AP Calculus Summer Packet

There are certain skills that have been taught to you over the previous years that are essential towards your success in AP Calculus. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you may understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra or trigonometry and not the calculus. This summer packet is intended for you to retain/review/relearn these topics. This packet can be found and printed off at my RHS AP Calculus website (al048.k12.sd.us).

Below is a list of several websites that may help you when you come across a difficult problem. If you are unsure of how to attempt these problems, please look online for help, or send me an email. Feel free to use all resources available to you via internet and textbooks. Please take these problems seriously. As stated before, students are weak in these skills and have a difficult time succeeding in calculus without them.

Work needs to be shown, when possible, in a neat, legible, organized manner. Do not always rely on your calculator. Half of your AP exam next year is taken without the calculator.

I believe you will benefit the most from this packet by starting it towards the end of June. You should try to complete a few problems each day, as if it was a daily journal. Do not do all of it now, and do not wait and do it a week before we start school in August. You are more likely to retain the information if you spread it out.

This summer assignment will not be graded when you return. But, there is a good chance I will give extra credit points for those of you that complete this. If you have any questions, please email me at anthony.lanham@k12.sd.us. I look forward to working with you next year.

Mr. Lanham

Helpful Websites

http://www.mathematicshelpcentral.com/index.html
http://www.mathtv.com/
http://archives.math.utk.edu/visual.calculus/

