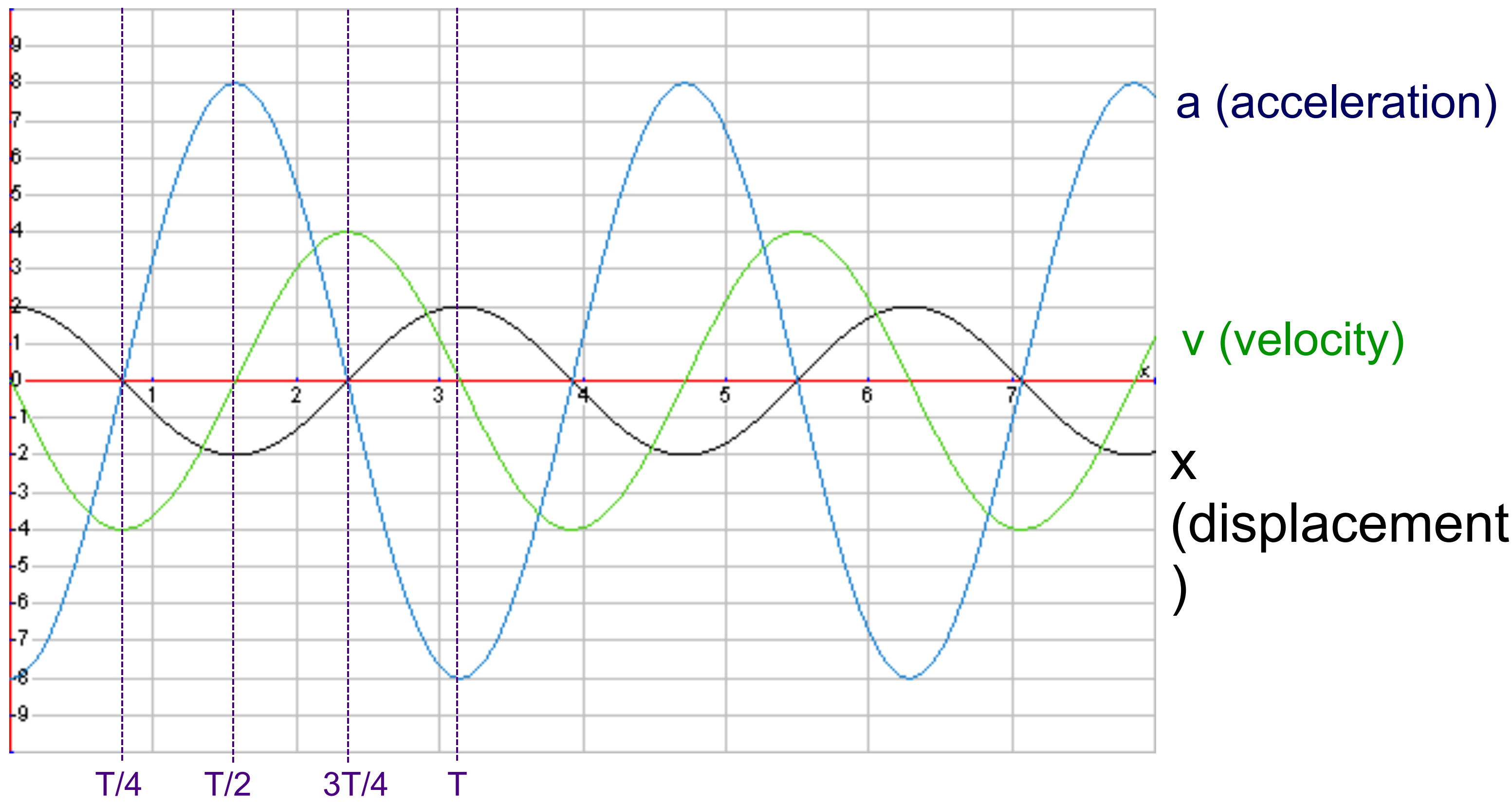
What things do you notice when you look at these graphs?

