AP Calculus Exam Prep Assignment #8 page 2

Problems

11) A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

A) Find the velocity at time t.

$v(t) = 2\pi - 2\pi \sin 2\pi t$

B) Find the acceleration at time t.

$a(t) = -4\pi^2 \cos 2\pi t$

C) What are all values of t, $0 \le t \le 3$, for which the particle is at rest?

$$v(t) = 0 \Rightarrow \sin 2\pi t = 1 \Rightarrow 2\pi t = \frac{\pi}{2} + 2n\pi \Rightarrow t = \frac{1}{4} + n$$

$$t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}$$

D) What is the maximum velocity?

$$a(t) = 0 \Rightarrow \cos 2\pi t = 0 \Rightarrow 2\pi t = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \Rightarrow t = \frac{1}{4}, \frac{3}{4}$$

$$v\left(\frac{1}{4}\right) = 0, v\left(\frac{3}{4}\right) = 4\pi$$

12) (1995 AB2) A particle moves along the y-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = t\cos t$. At time $t = 0$, the position of the particle is $y = 3$.

A) For what values of t, $0 \le t \le 5$, is the particle moving upward?

$$t\cos t > 0 \Rightarrow 0 < t < \frac{\pi}{2} \text{ and } \frac{3\pi}{2} < t \le 5 \text{ on } [0,5]$$

B) Write an expression for the acceleration of the particle in terms of t.

$a(t) = \cos t - t\sin t$

C) Write an expression for the position $y(t)$ of the particle.

$$y(t) = \int t\cos t\, dt \qquad \begin{matrix} u = t & dv = \cos t\, dt \\ du = dt & v = \sin t \end{matrix}$$

$$y(t) = t\sin t - \int \sin t\, dt = t\sin t + \cos t + C$$

$$3 = 0 + \cos 0 + C \Rightarrow C = 2 \quad y(t) = t\sin t + \cos t + 2$$

D) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

$$t\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + 2 = \frac{\pi}{2} + 2$$