Pages 262 – 263 #47, 48, 53, 54, 58, 59, 60, 61, 62
AP Calculus Optimization/Related Rates Review Problem Solutions

47)

$S = x^2 + 4xh = 108 \Rightarrow h = \dfrac{108 - x^2}{4x}$

$V = x^2 h = x^2\left(\dfrac{108 - x^2}{4x}\right) = 27x - \dfrac{x^3}{4}$

$V' = 27 - \dfrac{3}{4}x^2 \quad 27 - \dfrac{3}{4}x^2 = 0 \Rightarrow x^2 = 36 \Rightarrow x = 6$

$V'' = -\dfrac{3}{2}x < 0,$ so the value $x = 6$ is a maximum

Dimensions: $6 \times 6 \times 3$

48)

$S = x^2 + 4xh$

$V = x^2 h = 32 \Rightarrow h = \dfrac{32}{x^2}$

$S = x^2 + 4x\left(\dfrac{32}{x^2}\right) = x^2 + \dfrac{128}{x}$

$S' = 2x - \dfrac{128}{x^2} \quad 2x - \dfrac{128}{x^2} = 0 \Rightarrow x^3 = 64 \Rightarrow x = 4$

$S'' = 2 + \dfrac{256}{x^3}x > 0,$ so the value $x = 4$ is a minimum

Dimensions: $4 \times 4 \times 2$

53)

Straight/Straight: $C = 12(40,000) + 20(30,000) = \$1,080,000$

All Water: $C = 40,000\sqrt{544} = \$932,952.30$

Hybrid: $C = 40,000\left(\sqrt{y^2 + 144}\right) + 30,000(20 - y)$

$C' = 20,000\left(y^2 + 144\right)^{-1/2}(2y) - 30,000 = \dfrac{40,000y - 30,000\sqrt{y^2 + 144}}{\sqrt{y^2 + 144}}$

$40,000y - 30,000\sqrt{y^2 + 144} = 0 \Rightarrow y^2 + 144 = \dfrac{16}{9}y^2 \Rightarrow \dfrac{7}{9}y^2 = 144 \Rightarrow y \approx 13.607$ miles

$x = \sqrt{y^2 + 44} \approx 18.142$ miles

54)

$$A = 2rx \quad P = 2x + 2\pi r \Rightarrow r = \frac{400 - 2x}{2\pi}$$

$$A = 2x\left(\frac{400 - 2x}{2\pi}\right) = \frac{1}{2\pi}\left(800x - 4x^2\right)$$

$$A' = \frac{1}{2\pi}(800 - 8x) \quad A' = 0 \Rightarrow 800 - 8x = 0 \Rightarrow x = 100$$

$A'' = \frac{1}{2\pi}(-8) < 0,$ so the value of x is a maximum

$x = 100$ meters, $r = \frac{100}{\pi}$ meters

58)

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(10)\left(\frac{-2}{\pi}\right) = -40$$

The area is decreasing at a rate of 40 m^2/sec

59)

$$d = \sqrt{x^2 + y^2} \Rightarrow \frac{dd}{dt} = \frac{1}{2}\left(x^2 + y^2\right)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

$$\frac{dd}{dt} = \frac{1}{2\sqrt{5^2 + 12^2}}\left(2(5)(-1) + 2(12)(-5)\right) = \frac{1}{26}(-10 - 120) = -5$$

The particle is approaching the origin at a rate of 5 m/sec

60)

$$V = e^3 \quad \frac{dV}{dt} = 3e^2\frac{de}{dt}$$

$$1200 = 3\left(20^2\right)\frac{de}{dt}$$

$$\frac{de}{dt} = \frac{1200}{1200} = 1$$

The edges are increasing at a rate of 1 cm/min

61)

a) $\frac{r}{h}=\frac{4}{10}\Rightarrow h=\frac{5}{2}r$

b)

Find $\frac{dh}{dt}$

$$V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h=\frac{4}{25}\pi h^3$$

$$\frac{dV}{dt}=\frac{4}{25}\pi h^2\frac{dh}{dt}\Rightarrow -5=\frac{4}{25}\pi(6)^2\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{-5}{\frac{4}{25}\pi(36)}\approx -0.276$$

The water is dropping at a rate of 0.276 ft/min