1. Simplify using only positive exponents.

a) -3^{-x}

b) $-5\left(\frac{9}{4}\right)^{-1/2}$

c) $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]$

d) $\left(16x^2y\right)^{3/4}$

2. Find the domain of the following functions. Make sure to use interval notation (ex: [0, 3)).

a) $y=\log(2x-12)$

b) $y=\frac{x^2-5x-6}{x^2-3x-18}$

c) $y=\frac{2^{2-x}}{x}$

d) $y=\frac{\sqrt{2x-9}}{2x+9}$

e) $y=\sqrt{x^2-5x-14}$

f) $y=\frac{x}{\cos x}$

3. Factor completely.

a) x^5+11x^3-80x

b) $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

c) $2x^2+50y^2-20xy$

4. Solve the following inequalities by factoring and making sign charts.

a) $x^2-16>0$

b) $x^2+6x-16>0$

c) $x^2-3x\le 10$

d) $2x^2+5x\le 3$

5. Describe, in words, the transformations that would take place to $f(x)$ in each of the following.

a) $f(x)-4$ b) $f(x-4)$ c) $-f(x+2)$

d) $5f(x)+3$ e) $f(2x)$ f) $|f(x)|$

6. Solve each equation by factoring, graphing, or using the quadratic formula.

a) $7x^2-3x=0$ b) $4x(x-2)-5x(x-1)=2$ c) $x^2+6x+4=0$

d) $2x^2-3x+3=0$ e) $2x^2-(x+2)(x-3)=12$ f) $x+\frac{1}{x}=\frac{13}{6}$

g) $x^4-9x^2+8=0$ h) $x-10\sqrt{x}+9=0$ i) $\frac{1}{x^2}-\frac{1}{x}=6$

7. Find the equations of all vertical (x = ?) and horizontal (y = ?) asymptotes (if they exist).

a) $y=\frac{x}{x-3}$ b) $y=\frac{x+4}{x^2-1}$ c) $y=\frac{x+4}{x^2+1}$

d) $y=\frac{x^2-9}{x^3+3x^2-18x}$ e) $y=\frac{2x^3}{x^3-1}$

8. Simplify the following.

a) $\dfrac{x^3-4x^2+x}{6x+x^5}$

b) $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}$

c) $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x+y}{y}}$

d) $\dfrac{x^3-x+1}{x}$

e) $\dfrac{3x}{2x^3-4x+10}$

9. If $f(x)=x^2$, $g(x)=2x-1$, and $h(x)=2^x$, find the following.

a) $f(g(2))$

b) $g(f(2))$

c) $f(h(-1))$

d) $g\left(f\left(h\left(\dfrac{1}{2}\right)\right)\right)$

10. Solve each equation.

a) $\dfrac{2}{3}-\dfrac{5}{6}=\dfrac{1}{x}$

b) $x+\dfrac{6}{x}=5$

c) $\dfrac{x+1}{3}-\dfrac{x-1}{2}=1$

d) $\dfrac{x-5}{x+1}=\dfrac{3}{5}$

11. Solve each equation on the interval $[0, 2\pi)$. Give exact values $\left(\text{ex: } \frac{\pi}{3}\right)$ if possible.

a) $\cos^2 x = \cos x$

b) $2\cos x + \sqrt{3} = 0$

c) $4\sin^2 x = 1$

d) $2\sin^2 x + \sin x = 1$

e) $2\sin x \cos x + \sin x = 0$

f) $\sin^2 x - \cos^2 x = 0$

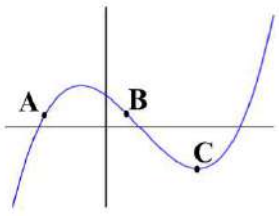
12. Let f be a linear function where $f(2) = -5$ and $f(-3) = 1$. Find $f(x)$.

13. Find an equation for the line, in point-slope form, that contains $(5,1)$ and is perpendicular to $6x - 3y = 2$.

14. Use the table to calculate the average rate of change from t = 1 to t = 4.

t	0	1	2	3	4
$x(t)$	8	7	5	1	2

15. Order the points A, B, and C, from least to greatest, by their rates of change.



16. If $g(x) = \frac{x}{x+3}$, find $g^{-1}(x)$ (*the inverse of g*).

17. Find the points of intersection in the graphs of $y = x - 1$ and $y^2 = 2x + 6$.

18. Evaluate the following using your unit circle (back page of packet).

a) $\sin\left(\frac{7\pi}{6}\right)$ b) $\csc(60°)$ c) $\cos(120°)$

d) $\sec\left(-\frac{2\pi}{3}\right)$ e) $\tan\left(\frac{\pi}{2}\right)$ f) $\cot(-135°)$

19. Sketch a graph of the piecewise function $f(x) = \begin{cases} x^2 - 5, & x < -1 \\ 0, & x = -1 \\ 6 - 4x, & x > -1 \end{cases}$.

20. Describe the left and right end-behavior of the function $f(x) = -3^x$.

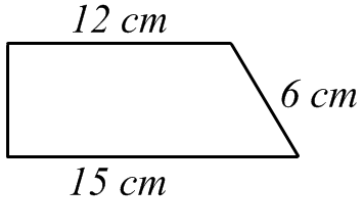
21. Find the domain and range of each function (without a calculator if possible).

a) $f(x) = (x-3)^2 + 2$ b) $f(x) = 2|x-4| - 3$ c) $f(x) = \sqrt[3]{1-x}$

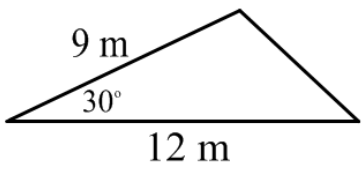
d) $f(x) = 5\sin x$ e) $f(x) = e^{-x}$

22. Find the area and circumference of a circle with diameter of 6ft.

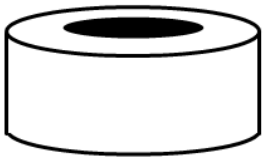
23. Find the area of the trapezoid.



24. Find the missing sides and angles of the triangle. Then find its area.

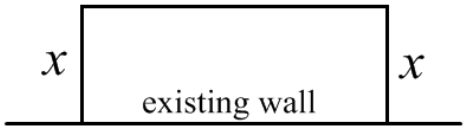


25. Find the volume of a washer with outer radius of 18 ft., inner radius of 15 ft., and height of 3 ft.



26. Rewrite $\log_5(x+3)$ into an equivalent expression using only natural logarithms.

27. Three sides of a fence and an existing wall form a rectangular enclosure. The total length of fence used for the three sides is 240 ft. Find x if the area enclosed is 5500 ft^2.



