AP Calculus Summer Packet

A. Basic functions: There are certain graphs that occur regularly in calculus and students should know the general shape of them, where they hit the x-axis (zeros) and y-axis (y-intercept), as well as the domain and range.

For each of the following functions, a) sketch a graph, b) list any zero(s), c) y-intercept, d) state the domain and range.

1. $y = x$	2. $y = x^2$	3. $y = x^3$
4. $y = \sqrt{x}$	5. $y = \|x\|$	6. $y = \frac{1}{x}$
7. $y = lnx$	8. $y = e^x$	9. $y = sinx$
10. $y = cosx$		

B. Linear Functions: Probably the most important concept from precalculus that is required for differential calculus is that of linear functions. Students need to know these formulas backwards and forwards.

Slope: Given two points (x_1, y_1) and (x_2, y_2), the slope of the line passing through the points can be written as: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Slope intercept form: the equation of a line with slope m and y-intercept b is given by $y = mx + b$.

Point-slope form: the equation of a line passing through the points (x_1, y_1) and slope m is given by $y - y_1 = m(x - x_1)$. While you might have preferred the simplicity of the $y = mx + b$ form in your algebra course, the $y - y_1 = m(x - x_1)$ form is far more useful in calculus.

Intercept form: the equation of a line with x-intercept a and y-intercept b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

General form: $Ax + By + C = 0$ where A, B and C are integers. While your algebra teacher might have required your changing the equation $y - 1 = 2(x - 5)$ to general form $2x - y - 9 = 0$, you will find that on the AP calculus test, it is sufficient to leave equations for a lines in point-slope form and it is recommended not to waste time changing it unless you are specifically told to do so.

Parallel lines Two distinct lines are parallel if they have the same slope: $m_1 = m_2$.

Normal lines: Two lines are normal (perpendicular) if their slopes are negative reciprocals: $m_1 \cdot m_2 = -1$.

Horizontal lines have slope zero. **Vertical lines** have no slope (slope is undefined).

11. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

a. $m = -7, (-3, -7)$

b. $m = -\frac{1}{2}, (2, -8)$

c. $m = \frac{2}{3}, \left(-6, \frac{1}{3}\right)$

12. Find the equation of the line in point-slope form, passing through the following points.

a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

13. Write equations of the line through the given point a) parallel and b) normal to the given line.

a. $(5,-3)$, $x+y=4$ b. $(-6,2)$, $5x+2y=7$ c. $(-3,-4)$, $y=-2$

14. Find an equation of the line containing (4, -2) and parallel to the line containing (-1, 4) and (2, 3). Put your answer in general form.

15. Find k if the lines 3x – 5y = 9 and 2x + ky = 11 are a) parallel and b) perpendicular.

C. Eliminating Complex Fractions

Calculus frequently uses **complex fractions**, which are fractions within fractions. Answers are never left with complex fractions and they must be eliminated. There are two methods to eliminate complex fractions:

When the problem is in the form of $\frac{\frac{a}{b}}{\frac{c}{d}}$, we can "flip the denominator" and write it as $\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$.
However, this does not work when the numerator and denominator are not single fractions. The best way is to eliminate the complex fractions in all cases is to find the LCD (lowest common denominator) of all the fractions in the complex fraction. Multiply all terms by this LCD and you are left with a fraction that is magically no longer complex. **Important:** Note that $\frac{x^{-1}}{y^{-1}}$ can be written as $\frac{y}{x}$ but $\frac{1+x^{-1}}{y^{-1}}$ must be written as $\frac{1+\frac{1}{x}}{\frac{1}{y}}$.

