Welcome to AP Calculus!

You are receiving this summer work packet because you are on the list of students registered for AP Calculus in the 2024-2025 school year. This packet is a review of the prerequisite concepts for AP Calculus. It is to be done NEATLY. All problems must be attempted and **all work must be shown**. If your work is messy and/or unorganized then you will not receive credit for that problem.

AP Calculus is a challenging and fast-paced course that will keep you thinking and on your toes at all times. It is also a fascinating course as you will see all of the concepts from Algebra 1, Geometry, Algebra 2, and Precalculus come to life. Remember all of those times when you were sitting in math class thinking "When am I going to use this?". The answer is NOW! You're going to use it now.

I honestly can say that I cannot wait to begin our Calculus journey together. I am looking forward to the wonderful mathematical insights and challenges we will explore together. Please feel free to contact me over the summer if you have questions about the directions. However, there will be no help with any of the questions. You must be resourceful and look things up if you forgot how to do something. Asking a friend for help is acceptable, however, copying a friend's work is not!

Have a wonderful summer and I will see you all in the fall!

~Mrs. Joyce

Due Date: September 4, 2024 (the first day of school) NO EXCUSES!
You will hand this packet in at the beginning of the period on the first class of the year…with a big smile. ☺
The packet itself will count as a 50-point homework grade.
There will be a TEST on the concepts in this packet during the 2nd week of school (the week of 9/9/24) so please take this review seriously!

AP Calculus
SUMMER PACKET

Name: _____

MUST SHOW ALL WORK FOR FULL CREDIT!

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x + 2) =$

3. $f(x + h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

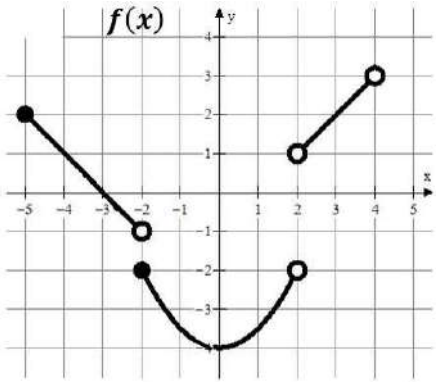
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -2$ when $x =$?



Write the equation of the line that meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

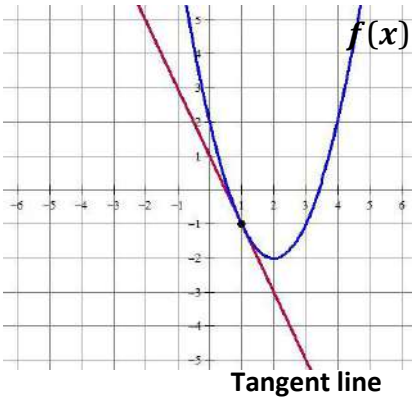
5. slope = 3 and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

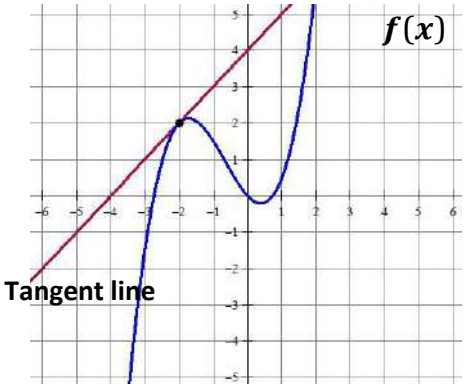
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



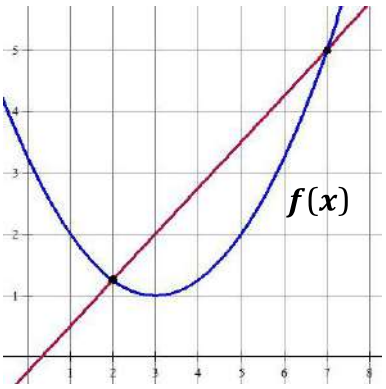
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y2-y1}{x_2-x_1}$ YOU MUST EXPLAIN YOUR ANSWERS!

