# AP Calculus Exam Prep Assignment #3 KEY

Multiple Choice

1) $\int_0^1 \frac{x^2}{x^2+1}\,dx =$ $\quad \frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1} \quad \int_0^1 \left(1 - \frac{1}{x^2+1}\right)dx = \left[x - \tan^{-1} x\right]_0^1$

$= \left(1 - \frac{\pi}{4}\right) - \left(0 - 0\right) = 1 - \frac{\pi}{4}$ **A)**

2) $\int \frac{5\,dx}{1+x^2} = 5\int \frac{dx}{1+x^2} = 5\tan^{-1} x + C$ **D)**

3) (GC) The average value of $f(x) = e^{-x^2}$ on the interval $-1 \le x \le 1$ is nearest to:

**D)** 0.75

4)

$u = x^2 + 2x \quad du = \left(2x+2\right)dx \Rightarrow (x+1)dx = \frac{1}{2}du$ **B)**

$\frac{1}{2}\int_0^3 e^u du = \frac{1}{2}\left[e^u\right]_0^3 = \frac{1}{2}\left(e^3 - 1\right)$

5) If $F$ and $f$ are continuous functions such that $F'(x) = f(x)$ for every $x$ then $\int_a^b f(x)\,dx$ is:

**D)** $F(b) - F(a)$

6) $\int_0^1 \sqrt{x^2 - 2x + 1}\,dx = \int_0^1 \sqrt{\left(x-1\right)^2}\,dx = \int_0^1 \left(x-1\right)dx = \left[\frac{x^2}{2} - x\right]_0^1 = -\frac{1}{2}$ **B)**

7)

$\int_{\sqrt{2}}^2 \frac{u}{u^2-1}\,du = \quad x = u^2 - 1 \quad dx = 2u\,du \Rightarrow u\,du = \frac{dx}{2}$ **A)**

$\int_1^3 \frac{dx}{2x} = \left[\frac{1}{2}\ln|x|\right]_1^3 = \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 3$

8) $\int_0^1 \frac{e^{-x}+1}{e^{-x}}\,dx = \int_0^1 \left(1 + e^x\right)dx = \left[x + e^x\right]_0^1 = 1 + e - \left(0+1\right) = e$ **A)**

9) What is the average value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$?

$$\frac{1}{2-\left(-1\right)}\int_{-1}^2 \left(3t^3 - t^2\right)dt = \frac{1}{3}\left[\frac{3t^4}{4} - \frac{t^3}{3}\right]_{-1}^2 = \frac{1}{3}\left[\left(12 - \frac{8}{3}\right) - \left(\frac{3}{4} + \frac{1}{3}\right)\right] = \frac{11}{4}$$ **A)**

10) $\dfrac{1}{\frac{\pi}{4}-\left(\frac{\pi}{6}\right)}\int_{\pi/6}^{\pi/4}\left(\csc^2 x\right)dx=\dfrac{12}{\pi}\left[-\cot x\right]_{\pi/6}^{\pi/4}=\dfrac{12}{\pi}\left(-1-\left(-\sqrt{3}\right)\right)$ **C)**

11)

$$y=\pm\sqrt{x^3}\quad L=2\int_0^4\sqrt{1+\left(\frac{3}{2}\sqrt{x}\right)^2}\,dx=2\int_0^4\sqrt{1+\frac{9}{4}x}\,dx\quad u=1+\frac{9}{4}x\quad du=\frac{9}{4}dx$$

$$=2\left(\frac{4}{9}\right)\int_1^{10}\sqrt{u}\,du=\frac{8}{9}\left[\frac{2}{3}u^{3/2}\right]_1^{10}=\frac{16}{27}\left[10^{3/2}-1\right]$$

**C)**

12)

$$\frac{dy}{dx}=\frac{1}{\cos x}\left(-\sin x\right)=-\tan x$$

$$L=\int_{\pi/4}^{\pi/3}\sqrt{1+\left(-\tan x\right)^2}\,dx=\int_{\pi/4}^{\pi/3}\sqrt{1+\tan^2 x}\,dx=\int_{\pi/4}^{\pi/3}\sqrt{\sec^2 x}\,dx=\int_{\pi/4}^{\pi/3}\sec x\,dx$$

**A)**

$$\ln\left|\sec x+\tan x\right|\Big|_{\pi/4}^{\pi/3}=\ln\left|\sec\frac{\pi}{3}+\tan\frac{\pi}{3}\right|-\ln\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|=\ln\left|2+\sqrt{3}\right|-\ln\left|\sqrt{2}+1\right|=\ln\left(\frac{2+\sqrt{3}}{\sqrt{2}+1}\right)$$

13)

$$\frac{dy}{dx}=x^{1/2}$$

$$L=\int_0^3\sqrt{1+\left(\sqrt{x}\right)^2}\,dx=\int_0^3\sqrt{1+x}\,dx=\left[\frac{2}{3}\left(1+x\right)^{3/2}\right]_0^3=\frac{16}{3}-\frac{2}{3}=\frac{14}{3}$$

**C)**

14)

**A)** $2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$

Problems

15) Let $f$ be the function defined by $f(x)=x^3+ax^2+bx+c$ and have the following properties:

(i) The graph of $f$ has a point of inflection at (0,-2)

(ii) The average value of $f(x)$ on the closed interval [0,2] is -3

$$f'(x)=3x^2+2ax+b\quad f''(x)=6x+2a$$

A) Determine the values of $a$, $b$, and $c$. $0=6(0)+2a\Rightarrow a=0\quad -2=0^3+a(0)^2+b(0)+c\Rightarrow c=-2$

$$-3=\frac{\left(8+0+2b-2\right)-\left(-2\right)}{2}\Rightarrow -6=2b+8\Rightarrow b=-7$$

B) Determine the value of $x$ that satisfies the conclusion of the Mean Value Theorem for $f$ on the closed interval [0,3]

$$f'(x)=3x^2-7=-3\Rightarrow 3x^2-4=0\Rightarrow c=\pm\sqrt{\frac{4}{3}}$$

On $\left[0,2\right]$ $c=\sqrt{\frac{4}{3}}\approx 1.155$

16) [1991 BC 4] Let $F(x) = \int_1^{2x} \sqrt{t^2 + t}\, dt$.

A) Find $F'(x)$. $F'(x) = 2\sqrt{4x^2 + 2x}$

B) Find the domain of $F$. $4x^2 + 2x \ge 0 \Rightarrow x \ge 0$ or $x \le -\frac{1}{2}$ $t^2 + t \ge 0 \Rightarrow t \le -1$ or $t \ge 0$

$x \ge 0,$ as $t$ is not continuous when $x < 0$.

C) Find $\lim\limits_{x \to 1/2} F(x)$ $\lim\limits_{x \to 1/2} \int_1^{2x} \sqrt{t^2 + t}\, dt = 0$

D) Find the length of the curve $y = F(x)$ for $1 \le x \le 2$.

$$L = \int_1^2 \sqrt{1 + \left(2\sqrt{4x^2 + 2x}\right)^2}\, dx = \int_1^2 \sqrt{1 + 4\left(4x^2 + 2x\right)}dx = \int_1^2 \sqrt{16x^2 + 8x + 1}dx = \int_1^2 \left(4x + 1\right)dx$$

$$= \left[2x^2 + x\right]_1^2 = \left(8 + 2\right) - (2 + 1) = 7$$