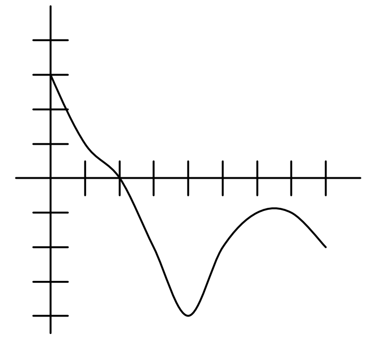
AP Calculus Exam Prep Assignment #6 page 3 KEY

14) **E)** 16



15) If the trapezoidal rule is used with $n = 5$ then $\int_0^1 \frac{dx}{1+x^2}$ = _______(to 3 decimal places)

x	0	0.2	0.4	0.6	0.8	1.0
y	1	0.962	0.862	0.735	0.610	0.5

$0.2\left(0.5\left(1+2(0.962)+2(0.862)+2(0.735)+2(0.610)+0.5\right)\right)$ **A)**

≈ 0.784

16) The table shows the speed of an object in feet per second during a 3-second period. Estimate the distance the object travels, using the trapezoidal method.

Time (sec)	0	1	2	3
Speed (ft/sec)	30	22	12	0

D) 49 ft.

17) If $M(4)$ is used to approximate $\int_0^1 \sqrt{1+x^3}\,dx$, then the definite integral is equal, to two decimal places, to:

x	0.125	0.375	0.625	0.875
y	1.001	1.026	1.115	1.292

$\int_0^1 \sqrt{1+x^3}\,dx \approx 0.25(1.001+1.026+1.115+1.292) \approx 1.1085 \approx 1.11$ **B)**

18) If $\int_0^1 \sqrt{1+x^3}\,dx$ is approximated by Riemann sums and the same number of subdivisions, and if *L, R, M,* and *T* denote respectively Left, Right, Midpoint, and Trapezoidal sums, then:

$$y=\sqrt{1+x^3} \Rightarrow y'=\frac{3x}{2\sqrt{1+x^3}} \Rightarrow y''=\frac{6\sqrt{1+x^3}-3x\left(\frac{3x}{\sqrt{1+x^3}}\right)}{4(1+x^3)}=\frac{\frac{6(1+x^3)-9x^2}{\sqrt{1+x^3}}}{4(1+x^3)}=\frac{6-9x^2+6x^3}{4(1+x^3)\sqrt{1+x^3}}$$

$y'' > 0$ on $[0,1]$ so

D) $\boldsymbol{L \le M \le T \le R}$

Problems

19) [1994 AB6] Let $F(x) = \int_0^x \sin\left(t^2\right) dt$ for $0 \le x \le 3$.

A)

$$T = \frac{1/4}{2}\left(\sin\left(0^2\right) + 2\left(\sin\left(.25^2\right) + \sin\left(.5^2\right) + \sin\left(.75^2\right)\right) + \sin\left(1\right)\right)$$
$$\approx 0.316$$

B)

Increases when $F'(x) > 0$. $F'(x) = \sin\left(x^2\right)$. $\sin\left(x^2\right) > 0$

$\Rightarrow 0 < x^2 < \pi$ and $2\pi < x^2 < 3\pi \Rightarrow 0 < x < 1.772$ and $2.507 < x < 3.070$

Increasing on $(0,1.772) \cup (2.507,3)$

C) If the average rate of change of *F* on the closed interval [1,3] is *k*, find $\int_1^3 \sin(t^2)\, dt$ in terms of *k*.

$\frac{1}{3\quad 1}\int_1^3 F(x)dx = k \qquad \int_1^3 F(x)dx = 2k$