The Period and Sinusoidal Nature of SHM

Use this graph to answer the following questions.

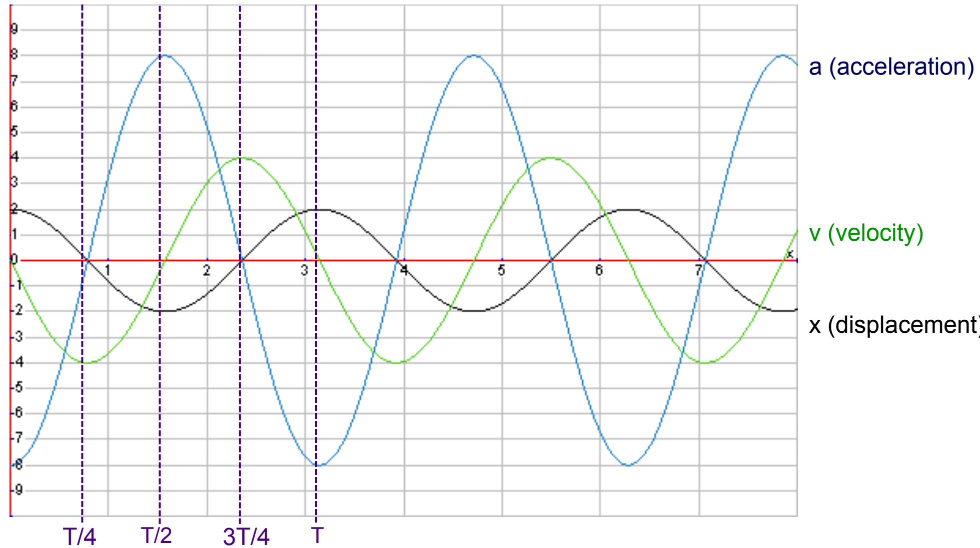


The Period and Sinusoidal Nature of SHM



30 What is the acceleration when x = 0?

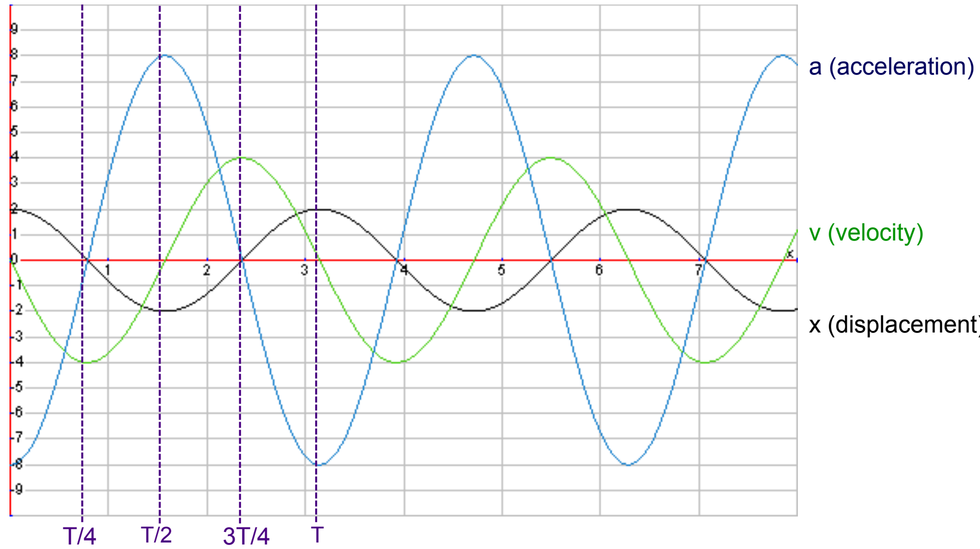
- A a < 0
- B a = 0
- C a > 0
- D It varies.
- E I need help



