23) $$\frac{dy}{dt} = -10\left(1+x^2\right)^{-2}(2x)\frac{dx}{dt} = \frac{-20x\frac{dx}{dt}}{\left(1+x^2\right)^2}$$

a) $$\frac{dy}{dt} = \frac{-20(-2)(3)}{\left(1+(-2)^2\right)^2} = \frac{120}{25} = 4.8 \text{ cm/sec}$$

b) $$\frac{dy}{dt} = \frac{-20(0)(3)}{\left(1+(0)^2\right)^2} = 0 \text{ cm/sec}$$

c) $$\frac{dy}{dt} = \frac{-20(20)(3)}{\left(1+(20)^2\right)^2} = \frac{-1200}{401^2} \text{ cm/sec}$$

24) $$\frac{dy}{dx} = 3x^2\frac{dx}{dt} - 4\frac{dx}{dt}$$

a) $$\frac{dy}{dx} = 3(-3)^2(-2) - 4(-2) = -46 \text{ cm/sec}$$

b) $$\frac{dy}{dx} = 3(1)^2(-2) - 4(-2) = 2 \text{ cm/sec}$$

c) $$\frac{dy}{dx} = 3(4)^2(-2) - 4(-2) = -88 \text{ cm/sec}$$

25)

$$\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

when $x = 3,\ y = 9$

$$y = x^2 \Rightarrow \frac{dy}{dt} = 2x\frac{dx}{dt} \quad \text{when } x = 3: \quad \frac{dy}{dt} = 2(3)(10) = 60$$

$$\left(\frac{\sqrt{90}}{3}\right)^2\frac{d\theta}{dt} = \frac{3(60) - 9(10)}{3^2} \Rightarrow \frac{d\theta}{dt} = \frac{9}{90}\left(\frac{90}{9}\right) = 1 \text{ radian/sec}$$

26)

$$\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

when $x = -4,\ y = 2$

$$y = \sqrt{-x} \Rightarrow \frac{dy}{dt} = \frac{1}{2}(-x)^{-1/2}(-1)\frac{dx}{dt} \quad \text{when } x = -4: \quad \frac{dy}{dt} = -\frac{1}{2}(4)^{-1/2}(-8) = 2$$

$$\left(\frac{\sqrt{20}}{-4}\right)^2 \frac{d\theta}{dt} = \frac{-4(2) - 2(-8)}{(-4)^2} \Rightarrow \frac{d\theta}{dt} = \frac{16}{20}\left(\frac{8}{16}\right) = 0.4 \text{ radian/sec}$$

27)

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-8 = 4\pi\left(10^2\right)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-8}{400\pi} = \frac{-1}{50\pi}$$

$$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(10)\left(\frac{-1}{50\pi}\right) = -1.6$$

The surface area is decreasing at a rate of 1.6 cm^2/min

28)

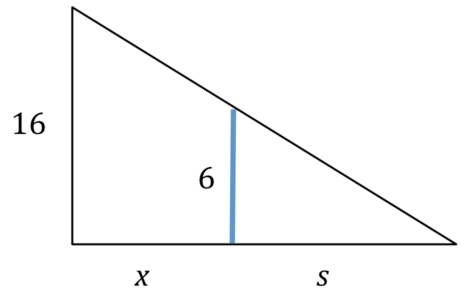
$$d = \sqrt{x^2 + y^2} \Rightarrow \frac{dd}{dt} = \frac{1}{2}\left(x^2 + y^2\right)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

$$\frac{dd}{dt} = \frac{1}{2}\left((5)^2 + (12)^2\right)^{-1/2}\left(2(5)(-1) + 2(12)(-5)\right)$$

$$= \frac{1}{2\sqrt{169}}(-10 - 120) = \frac{-130}{2(13)} = -5$$

The distance is decreasing at a rate of 5 m/sec

29) by similar triangles: $\frac{x+s}{16} = \frac{s}{6} \Rightarrow s = \frac{3}{5}x$



$$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt} = \frac{3}{5}(-5) = -3$$

The shadow is growing shorter at a rate of 3 ft/sec

31)

$$\tan\theta = \frac{x}{y} \quad \sec^2\theta\frac{d\theta}{dt} = \frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}$$

When the car is right in front of you, $\theta = 0$.

$$\sec^2(0)\frac{d\theta}{dt} = \frac{132(264) - 0(0)}{132^2} \Rightarrow \frac{d\theta}{dt} = 2 \text{ radians/sec}$$

$\frac{1}{2}$ second later : $x = 132$ ft $\quad \tan\theta = \frac{132}{132} = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\sec^2\left(\frac{\pi}{4}\right)\frac{d\theta}{dt} = \frac{132(264) - 132(0)}{132^2} \Rightarrow 2\frac{d\theta}{dt} = 2 \Rightarrow \frac{d\theta}{dt} = 1 \text{ radian/sec}$$