AP Calculus
Summer Packer Key

1 a) Remember, the negative in the front and the negative in the exponent have nothing to do w/ each other. Answer: $\frac{-1}{3^x}$

b) Answer: $-5\sqrt{\frac{4}{9}} = -5\left(\frac{2}{3}\right)$ or $-10/3$.

c) Answer: $\frac{-8}{(x-2)^3}$

d) The 16, x^2, and the *y* are all bases to the exponent. When you take a power to another power, multiply the exponents. Answer: $8x^{3/2}y^{3/4}$. Also, $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

2. a) You can only take the log of a positive number, not zero. Remember, in correct domain notation, brackets indicate inclusion. Answer: $(6,\infty)$

b) Look at the denominator, factor it. $(x-6)(x+3)$, Answer: $(-\infty,-3)\cup(-3,6)\cup(6,\infty)$

c) You can take 2 to any power, once again, the denominator can't be zero. Answer: $(-\infty,0)\cup(0,\infty)$

d) Here's one where you have to look at both. The numerator can't be negative, the denominator can't be zero. The top's domain is 4.5 or greater and the denominator can't be -4.5. Answer: $[4.5,\infty)$

e) You must factor first, then make a sign chart. $(x-7)(x+2)$. When *x* is smaller than -2, $x^2-5x-14$ is positive. When *x* is in between -2 and 7, $x^2-5x-14$ is negative. When *x* is larger than 7, $x^2-5x-14$ is positive. Answer: $(-\infty,-2]\cup[7,\infty)$

f) The numerator of a fraction can be anything, so look to the denominator. The denominator can not be zero. Answer, from $[0,2\pi)$: $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$

3. a) Answer: $x^5+11x^3-80x = x\left(x^4+11x^2-80\right) = x\left(x^2+16\right)\left(x^2-5\right)$

b) Look at this problem as two terms, factor out what is common from each term. $(x-3)^2(2x+1)^2$ is common in both terms. If you take those out, you are left with $(2x+1)+(x-3)$ which equals $3x-2$. Answer: $(x-3)^2(2x+1)^2(3x-2)$

c) Answer: $2x^2-20xy+50y^2 = 2\left(x^2-10xy+25y^2\right) = 2(x-5y)(x-5y) = 2(x-5y)^2$

4. a) Factor, solutions are 4 and -4, make a sign chart. $+\ (-4)\ -\ (4)\ +$ Also check to see if the inequality sign has a bar under it to determine if you use [ ] or ( ). Answer: $(-\infty,-4)\cup(4,\infty)$

   b) Same procedure as 4a. Answer: $(-\infty,-8)\cup(2,\infty)$

   c) Subtract the 10 first. Then it is like 4a and 4b. This one is inclusive. Answer: $[-2,5]$

   d) Same as 4c. Answer: $\left[-3,\frac{1}{2}\right]$

5. a) Down 4
   b) Right 4
   c) First reflect it in the *x*-axis (upside down), then left 2 (Remember, always stretch/reflect first)
   d) First vertically stretch by 5, then move up 3
   e) Horizontally shrink by 2
   e) All parts of the graph that are below the *x*-axis need to be reflect up. No part of the graph should be below the *x*-axis.

6. a) $x(7x-3)=0$, Answer: $x=\frac{3}{7}$ and $x=0$

   b) Distribute AND subtract the 2, you must always have a 0 on one side of the equals sign. $4x^2-8x-5x^2+5x-2=-x^2-3x-2=x^2+3x+2=(x+1)(x+2)=0$, Answer: $x=-1$ and $x=-2$

   c) You must use the Quadratic Formula. Answer: $x=-0.763$ and $x=-5.236$

   d) This one doesn't factor and gives nonreal solutions. Answer: No solution

   e) $2x^2-x^2+x+6-12=x^2+x-6=(x+3)(x-2)=0$, Answer: $x=-3$ and $x=2$

   f) Since fractions are involved, try to get common denominators, then use proportional rules. $\frac{x^2}{x}+\frac{1}{x}=\frac{13}{6},\ \frac{x^2+1}{x}=\frac{13}{6},\ 6x^2+6=13x,\ 6x^2-13x+6=0,\ (3x-2)(2x-3)=0$

   Answer: $x=\frac{2}{3}$ and $x=\frac{3}{2}$

   g) $(x^2-8)(x^2-1)=0$, Answer: $x=\pm\sqrt{8}$ and ±1

   h) $(x^{1/2}-9)(x^{1/2}-1)=0$, Answer: $x=81$ and $x=1$

   i) $\frac{1-x}{x^2}=6,\ 1-x=6x^2,\ 6x^2+x-1=0,\ (3x-1)(2x+1)=0$, Answer: $x=\frac{1}{3}$ and $x=-\frac{1}{2}$

7. a) Vertical asymptotes are found at values that make ONLY the denominator zero. If a value of *x* makes both the denominator and numerator zero, there is a hole. Horizontal asymptotes are also known as end behavior. Answer: VA: $x=3$, HA: $y=1$

   b) For the horizontal asymptote, since the bigger exponent is in the denominator, the function will always approach zero. Answer: VA: $x=1$ and $x=-1$, HA: $y=0$

   c) Answer: VA: none, HA: $y=0$

   d) Factor before deciding either. $y=\frac{(x-3)(x+3)}{x(x-3)(x+6)}=\frac{(x+3)}{x(x+6)}$, at *x* = 3, there is a hole.