16. $\dfrac{\frac{5}{8}}{\frac{-2}{3}}$

17. $\dfrac{4-\frac{2}{9}}{3+\frac{4}{3}}$

18. $\dfrac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

19. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

20. $\dfrac{1+x^{-1}}{1-x^{-2}}$

21. $\dfrac{x^{-1}+y^{-1}}{x+y}$

22. $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

23. $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{\frac{-3}{4}}$

24. $\dfrac{2x(2x-1)^{1/2}-2x^{2}(2x-1)^{-1/2}}{(2x-1)}$

D. Adding Fractions

25. $\frac{2}{3}-\frac{1}{x}$

26. $\frac{1}{x-3}+\frac{1}{x+3}$

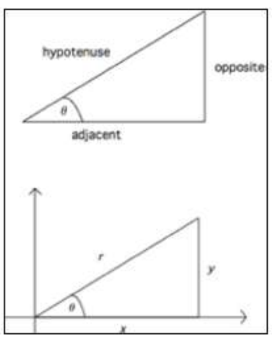
27. $\frac{5}{2x}-\frac{5}{3x+15}$

28. $\frac{2x-1}{x-1}-\frac{3x}{2x+1}$

E. Right Angle Trigonometry

Trigonometry is an integral part of AP calculus. Students must know the basic trig function definitions in terms of opposite, adjacent and hypotenuse as well as the definitions if the angle is in standard position.

Given a right triangle with one of the angles named θ, and the sides of the triangle relative to θ named opposite (y), adjacent (x), and hypotenuse (r) we define the 6 trig functions to be:



$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}=\frac{y}{r} \qquad \csc\theta=\frac{\text{hyptotenuse}}{\text{opposite}}=\frac{r}{y}$$

$$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}=\frac{x}{r} \qquad \sec\theta=\frac{\text{hyptotenuse}}{\text{adjacent}}=\frac{r}{x}$$

$$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}=\frac{y}{x} \qquad \cot\theta=\frac{\text{adjacent}}{\text{opposite}}=\frac{x}{y}$$

The Pythagorean theorem ties these variables together: $x^2+y^2=r^2$. Students should recognize right triangles with integer sides: 3-4-5, 5-12-13, 8-15-17, 7-24-25. Also any multiples of these sides are also sides of a right triangle. Since r is the largest side of a right triangle, it can be shown that the range of $\sin\theta$ and $\cos\theta$ is $[-1,1]$, the range of $\csc\theta$ and $\sec\theta$ is $(-\infty,-1]\cup[1,\infty)$ and the range of $\tan\theta$ and $\cot\theta$ is $(-\infty,\infty)$.

Also vital to master is the signs of the trig functions in the four quadrants. A good way to remember this is A – S – T – C where <u>A</u>ll trig functions are positive in the 1st quadrant, <u>S</u>in is positive in the 2nd quadrant, <u>T</u>an is positive in the 3rd quadrant and <u>C</u>os is positive in the 4th quadrant.

