8) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(50)(0.01) = \pi$$

The area is increasing at a rate of π cm^2 / sec

9a) $A = lw$

$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$

$$\frac{dA}{dt} = 12(2) + 5(-2) = 14$$

The area is increasing at a rate of 14 cm^2 / sec

9b) $P = 2l + 2w$

$$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$$

$$\frac{dP}{dt} = 2(-2) + 2(2) = 0$$

The perimeter is not changing.

9c) $D = \sqrt{l^2 + w^2}$

$$\frac{dD}{dt} = \frac{1}{2}\left(l^2 + w^2\right)^{-1/2}\left(2l\frac{dl}{dt} + 2w\frac{dw}{dt}\right)$$

$$\frac{dD}{dt} = \frac{1}{2\sqrt{144 + 25}}\left(2(12)(-2) + 2(5)(2)\right)$$

$$\frac{dD}{dt} = \frac{-28}{26}$$

The diagonal is decreasing at a rate of $\frac{14}{13}$ cm/sec.

10a) $V = lwh \Rightarrow \dfrac{dV}{dt} = lw\dfrac{dh}{dt} + h\left(l\dfrac{dw}{dt} + w\dfrac{dl}{dt}\right)$

$\dfrac{dV}{dt} = 4(3)(1) + 2\left(4(-2) + 3(1)\right) = 2$

The volume is increasing at a rate of 2 m^3/sec.

10b) $T = 2lw + 2wh + 2lh$

$\dfrac{dT}{dt} = 2l\dfrac{dw}{dt} + 2w\dfrac{dl}{dt} + 2w\dfrac{dh}{dt} + 2h\dfrac{dw}{dt} + 2l\dfrac{dh}{dt} + 2h\dfrac{dl}{dt}$

$\dfrac{dT}{dt} = 8(-2) + 6(1) + 6(1) + 4(-2) + 8(1) + 4(1) = 0$

The surface are is not changing.

10c) $s = \sqrt{x^2 + y^2 + z^2}$

$\dfrac{ds}{dt} = \dfrac{1}{2}\left(x^2 + y^2 + z^2\right)^{-1/2}\left(2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} + 2z\dfrac{dz}{dt}\right)$

$\dfrac{ds}{dt} = \dfrac{1}{2\sqrt{4^2 + 3^2 + 2^2}}\left(2(4)(1) + 2(3)(-2) + 2(2)(1)\right)$

$\dfrac{ds}{dt} = \dfrac{1}{2\sqrt{29}}(0) = 0$

The length of the diagonal is not changing.

11a) $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

$100\pi = 4\pi(5)^2\dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{100\pi}{100\pi} = 1$

The radius of the balloon is increasing at a rate of 1 ft/min.

11b) $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(5)(1)$$

$$\frac{dr}{dt} = 40\pi$$

The surface area of the balloon is increasing at a rate of 40 ft^2/min.

12) $$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = k\left(4\pi r^2\right)$$

$$\frac{dr}{dt} = k$$

15) $V = \pi r^2 h = 6\pi r^2$

$$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$$

$$\frac{dV}{dt} = 12\pi(1.900)\left(\frac{0.001}{3}\right) \approx 0.0239$$

The volume is increasing at 0.0239 in^3/min

16a/b) $$V = \frac{1}{3}\pi r^2 h \qquad h = \frac{3}{4}r \Rightarrow r = \frac{4}{3}h$$

To find $\frac{dh}{dt}$: $V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h = \frac{16}{27}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$

$$10 = \frac{16}{9}\pi(4)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{90}{256\pi} \approx 0.1119 \text{ m/min} = 11.19 \text{ cm/min}$$

To find $\frac{dr}{dt}$: $V = \frac{1}{3}\pi r^2\left(\frac{3}{4}r\right) = \frac{1}{4}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{3}{4}\pi r^2 \frac{dr}{dt}$

$$10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{30}{64\pi} \approx 0.1492 \text{ m/min} = 14.92 \text{ cm/min}$$

17) $V = \frac{1}{3}\pi r^2 h \qquad \frac{DV}{dt} = -50 \qquad \frac{r}{h} = \frac{45}{6} \Rightarrow r = \frac{15}{2}h \Rightarrow h = \frac{2}{15}r$

To find $\frac{dh}{dt}$: $V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h = \frac{75}{4}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi(5)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-8}{225\pi} \approx -0.0113$ m/min = -1.13 cm/min

To find $\frac{dr}{dt}$: $V = \frac{1}{3}\pi r^2\left(\frac{2}{15}r\right) = \frac{2}{45}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{2}{15}\pi r^2 \frac{dr}{dt}$

$-50 = \frac{2}{15}\pi\left(\frac{75}{2}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-4}{15\pi} \approx -0.0849$ m/min = -8.49 cm/min

18) $V = \frac{\pi}{3}y^2(39 - y) = 13\pi y^2 - \frac{\pi}{3}y^3 \qquad \frac{DV}{dt} = -6$

a) To find $\frac{dy}{dt}$: $\frac{dV}{dt} = 26\pi y \frac{dy}{dt} - \pi y^2 \frac{dy}{dt}$

$-6 = 26\pi(8)\frac{dy}{dt} - \pi(64)\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{-6}{208\pi - 64\pi} \approx -0.0132$ m/min = -1.32 cm/min

b) To find r: $r^2 + (13 - y)^2 = 13^2 \Rightarrow r = \sqrt{169 - (13 - y)^2} = \sqrt{26y - y^2}$

c) $\frac{dr}{dt} = \frac{26 - 2y}{2\sqrt{26y - y^2}}\frac{dy}{dt} = \frac{13 - y}{\sqrt{26y - y^2}}\frac{dy}{dt}$

when $y = 8$: $\frac{dr}{dt} = \frac{13 - 8}{\sqrt{26(8) - 8^2}}(-0.0132) \approx -0.0055$ m/min = -0.55 cm/min