10. Which choice represents the slope of the secant line shown?
Explain your answer.

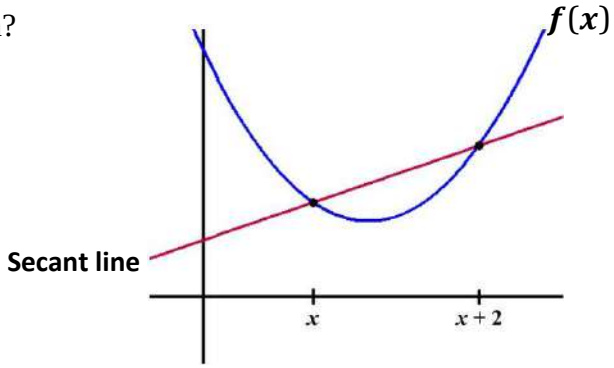
A) $\frac{7-2}{\mathbf{f}(7)-\mathbf{f}(2)}$ B) $\frac{\mathbf{f}(7)-2}{7-\mathbf{f}(2)}$ C) $\frac{7-\mathbf{f}(2)}{\mathbf{f}(7)-2}$ D) $\frac{\mathbf{f}(7)-\mathbf{f}(2)}{7-2}$



11. Which choice represents the slope of the secant line shown?
Explain your answer.

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

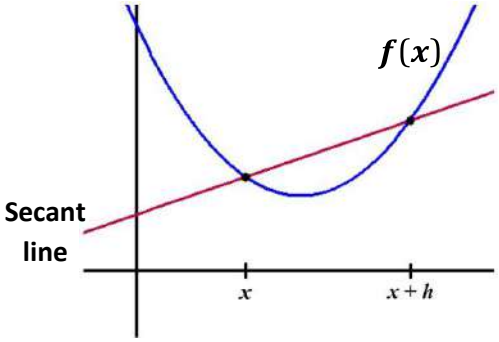
D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?
Explain your answer.

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

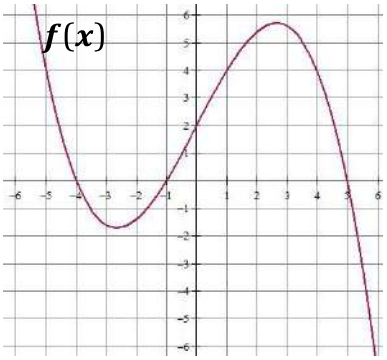
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?
Explain your answer.

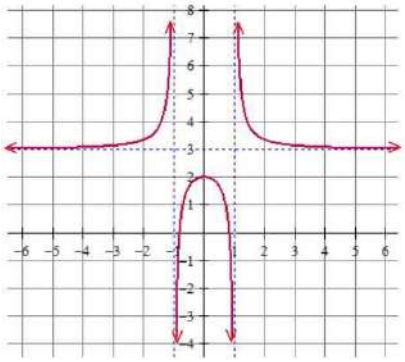
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes. (Write asymptotes as the equation of a vertical/horizontal line)

14.



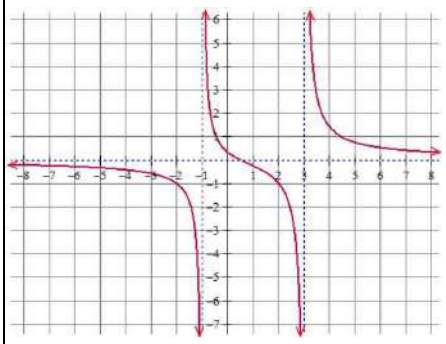
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



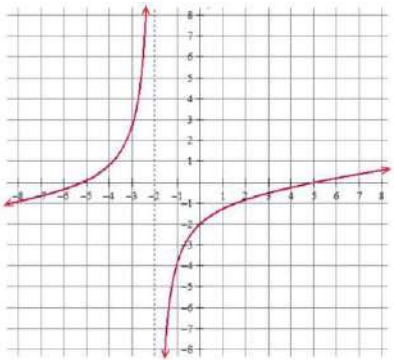
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE! YOU MUST EXPLAIN YOUR ANSWERS!