29. Let be a point on the terminal side of θ. Find the 6 trig functions of θ. (Answers need not be rationalized).

a) $P(15,8)$

b. $P(-2,3)$

c. $P\left(-2\sqrt{5},-\sqrt{5}\right)$

30. If $\tan\theta=\frac{12}{5}$, θ in quadrant III, find $\sin\theta$ and $\cos\theta$

31. If $\csc\theta=\frac{6}{5}$, θ in quadrant II, find $\cos\theta$ and $\tan\theta$

32. $\cot\theta=\frac{-2\sqrt{10}}{3}$ find $\sin\theta$ and $\cos\theta$

33. Find the quadrants where the following is true: Explain your reasoning.

a. $\sin\theta > 0$ and $\cos\theta < 0$

b. $\csc\theta < 0$ and $\cot\theta > 0$

c. all functions are negative

34. Which of the following is possible? Explain your reasoning.

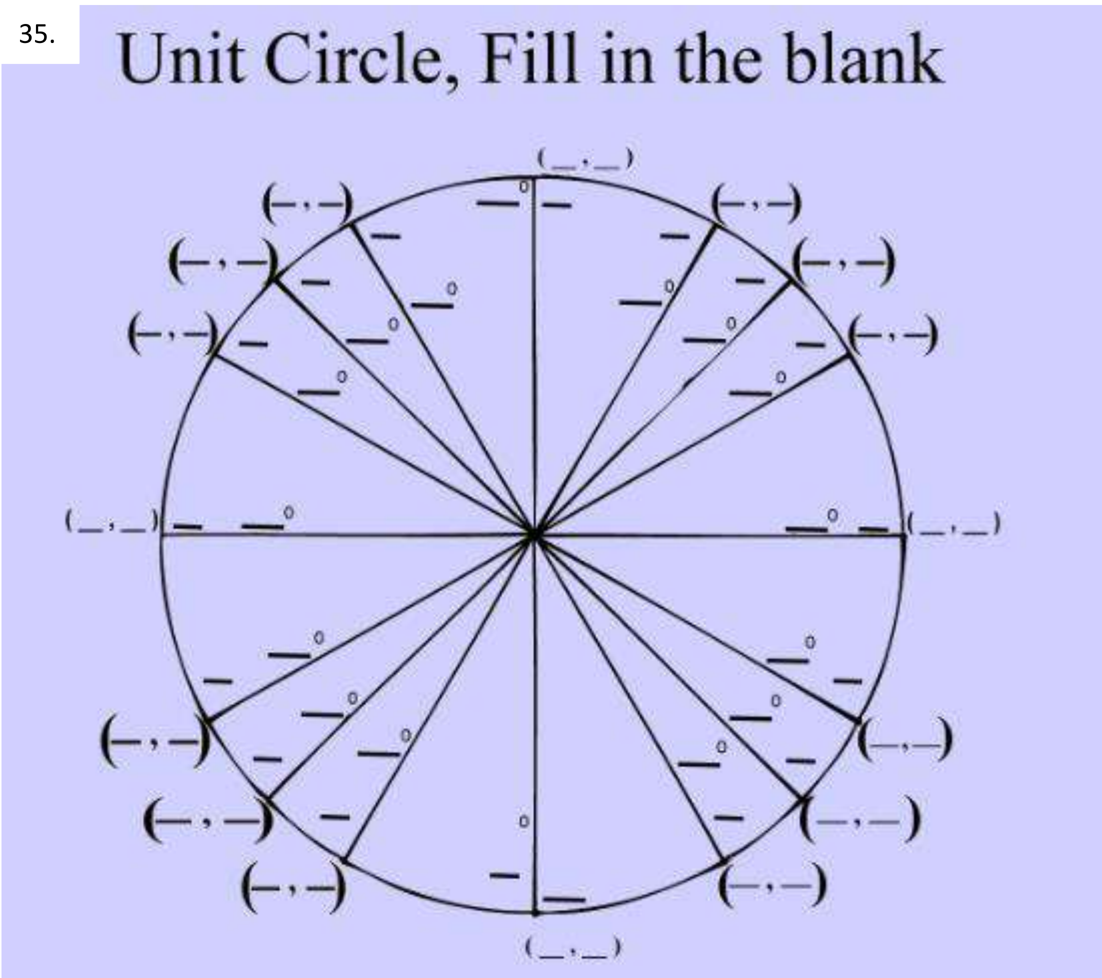
a. $5\sin\theta = -2$

b. $3\sin\alpha + 4\cos\beta = 8$

c. $8\tan\theta + 22 = 85$

F. Special Angles

35.



36. Use the unit circle to fill in the chart.

θ(deg)	θ(rad)	Sin θ	Cos θ	Tan θ	Csc θ	Sec θ	Cot θ
0							
30							
45							
60							
90							
120							
135							
150							
180							
210							
225							
240							
270							
300							
315							
330							
360							

- Evaluate each of the following without looking at a chart.

37. $\sin^2 120° + \cos^2 120°$

38. $2\tan^2 300° + 3\sin^2 150° - \cos^2 180°$

39. $\cot^2 135° - \sin 210° + 5\cos^2 225$

40. $\cot(-30°) + \tan(600°) - \csc(-450°)$

41. $\left(\cos\frac{2\pi}{3} - \tan\frac{3\pi}{4}\right)^2$

42. $\left(\sin\frac{11\pi}{6} - \tan\frac{5\pi}{6}\right)\left(\sin\frac{11\pi}{6} + \tan\frac{5\pi}{6}\right)$

• Determine whether each of the following statements are true or false.

43. $\sin\frac{\pi}{6}+\sin\frac{\pi}{3}=\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$

44. $\frac{\cos\frac{5\pi}{3}+1}{\tan^2\frac{5\pi}{3}}=\frac{\cos\frac{5\pi}{3}}{\sec\frac{5\pi}{3}-1}$

45. $2\left(\frac{3\pi}{2}+\sin\frac{3\pi}{2}\right)\left(1+\cos\frac{3\pi}{2}\right)>0$

46. $\frac{\cos^3\frac{4\pi}{3}+\sin\frac{4\pi}{3}}{\cos^2\frac{4\pi}{3}}>0$