Answer

31 What is the acceleration when x = A?

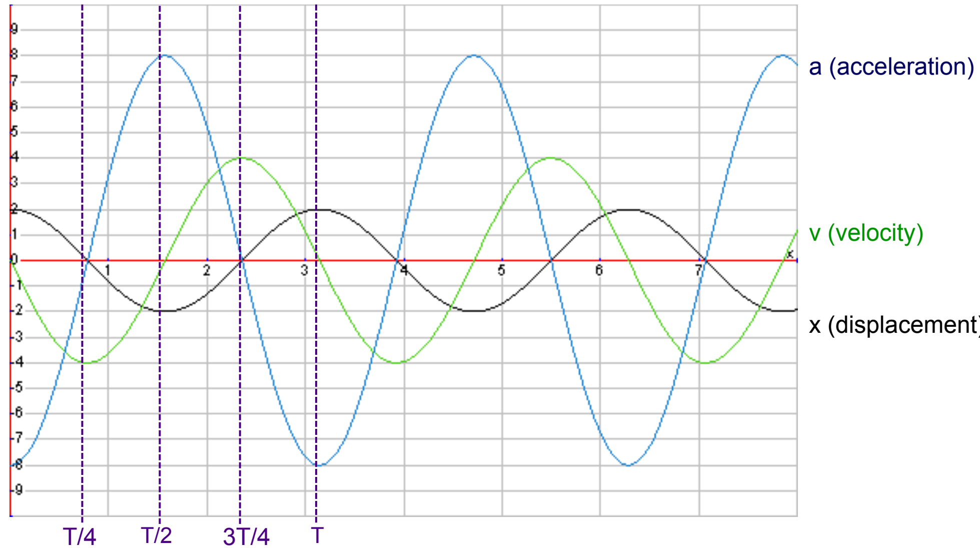
- a < 0
- a = 0
- a > 0
- It varies.
- E I need help



Answer

32 What is the acceleration when x = -A?

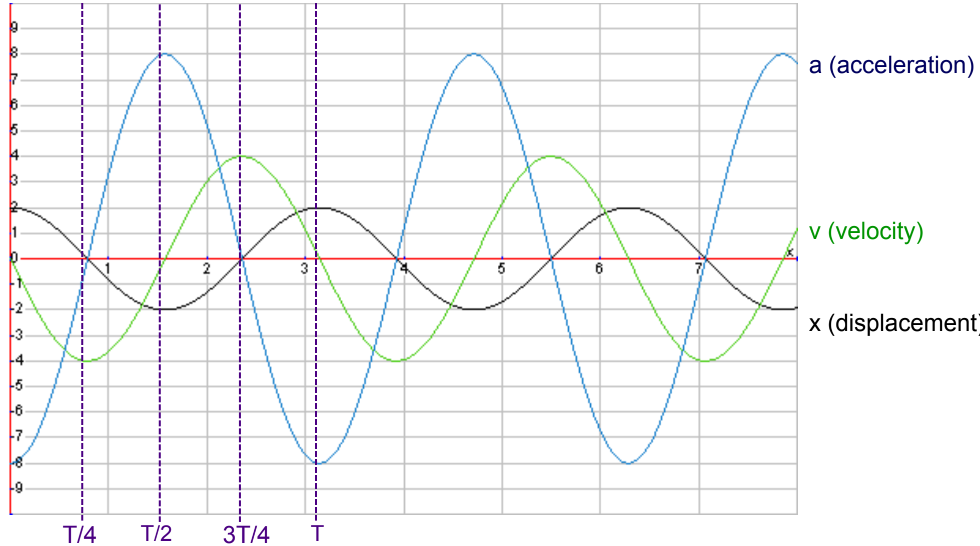
- A $a < 0$
- B $a = 0$
- C $a > 0$
- D It varies.
- E I need help