   Answer: VA: $x=0$ and $x=-6$, HA: $y=0$

   e) Since the largest exponent is in the numerator and denominator, the horizontal asymptote can be found by looking at the leading terms, $\frac{2x^3}{x^3}=2$. Answer: VA: $x=1$, HA: $y=2$

8. a) Since there is an '*x*' in every term, you can cross one *x* off. Answer: $\dfrac{x^2-4x+1}{6+x^4}$

b) $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}=\dfrac{\dfrac{x^2-1}{x}}{\dfrac{x^2+1}{x}}=\dfrac{x^2-1}{x}\cdot\dfrac{x}{x^2+1}$, Answer: $\dfrac{x^2-1}{x^2+1}$ (You can not simplify any further)

c) $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x+y}{y}}=\dfrac{x^2-y^2}{xy}\cdot\dfrac{y}{x+y}=\dfrac{(x+y)(x-y)y}{xy(x+y)}$, Answer: $\dfrac{x-y}{x}$ or $1-\dfrac{y}{x}$

d) Answer: x^2-1+x^{-1} or $x^2-1+\dfrac{1}{x}$

e) You can't do anything to this one. Answer: $\dfrac{3x}{2x^3-4x+10}$

9. a) $g(2)=3,\ f(3)=9$, Answer: 9

b) $f(2)=4,\ g(4)=7$, Answer: 7

c) $h(-1)=0.5,\ f(0.5)=0.25$, Answer: 0.25

d) $h(0.5)=\sqrt{2},\ f(\sqrt{2})=2,\ g(2)=3$, Answer: 3

10. a) Common denominators and proportion/ratio rules. $\dfrac{2}{3}-\dfrac{5}{6}=\dfrac{1}{x},\ \dfrac{4}{6}-\dfrac{5}{6}=\dfrac{1}{x},\ -\dfrac{1}{6}=\dfrac{1}{x},\ -x=6$
Answer: $x=-6$

b) $x+\dfrac{6}{x}=5,\ \dfrac{x^2}{x}+\dfrac{6}{x}=5,\ \dfrac{x^2+6}{x}=5,\ x^2+6=5x,\ x^2-5x+6=0,\ (x-3)(x-2)=0$
Answer: $x=2$ and $x=3$

c) With this, multiply all terms by 6, that would get rid of the fractions.
$6\left(\dfrac{x+1}{3}-\dfrac{x-1}{2}=1\right),\ 2(x+1)-3(x-1)=6,\ 2x+2-3x+3-6=0,\ -x-1=0$, Answer: $x=-1$

d) This should be the easiest of the group, cross-multiply and solve. $5x-25=3x+3,\ 2x=28$
Answer: $x=14$

11. a) KNOW HOW TO FACTOR. $\cos^2 x - \cos x = 0$, $\cos x(\cos x - 1) = 0$, Answer: $x = \frac{\pi}{2}, \frac{3\pi}{2}, 0$

b) Give exact answers when possible. $\cos x = -\frac{\sqrt{3}}{2}$, Answer: $x = \frac{5\pi}{6}$ and $\frac{7\pi}{6}$

c) Don't forget the plus/minus sign. $\sin x = \pm\frac{1}{2}$, Answer: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

d) Always have a quadratic trinomial set equal to zero before you try to factor.
$2\sin^2 x + \sin x - 1 = 0$, $(2\sin x - 1)(\sin x + 1) = 0$, $\sin x = \frac{1}{2}$ and $\sin x = -1$,
Answer: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

e) $\sin x(2\cos x + 1) = 0$, Answer: $x = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

f) Factor first (difference of 2 squares), $(\sin x - \cos x)(\sin x + \cos x) = 0$. If we set each factor equal to zero, it's hard to solve when there are two trig. functions involved. But, if you use a little common sense, and your unit circle, it's not that hard. $\sin x = \cos x$ and $\sin x = -\cos x$.
There are 4 places where sine and cosine are equal or opposites. Answer: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

12. Function notation, this information is the same as (2, -5) and (-3, 1). Answer: $y = -\frac{6}{5}x - \frac{13}{5}$, or if you want to get rid of all fractions, and write it in standard form, $6x + 5y = 13$

13. ***Point-slope needs to be used as much as possible***. It is the easiest form b/c the only math involved is finding slope. Many people make mistakes when finding 'b', that doesn't happen in this form.
Answer: $y - 1 = -\frac{1}{2}(x - 5)$

