Key

AP Calculus – Final Review Sheet

When you see the words ….	This is what you think of doing
1. Find the zeros	Set function = 0, factor or use quadratic equation if quadratic, graph to find zeros on calculator
2. Find equation of the line tangent to $f(x)$ on $[a,b]$	Take derivative - $f'(a)=m$ and use $y-y_1=m(x-x_1)$
3. Find equation of the line normal to $f(x)$ on $[a,b]$	Same as above but $m=\frac{-1}{f'(a)}$
4. Show that $f(x)$ is even	Show that $f(-x)=f(x)$ - symmetric to y-axis
5. Show that $f(x)$ is odd	Show that $f(-x)=-f(x)$ - symmetric to origin
6. Find the interval where $f(x)$ is increasing	Find $f'(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.
7. Find interval where the slope of $f(x)$ is increasing	Find the derivative of $f'(x)=f''(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where it is positive.
8. Find the minimum value of a function	Make a sign chart of $f'(x)$, find all relative minimums and plug those values back into $f(x)$ and choose the smallest.
9. Find the minimum slope of a function	Make a sign chart of the derivative of $f'(x)=f''(x)$, find all relative minimums and plug those values back into $f'(x)$ and choose the smallest.
10. Find critical values	Express $f'(x)$ as a fraction and set both numerator and denominator equal to zero.
11. Find inflection points	Express $f''(x)$ as a fraction and set both numerator and denominator equal to zero. Make sign chart of $f''(x)$ to find where $f''(x)$ changes sign. (+ to – or – to +)
12. Show that $\lim_{x\to a} f(x)$ exists	Show that $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$
13. Show that $f(x)$ is continuous	Show that 1) $\lim_{x\to a} f(x)$ exists $\left(\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)\right)$ 2) $f(a)$ exists 3) $\lim_{x\to a} f(x)=f(a)$
14. Find vertical asymptotes of $f(x)$	Do all factor/cancel of $f(x)$ and set denominator = 0
15. Find horizontal asymptotes of $f(x)$	Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
16. Find the average rate of change of $f(x)$ on $[a,b]$	Find $\frac{f(b)-f(a)}{b-a}$

17. Find instantaneous rate of change of $f(x)$ at a	Find $f'(a)$
18. Find the average value of $f(x)$ on $[a,b]$	Find $\frac{\int_a^b f(x)dx}{b-a}$
19. Find the absolute maximum of $f(x)$ on $[a,b]$	Make a sign chart of $f'(x)$, find all relative maximums and plug those values back into $f(x)$ as well as finding $f(a)$ and $f(b)$ and choose the largest.
20. Show that a piecewise function is differentiable at the point a where the function rule splits	First, be sure that the function is continuous at $x = a$. Take the derivative of each piece and show that $\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$
21. Given $s(t)$ (position function), find $v(t)$	Find $v(t) = s'(t)$
22. Given $v(t)$, find how far a particle travels on $[a,b]$	Find $\int_a^b \lvert v(t) \rvert dt$
23. Find the average velocity of a particle on $[a,b]$	Find $\frac{\int_a^b v(t)\,dt}{b-a} = \frac{s(b)-s(a)}{b-a}$
24. Given $v(t)$, determine if a particle is speeding up at $t = k$	Find $v(k)$ and $a(k)$. Examine their signs. If both positive, the particle is speeding up, if different signs, then the particle is slowing down.
25. Given $v(t)$ and $s(0)$, find $s(t)$	$s(t) = \int v(t)\,dt + C$ Plug in $t = 0$ to find C
26. Show that Rolle's Theorem holds on $[a,b]$	Show that f is continuous and differentiable on the interval. If $f(a) = f(b)$, then find some c in $[a,b]$ such that $f'(c) = 0$.
27. Show that Mean Value Theorem holds on $[a,b]$	Show that f is continuous and differentiable on the interval. Then find some c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
28. Find domain of $f(x)$	Assume domain is $(-\infty,\infty)$. Restrictable domains: denominators $\neq 0$, square roots of only non negative numbers, log or ln of only positive numbers.
29. Find range of $f(x)$ on $[a,b]$	Use max/min techniques to rind relative max/mins. Then examine $f(a), f(b)$
30. Find range of $f(x)$ on $(-\infty,\infty)$	Use max/min techniques to rind relative max/mins. Then examine $\lim_{x \to \pm\infty} f(x)$.
31. Find $f'(x)$ by definition	$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ or $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$
32. Find derivative of inverse to $f(x)$ at $x = a$	Interchange x with y. Solve for $\frac{dy}{dx}$ implicitly (in terms of y). Plug your x value into the inverse relation and