17. Which of the following functions has a vertical asymptote at $x = 4$? **Explain how you know.**

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $f(x)=\frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true? **Explain your answer.**

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$		
19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$
Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$		
25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

Factor Completely.

64. $x^2 - 4x - 21$	65. $2x^2 + 7x - 15$	66. $x^3 - 2x^2 - 4x + 8$
67. $18x^2 - 8$	68. $-4x^3 - 2x^2 + 30x$	

Simplify.

69. $\frac{\sqrt{x}}{x}$

70. $e^{\ln x}$

71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^{7}$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$, then determine each of the following.
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$, then determine each of the following.
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$

87. $f^{-1}(4)$

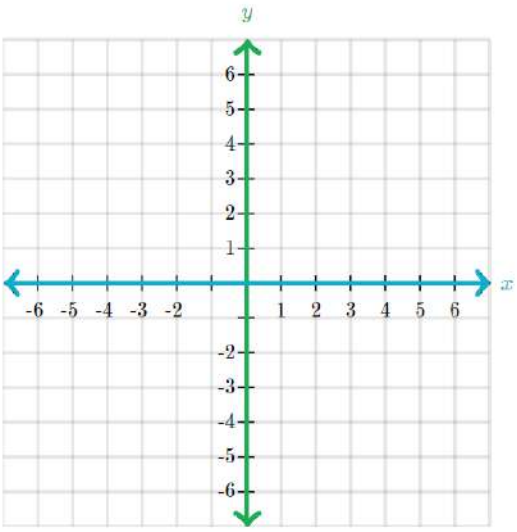
88. $k^{-1}(x)$

89. $k(g(x))$

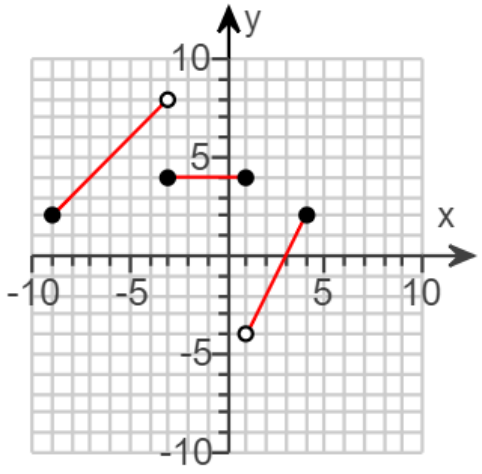
90. $g(f(2))$

91. Graph the piecewise function

$$y = \begin{cases} x^2 & \text{if } x < 0 \\ x+2 & \text{if } 0 \le x \le 3 \\ 4 & \text{if } x > 3 \end{cases}$$



92. Write an equation for the piecewise function graphed below.



Simplify by rationalizing the denominator:

93. $\dfrac{2}{\sqrt{3}}$

94. $\dfrac{3}{2-\sqrt{5}}$

95. $\dfrac{2-\sqrt{3}}{4-\sqrt{6}}$

Simplify each complex fraction:

96. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x^2}}$

97. $\dfrac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

98. $\dfrac{\frac{2}{x}-\frac{2}{3x}}{\frac{1}{x}-\frac{5}{6x}}$

Determine if each function is continuous. If yes, explain why. If no, identify the type of discontinuity.

99. $g(x)=\begin{cases}2x-4, & x<3\\ -x+5, & x\geq 3\end{cases}$

100. $b(x)=\dfrac{x(3x+1)}{3x^2-5x-2}$

101. $h(x)=\dfrac{\sqrt{x^2-10x+25}}{x-5}$

102. How do you know if a rational function has a vertical asymptote?

103. How do you know if a rational function has a hole in the graph?

104. How do you know if a rational function has a horizontal asymptote?

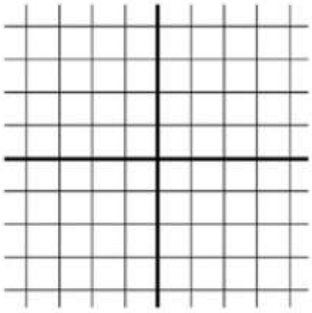
105. For each function below, identify all asymptotes and holes in the graph. Write each asymptote as the equation of a horizontal or vertical line and write each hole as a coordinate point.