Answer

33 What is the velocity when x = 0?

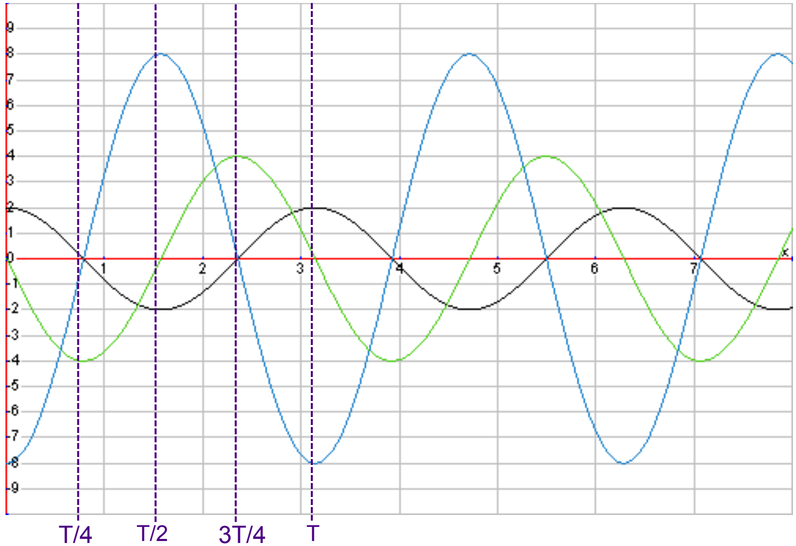
- v < 0
- v = 0
- v > 0
- A or C
- E I need help



Answer

34 What is the velocity when x = A?

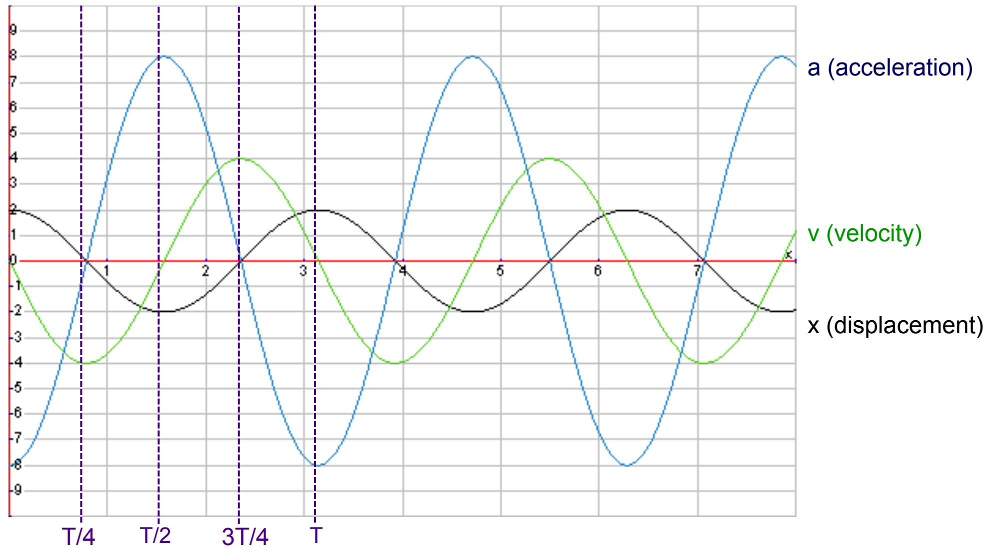
- A v < 0
- B v = 0
- C v > 0
- D A or C
- E I need help