	solve for y. Finally, plug that y into your $\frac{dy}{dx}$.
33. y is increasing proportionally to y	$\frac{dy}{dt} = ky$ translating to $y = Ce^{kt}$
34. Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\int_a^c f(x)dx = \int_c^b f(x)dx$
35. $\frac{d}{dx}\int_a^x f(t)dt =$	2nd FTC: Answer is $f(x)$
36. $\frac{d}{dx}\int_a^u f(t)\,dt$	2nd FTC: Answer is $f(u)\frac{du}{dx}$
37. The rate of change of population is …	$\frac{dP}{dt} = \ldots$
38. The line $y = mx + b$ is tangent to $f(x)$ at (x_1, y_1)	Two relationships are true. The two functions share the same slope ($m = f'(x)$) and share the same y value at x_1.
39. Find area using left Riemann sums	$A = base[x_0 + x_1 + x_2 + \ldots + x_{n-1}]$
40. Find area using right Riemann sums	$A = base[x_1 + x_2 + x_3 + \ldots + x_n]$
41. Find area using midpoint rectangles	Typically done with a table of values. Be sure to use only values that are given. If you are given 6 sets of points, you can only do 3 midpoint rectangles.
42. Find area using trapezoids	$A = \frac{base}{2}[x_0 + 2x_1 + 2x_2 + \ldots + 2x_{n-1} + x_n]$ This formula only works when the base is the same. If not, you have to do individual trapezoids.
43. Solve the differential equation …	Separate the variables – x on one side, y on the other. The dx and dy must all be upstairs.
44. Meaning of $\int_a^x f(t)dt$	The accumulation function – accumulated area under the function $f(x)$ starting at some constant a and ending at x.
45. Given a base, cross sections perpendicular to the x-axis are squares	The area between the curves typically is the base of your square. So the volume is $\int_a^b (base^2)dx$
46. Find where the tangent line to $f(x)$ is horizontal	Write $f'(x)$ as a fraction. Set the numerator equal to zero.
47. Find where the tangent line to $f(x)$ is vertical	Write $f'(x)$ as a fraction. Set the denominator equal to zero.
48. Find the minimum acceleration given $v(t)$	First find the acceleration $a(t) = v'(t)$. Then minimize the acceleration by examining $a'(t)$.
49. Approximate the value of $f(0.1)$ by using the	Find the equation of the tangent line to f using

tangent line to f at $x = 0$	$y - y_1 = m(x - x_1)$ where $m = f'(0)$ and the point is $(0, f(0))$. Then plug in 0.1 into this line being sure to use an approximate $(\approx)$sign.
50. Given the value of $F(a)$ and the fact that the anti-derivative of f is F, find $F(b)$1	Usually, this problem contains an antiderivative you cannot take. Utilize the fact that if $F(x)$is the antiderivative of f, then $\int_a^b F(x)dx = F(b) - F(a)$. So solve for $F(b)$ using the calculator to find the definite integral.
51. Find the derivative of $f(g(x))$	$f'(g(x)) \cdot g'(x)$
52. Given $\int_a^b f(x)dx$, find $\int_a^b [f(x) + k]dx$	$\int_a^b [f(x) + k]dx = \int_a^b f(x)dx + \int_a^b kdx$
53. Given a picture of $f'(x)$, find where $f(x)$ is increasing	Make a sign chart of $f'(x)$ and determine where $f'(x)$ is positive.
54. Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a, b]$	Generate a sign chart of $v(t)$ to find turning points. Then integrate $v(t)$ using $s(0)$ to find the constant to find $s(t)$. Finally, find s(all turning points) which will give you the distance from your starting point. Adjust for the origin.
55. Given a water tank with g gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find a) the amount of water in the tank at m minutes	$g + \int_t^{t_2} (F(t) - E(t))dt$
56. b) the rate the water amount is changing at m	$\frac{d}{dt}\int_t^m (F(t) - E(t))dt = F(m) - E(m)$
57. c) the time when the water is at a minimum	$F(m) - E(m)$=0, testing the endpoints as well.
58. Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.	Straddle c, using a value k greater than c and a value h less than c. so $f'(c) \approx \frac{f(k) - f(h)}{k - h}$
59. Given $\frac{dy}{dx}$, draw a slope field	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the indicated slopes at the points.
60. Find the area between curves $f(x), g(x)$ on $[a, b]$	$A = \int_a^b [f(x) - g(x)]dx$, assuming that the f curve is above the g curve.
61. Find the volume if the area between $f(x), g(x)$ is rotated about the x-axis	$A = \pi\int_a^b \left[f(x)^2 - g(x)^2\right]dx$ assuming that the f curve is above the g curve.