a) $y=\dfrac{2x}{x-4}$

b) $y=\dfrac{x^2-2x+1}{x^2+x-2}$

106. For each function below, sketch an accurate graph, identify any important points (label intercepts, give dashed lines for asymptotes, etc.), then state the domain and range. (Graph sinx/cosx on (-2π, 2π)

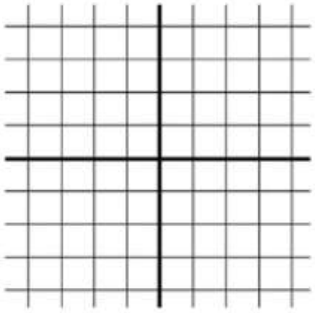
$f(x) = \frac{1}{x}$



Domain: _____

Range: _____

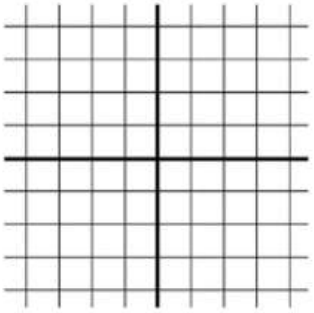
$f(x) = \sin x$



Domain: _____

Range: _____

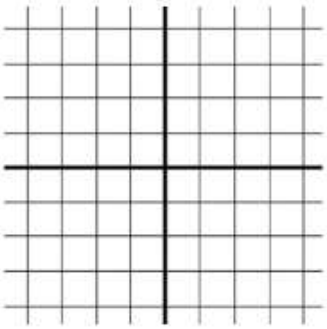
$f(x) = |x|$



Domain: _____

Range: _____

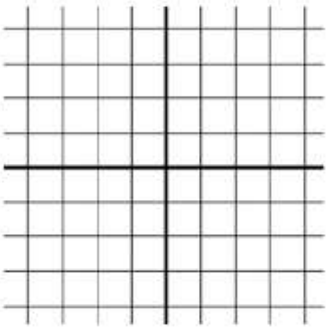
$f(x) = \sqrt{x}$



Domain: _____

Range: _____

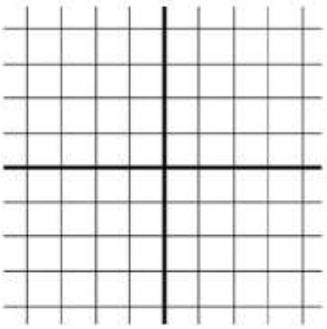
$f(x) = \cos x$



Domain: _____

Range: _____

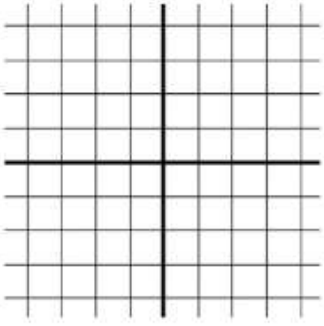
$f(x) = x$



Domain: _____

Range: _____

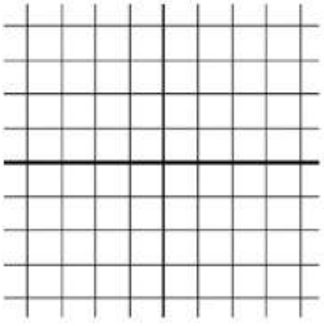
$f(x) = x^2$



Domain: _____

Range: _____

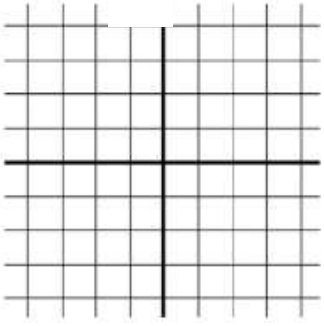
$f(x) = \ln x$



Domain: _____

Range: _____

$f(x) = \frac{1}{e^x}$



Domain: _____

Range: _____