Answer

35 Where is the mass when acceleration is at a maximum magnitude?

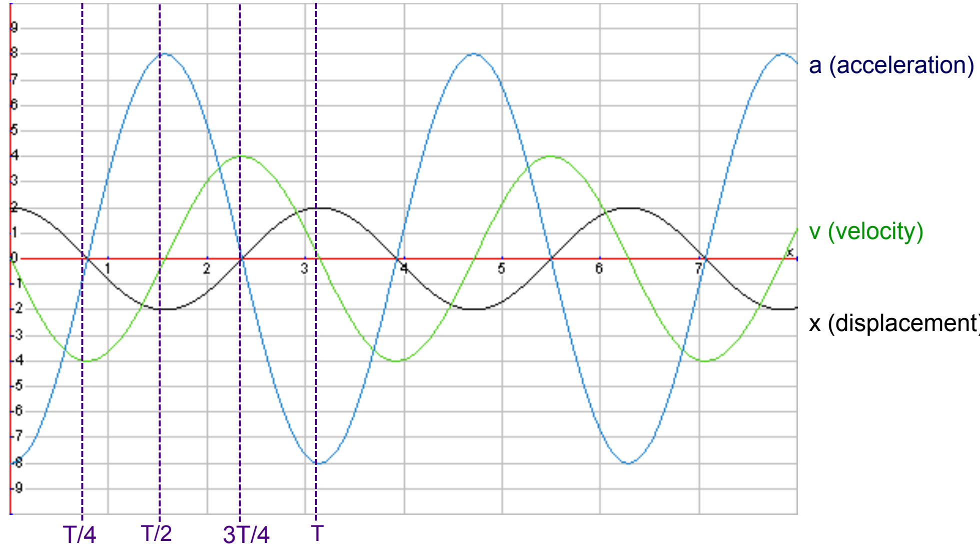
- A x = A
- B x = 0
- C x = -A
- D A or C
- E I need help



Answer

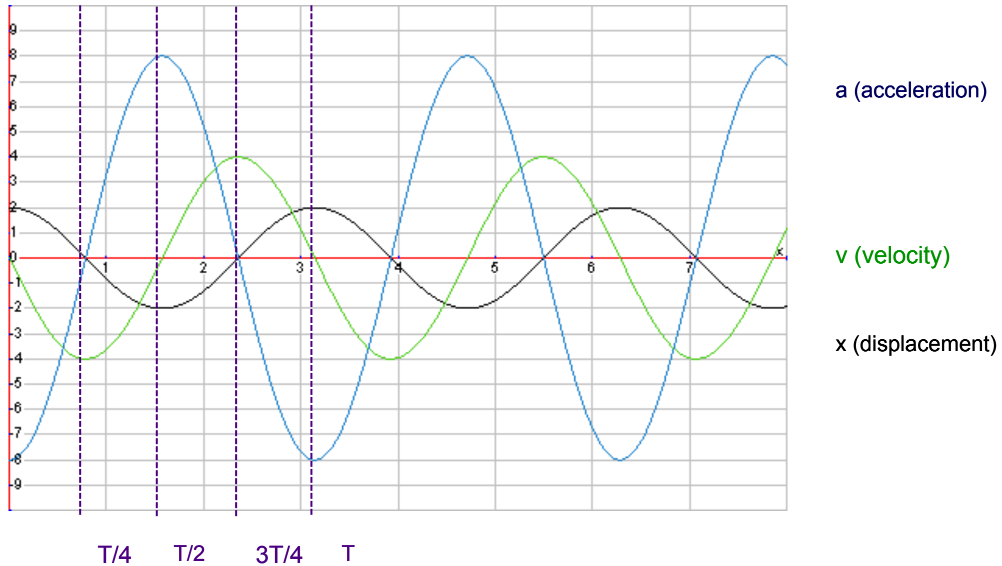
36 Where is the mass when velocity is at a maximum?

