# AP Calculus Exam Prep Assignment #8 KEY

1) $\dfrac{dy}{dx} = \dfrac{1}{x\sqrt{x^2+1}}\left(\sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}}\right) = \dfrac{1}{x\sqrt{x^2+1}}\left(\dfrac{2x^2+1}{\sqrt{x^2+1}}\right) = \dfrac{2x^2+1}{x(x^2+1)}$ **D)**

2)

$x = \sqrt{u^2+1} \Rightarrow x^2 - 1 = u^2 \quad \dfrac{d}{du}\sin u^2 = 2u\cos u^2$ or $\dfrac{dy}{du} = \dfrac{dy}{dx}\cdot\dfrac{dx}{du} = 2x\cos(x^2-1)\left[\dfrac{u}{\sqrt{u^2+1}}\right]$ **D)**

$= 2\sqrt{u^2+1}\left(\cos u^2\right)\left[\dfrac{u}{\sqrt{u^2+1}}\right] = 2u\cos u^2$

3) $dy = \left[\sqrt{1+x^2} + \dfrac{x^2}{\sqrt{1+x^2}}\right]dx \Rightarrow dy = (1+0)2 = 2$ **E)**

4) $\sec^2(xy)\left[y + x\dfrac{dy}{dx}\right] = 1 \Rightarrow y + x\dfrac{dy}{dx} = \cos^2(xy) \Rightarrow \dfrac{dy}{dx} = \dfrac{\cos^2(xy) - y}{x}$ **E)**

5) $\dfrac{dy}{dx} = \dfrac{\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right) - \left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2} = \dfrac{e^{2x}+2+e^{-2x} - \left(e^{2x}-2+e^{-2x}\right)}{\left(e^x+e^{-x}\right)^2} = \dfrac{4}{\left(e^x+e^{-x}\right)^2}$ **C)**

6)

$\dfrac{dy}{dx} = \dfrac{1}{x^2+y^2}\left[2x + 2y\dfrac{dy}{dx}\right] \Rightarrow \dfrac{dy}{dx}\left(x^2+y^2\right) - 2y\dfrac{dy}{dx} = 2x \Rightarrow \dfrac{dy}{dx} = \dfrac{2x}{x^2-2y+y^2}$ **D)**

at (1,0) $\dfrac{dy}{dx} = \dfrac{2}{1-0-0} = 2$

7) $\dfrac{ds}{dt} = 4(t-2)^3 \quad 4(t-2)^3 = 0 \Rightarrow t = 2$ **B) 1**

8)

$s' = 3t^2 - 12t + 9 \quad s' = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$ **E)**

$s' < 0$ for $1 < t < 3$, so $s$ is increasing on $-\infty < t < 1$ and $3 < t < \infty$

9) $v = 3t^2 + 3 \quad a = 6t$ The speed is decreasing when $a < 0$. This occurs when $t < 0$ **C) $t < 0$**

10)

$v(t) = e^{-t}\left(4\cos 4t\right) + -e^{-t}\sin 4t$ **E)**

Distance $= \int_0^1 |v(t)|dt = \int_0^1 \left|e^{-t}\left(4\cos 4t\right) + -e^{-t}\sin 4t\right|dt \approx 1.671$