AP Calculus Exam Prep Assignment #6 page 4

20) [GC]The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table shows the water temperature as recorded every 3 days over a 15-day period

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

A) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) \approx \frac{21-24}{15-9} \approx -\frac{1}{2} \text{ °C/day}$$

B) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

Average temp (in °C) =

$$\frac{3\left(\frac{1}{2}\right)(31+20)+3\left(\frac{1}{2}\right)(28+31)+3\left(\frac{1}{2}\right)(24+28)+3\left(\frac{1}{2}\right)(22+24)+3\left(\frac{1}{2}\right)(21+22)}{15} = \frac{376.5}{15} \approx 25.1$$

C) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$$P'(t) = 10t\left(-\frac{1}{3}e^{-t/3}\right) + 10e^{-t/3}$$

$$P'(12) = 10(12)\left(-\frac{1}{3}e^{-4}\right) + 10e^{-4} = -\frac{30}{e^4} \approx -0.549$$

The would show that the temperature of the water is decreasing at a rate of approximately 0.549°C per day on the 12th day.

D) Use the function P defined in part (C) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

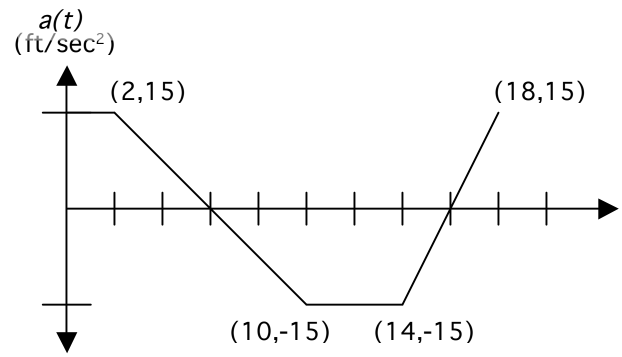
Average value = $\frac{1}{15-0}\int_0^{15}\left(20+10te^{-t/3}\right)dt = \frac{1}{15}\left[20t\right]_0^{15} + \frac{10}{15}\int_0^{15} te^{-t/3}dt$ $u = t \quad dv = e^{-t/3}dt$, $du = dt \quad v = -3e^{-t/3}$

$$= \frac{1}{15}\left[300-0\right] + \frac{2}{3}\left[-3te^{-t/3}\right]_0^{15} - \frac{2}{3}\int_0^{15} -3e^{-t/3}dt = 20 + \frac{2}{3}\left[-45e^{-5}-0\right] + 2\left[-3e^{-t/3}\right]_0^{15}$$

$$= 20 - 30e^{-5} + 2\left[-3e^{-5} - (-3)\right] = 26 - \frac{36}{e^5} \approx 25.757°C$$

21) A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration *a(t)*, in ft/sec^2, is the piecewise linear function defined by the graph at the right.

A) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?



The velocity of the car is increasing at $t = 2$, as the acceleration is positive from $t = 0$ to $t = 2$ and the velocity was positive at $t = 0$.

B) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

The velocity of the car is 55 ft/sec at $t = 12$. The area between $a(t)$ and the x-axis from $t = 0$ to $t = 6$ (which would be a positive quantity) represents the total (positive) acceleration, and would equal the area between $a(t)$ and the x-axis from $t = 10$ to $t = 12$ (which would be a negative quantity), which would represent the total (negative) acceleration, or deceleration. Thus the velocity would again be 55 ft/sec at $t = 12$.

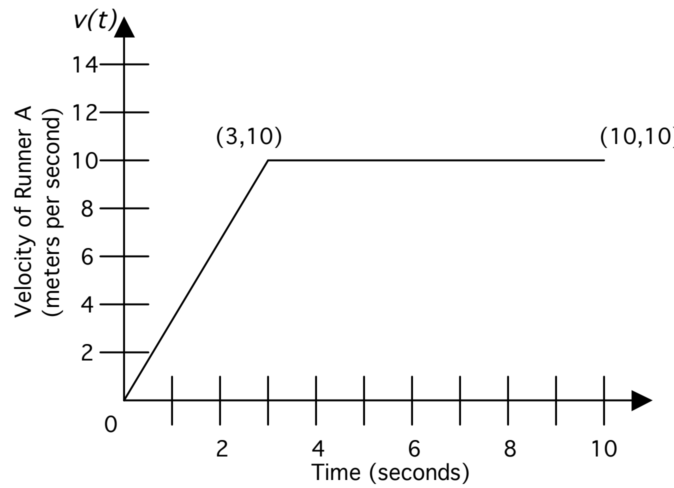
C) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify.

Maximum velocity occurs when its derivative, acceleration, equals 0. This occurs at $t = 6$ and $t = 16$.
Since at $t = 6$ the acceleration passes from positive to negative, this time is when the maximum velocity occurs.

D) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify.

$\int_{12}^{16} a(t)\,dt = 15(2) + \frac{1}{2}(15)(2) = 45$. This represents the total negative acceleration from 55ft/sec. Therefore, at $t = 16$, the velocity of the car is $55 - 45 = 10$ ft/sec. From $t = 16$ to $t = 18$, the acceleration is positive, so the velocity will be increasing. Thus the car's velocity will never equal zero.

21) Two runners, A and B, run on a straight racetrack $0 \leq t \leq 10$ seconds. The graph, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.



A) Find the velocity of each runner at time $t = 2$ seconds. Indicate units of measure.

Runner A: $v(t) - 0 = \dfrac{10}{3}(t-0) \Rightarrow v(t) = \dfrac{10}{3}t$

At $t = 2$, Runner A's velocity is 20/3 meters per second.

Runner B: $v(2) = \dfrac{24(2)}{2(2)+3} = \dfrac{48}{7}$ meters per second

B) Find the acceleration of each runner at time $t = 2$ seconds. Indicate units of measure.

Runner A: Acceleration is the slope of velocity, so at $t = 2$ Runner A's acceleration is 10/3 m/sec^2.

Runner B: $a(t) = v'(t) = \dfrac{(2t+3)24 - 24t(2)}{(2t+3)^2}$ $\qquad a(2) = \dfrac{(2(2)+3)24 - 24(2)(2)}{(2(2)+3)^2} = \dfrac{168-96}{49} = \dfrac{72}{49}$ m/sec^2.

C) Find the total distance run by each runner over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

Runner A: $\displaystyle\int_0^{10} v(t)\,dt = \frac{1}{2}(3)(10) + 10(7) = 85$ meters.

Runner B: $\displaystyle\int_0^{10} \frac{24t}{2t+3}\,dt \approx 83.336$ m