- A x = A
- B x = 0
- C x = -A
- D A or C
- E I need help



Answer

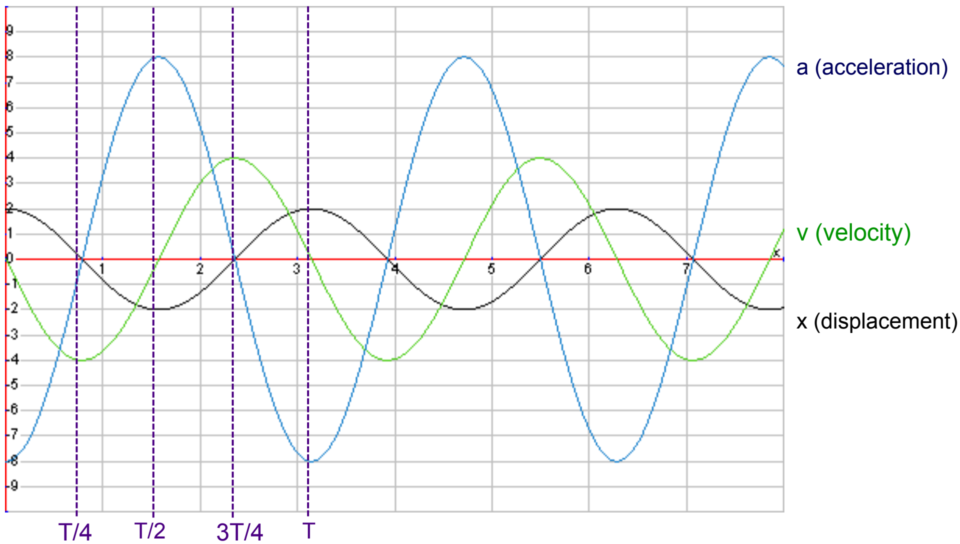
37 Which of the following represents the position as a function of time?



- x = 4 cos (2t)
- x = 2 sin (2t)
- x = 2 cos (2t)
- x = 8 cos (2t)
- E I need help

Answer

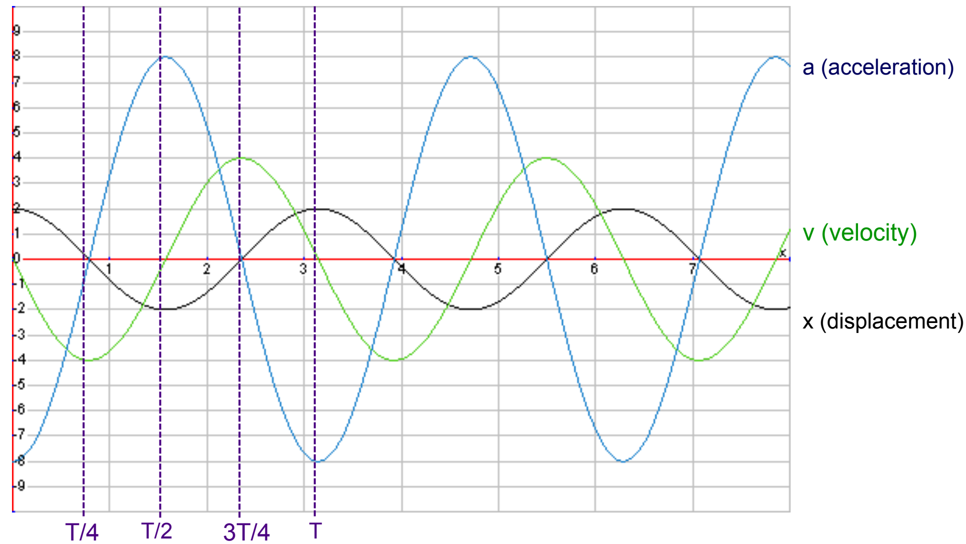
38 Which of the following represents the velocity as a function of time?



- v = -12 sin (2t)
- v = -4 cos (2t)
- v = -12 cos (2t)
- v = -4 sin (2t)
- E I need help

Answer

39 Which of the following represents the acceleration as a function of time?



A) a = -8 sin (2t)

B) a = -4 cos (2t)

C) a = -8 cos (2t)

D) a = -4 sin (2t)

E I need help

Answer