AP Calculus Integration by Parts Worksheet

Evaluate the following by hand.

1) $\int x(x+1)^8\,dx$

$$u = x \qquad dv = (x+1)^8\,dx$$
$$du = dx \qquad v = \frac{1}{9}(x+1)^9$$
$$\frac{1}{9}x(x+1)^9 - \int \frac{1}{9}(x+1)^9\,dx$$
$$\frac{1}{9}x(x+1)^9 - \frac{1}{9}\left(\frac{1}{10}\right)(x+1)^{10} + C$$

2) $\int xe^{-x}\,dx$

$$u = x \qquad dv = e^{-x}\,dx$$
$$du = dx \qquad v = -e^{-x}$$
$$-xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} - e^{-x} + C$$

3) $\int x\ln 2x\,dx$

$$u = \ln 2x \qquad dv = x\,dx$$
$$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$$
$$\frac{x^2\ln 2x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln 2x}{2} - \frac{x^2}{4} + C$$

4) $\int x(\ln x)^2\,dx$

$$u = (\ln x)^2 \qquad dv = x\,dx$$
$$du = \frac{2\ln x}{x}dx \qquad v = \frac{x^2}{2}$$
$$\frac{x^2(\ln x)^2}{2} - \int x\ln x\,dx \qquad \begin{matrix} u = \ln x & dv = x\,dx \\ du = \frac{1}{x}dx & v = \frac{x^2}{2} \end{matrix}$$
$$\frac{x^2(\ln x)^2}{2} - \left[\frac{x^2}{2}\ln x - \int \frac{x}{2}dx\right] = \frac{x^2(\ln x)^2}{2} - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$$

5) $\int x^2\sin x\,dx$

$$u = x^2 \qquad dv = \sin x\,dx$$
$$du = 2x\,dx \qquad v = -\cos x$$
$$-x^2\cos x + \int 2x\cos x\,dx \qquad \begin{matrix} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{matrix}$$
$$-x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right] = -x^2\cos x + 2x\sin x + 2\cos x + C$$

6) $\int e^{2x}\sin x\,dx$

$$u = e^{2x} \qquad dv = \sin x\,dx$$
$$du = 2e^{2x}dx \qquad v = -\cos x$$
$$-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx \qquad \begin{matrix} u = e^{2x} & dv = \cos x\,dx \\ du = 2e^{2x}dx & v = \sin x \end{matrix}$$
$$-e^{2x}\cos x + 2\left[e^{2x}\sin x - 2\int e^{2x}\sin x\,dx\right] = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$$
$$\int e^{2x}\sin x\,dx = \frac{1}{5}\left[-e^{2x}\cos x + 2e^{2x}\sin x\right] + C$$

7) $\int_0^{\pi} x\sin 2x\,dx$

$$u = x \qquad dv = \sin 2x\,dx$$
$$du = dx \qquad v = -\frac{1}{2}\cos 2x$$
$$\left[-\frac{1}{2}x\cos 2x\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi}\cos 2x\,dx = \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi} = -\frac{\pi}{2}$$

8) $\int_0^1 x^2 e^x\,dx$

$$u = x^2 \qquad dv = e^x dx$$
$$du = 2x\,dx \qquad v = e^x$$
$$x^2e^x\Big|_0^1 - 2\int_0^1 xe^x dx \qquad \begin{matrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{matrix}$$
$$x^2e^x\Big|_0^1 - 2\left[xe^x\Big|_0^1 - \int_0^1 e^x dx\right] = e - 2\left[e - e^x\Big|_0^1\right] = e - 2\left[e - (e-1)\right] = e - 2$$