28. The number of elk after t years in a state park is modeled by the function $P(t)=\dfrac{1216}{1+75e^{-0.03t}}$.

a) What was the initial population?

b) When will the number of elk be 750?

c) What is the maximum number of elk possible in the park?

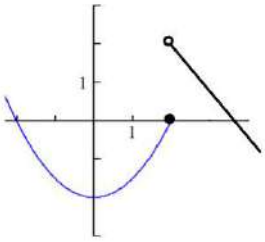
29. Simplify $\csc x - \tan x \sin x \cos x$.

30. Sketch a graph of the piecewise function $f(x)=\begin{cases}-x^2, & -2\le x<1\\ -2, & x=1\\ 3x+5, & 1<x\le 3\end{cases}$.

31. Use a graphing calculator to solve $e^{2x}=3x^2$.

32. Do the lines $-x+5y=22$ and $7x-2y=19$ intersect?

33. The function $f(x)$ is graphed below. Find the following.



a) $f(2)$ b) $f(0)$ c) $f(x)=0$

d) $\lim_{x\to 2^+} f(x)$ e) $\lim_{x\to 2^-} f(x)$ f) $\lim_{x\to 2} f(x)$

34. If the annual rate of salary increase averages 4.5% over the next 15 years, then the approximate salary *S* during any year in that period is $S(t)=30,000(1.045)^t$.

a) What was the starting salary?

b) When will the individual earn $50,000?

35. Expand the following logarithmic expressions.

a) $\ln\left(\frac{x^2-1}{x^3}\right)^3$ b) $\ln\left(3e^2\right)$

36. Condense the following logarithmic expressions.

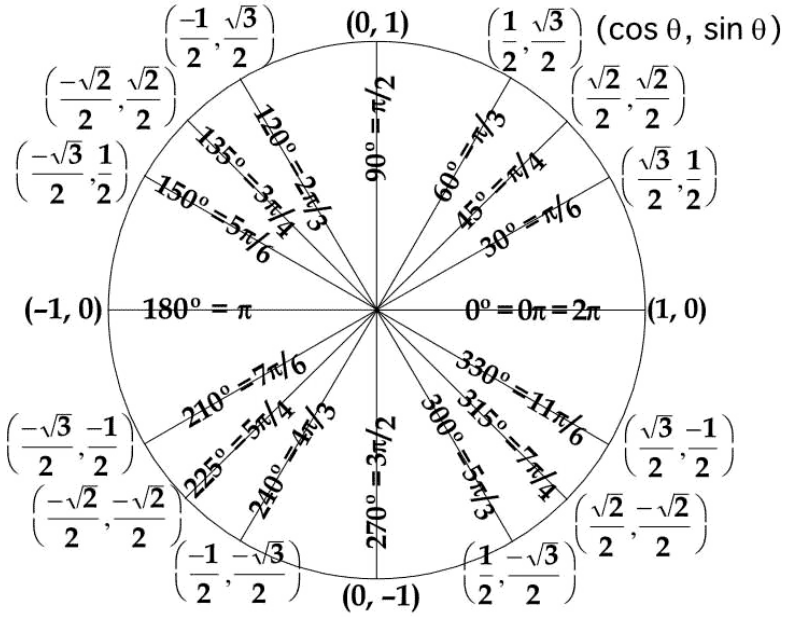
a) $\ln(x-2)-\ln(x+2)$ b) $3\ln x+2\ln y-4\ln z$

37. Find the following without using a calculator.

a) $\ln e$ b) $\ln 0$ c) $\ln 1$ d) $\ln e^3$

38. KNOW THE UNIT CIRCLE!!!!!

Remember, the sine of an angle is its *y*-coordinate and the cosine is the *x*-coordinate.



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	DNE	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\theta$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	DNE	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$