14. Use the slope formula, Answer: $\frac{x(1) - x(4)}{1 - 4} = \frac{7 - 2}{1 - 4} = \frac{5}{-3}$

15. Rate of change means "slope". Answer: B, C, A

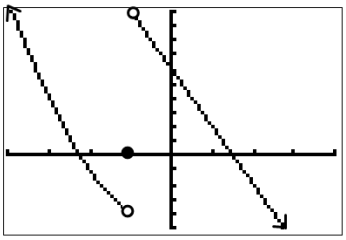
16. Switch all x's and y's, then solve for y. $x = \frac{y}{y + 3}$, $xy + 3x = y$, $xy - y = -3x$, $y(x - 1) = -3x$
Answer: $y = g^{-1}(x) = \frac{-3x}{x - 1}$

17. Two ways to do this, use your calculator or use algebra (substitution). Since y is solved for, substitute $x - 1$ into the other equation for y. $(x - 1)^2 = 2x + 6$, $x^2 - 2x + 1 = 2x + 6$, $x^2 - 4x - 5 = 0$, remember, always get zero on one side before factoring. $(x - 5)(x + 1) = 0$, $x = 5$ and $x = -1$,
Answer: (5, 4) and (-1, -2)

18. KNOW YOUR UNIT CIRCLE

Answers: a) $-\frac{1}{2}$ b) $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ c) $-\frac{1}{2}$ d) -2 e) Undefined f) 1

19. Piecewise functions are not as confusing as they look. They are just pieces of functions put together over a given domain, to make one function. Remember, all functions must pass the vertical line test, so keep that in mind. When *x* is less than -1, use the parabola x^2-5, when *x* = -1, put a point at 0, when *x* is greater than -1, use the line 6 – 4*x*. Also notice the open and closed circles. Answer:



20. You can plug numbers in, or look at the graph. Answer: Left $f(x)\to-\infty$, Right $f(x)\to 0$

21. a) Parabola opening upward with vertex (3, 2), Answer: Domain $(-\infty,\infty)$, Range $[2,\infty)$
    b) Shape of a V, with vertex (4, -3), Answer: Domain $(-\infty,\infty)$, Range $[-3,\infty)$
    c) You can take the cube root of any number, Answer: Domain = Range = $(-\infty,\infty)$
    d) The 5 is the amplitude, Answer: Domain $(-\infty,\infty)$, Range $[-5,5]$
    e) A positive base can be taken to any power, but will never yield a negative answer. Answer: Domain $(-\infty,\infty)$, Range $(0,\infty)$

22. Area: $9\pi\approx 28.274$ Circumference: $6\pi\approx 18.849$
(always round or truncate (cut off) to 3 decimal places)

23. Average of the bases times height, must solve for height first. $h=\sqrt{6^2-3^2}=\sqrt{27}$,
Answer: $\frac{12+15}{2}\sqrt{27}=70.148\text{ cm}^2$

24. Use law of cosines to find the missing side, then use law of sines to find the missing angles.
$c^2=9^2+12^2-2(9)(12)\cos 30°=6.159$, make sure to store this answer for later use.
$\frac{6.159}{\sin 30}=\frac{9}{\sin a}=\frac{12}{\sin b}$, Answer: Missing side is 6.159, missing angles are 46.936 and 103.064

25. Subtract the two volumes, volume of a cylinder is $\pi r^2 h$, Answer: 933.053 ft^3

26. Answer: $\frac{\ln(x+3)}{\ln 5}$

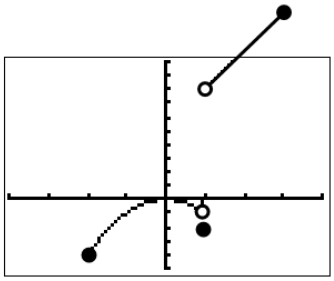
27. The missing side is $240-x$, find the area of the rectangle. $A=x(240-x)=550$, this problem has 2 solutions. Answer: x = 30.845 ft or x = 89.155 ft

28. Answer: a) 16 elk b) 159779 years c) 1216 elk

29. If you can't factor or see identities, change everything into sine and cosine. $\dfrac{1}{\sin x}-\dfrac{\sin x}{\cos x}\sin x\cos x$

Answer: $\dfrac{1}{\sin x}-\sin^2 x$ or $\csc x-\sin^2 x$ or $\dfrac{1-\sin^3 x}{\sin x}$

30. Answer:



31. Put one side into y_1, and other into y_2, find the point(s) where they intersect. Answer: x = -0.391
32. Solve for y and look at their slopes, if they are different, they must intersect. Answer: Yes

33. For those that didn't take accelerated precalc, don't panic. We will spend the first month talking about limits (lim). All that is asking if for the behavior or y-value near the given x-value. So, if you see $\lim\limits_{x\to 4^-} f(x)$ it is asking for the y-value just to the right of 4.

Answers

a) 0 b) -2 c) 2 and -2 d) 2 e) 0 f) DNE

34. a) 30000 b) 11.605 years

35. a) $3\ln\left(x^2-1\right)-9\ln x$ b) $\ln 3+2\ln e=\ln 3+2$

36. $\ln\left(\dfrac{x-2}{x+2}\right)$ b) $\ln\left(\dfrac{x^3y^2}{z^4}\right)$

37. a) 1 b) Can not do c) 0 d) 3