

AP Calculus Exam Prep Assignment #4 KEY page 3

15)

$$\frac{3x+5}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4} \Rightarrow \begin{cases} A+B=3 \\ 4A-3B=5 \end{cases} \Rightarrow A=2, B=1$$

$$2 \int_4^5 \left(\frac{1}{x-3} \right) dx + \int_4^5 \left(\frac{1}{x+4} \right) dx = \left[2 \ln|x-3| + \ln|x+4| \right]_4^5 \quad \mathbf{D)}$$

$$= (2 \ln 2 + \ln 9) - (2 \ln 1 + \ln 8) = \ln 36 - \ln 8 = \ln \frac{9}{2}$$

16) Which of these indefinite integral requires the method of REPEATED integration by parts?

$$\mathbf{D)} \int e^{2x} \cos 3x dx \quad \backslash$$

17)

$$\int \frac{x}{\sqrt{4-x^2}} dx + \int \frac{2}{\sqrt{4-x^2}} dx = \int \frac{2}{\sqrt{4-x^2}} dx = \int \frac{2}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} dx = \int \frac{2}{2\sqrt{\left(1-\frac{x^2}{4}\right)}} dx$$

$$u = 4 - x^2, du = -2x dx$$

B)

$$-\frac{1}{2} \int \frac{du}{\sqrt{u}} + \int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} dx = -\frac{1}{2} (2\sqrt{u}) + 2 \sin^{-1}\left(\frac{x}{2}\right) + C = -\sqrt{4-x^2} + 2 \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$18) \int_{-\infty}^{\infty} \frac{dx}{x^2+1} = \quad \mathbf{C)}$$

A) 0

B) $\pi/2$

C) π

D) 2π

E) N.O.T.

$$19) \int_2^4 \frac{dx}{(x-3)^{2/3}}$$

$$\left. \frac{(x-3)^{1/3}}{1/3} \right|_2^4 = 3 - (-3) = 6 \quad \mathbf{A)}$$

20) Which of the following improper integrals diverges?

D)

$$\lim_{a \rightarrow 0} \int_{-1}^a \frac{dx}{x^2} + \lim_{b \rightarrow 0} \int_b^1 \frac{dx}{x^2}$$

$$\lim_{a \rightarrow 0^-} \left[-\frac{1}{x} \right]_{-1}^a + \lim_{b \rightarrow 0^+} \left[-\frac{1}{x} \right]_b^1 = \lim_{a \rightarrow 0^-} \left[-\frac{1}{a} + 1 \right]_{-1}^a + \lim_{b \rightarrow 0^+} \left[-1 + \frac{1}{b} \right]_b^1 = \infty + \infty = \infty$$

$$23) \text{ C) } \pi \int_{-2}^2 \left[(k - x^2)^2 - (k - 4)^2 \right] dx$$